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# GENERIC EXPERIMENTAL AND NUMERICAL APPROACHES TO EXPLORE SOUND AND VIBRATION OF PERCUSSION INSTRUMENTS - TUNING FORK

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Percussion instrument is one of categories in varieties of musical instruments. This paper aims to show the systematic approaches, both experimental and numerical, in studying percussion instrument. Source-Path-Response (SPR) system block diagram is first introduced, in particular for a tuning fork's vibration and sound radiation aspects. Five kinds of tests and analyses to explore the vibration and acoustic response of a musical instrument are then presented. The example case study of the tuning fork is shown base on the experimental testing and numerical analysis. Depending on the need, this work lays out the four stages of the study on tuning fork. In the first stage, one can measure the sound and vibration response and interpret the correlation between sound and vibration. In the second stage, if the structural modification or design modification (DM) is required, the constructed finite element model needs to be validated, i.e. model verification (MV). In the third stage, the pure structure analysis, including both modal and harmonic analyses, can be performed to obtain structural modal properties and frequency response functions (FRFs), respectively. The fourth stage is to explore the structural sound radiation via vibroacoustic analysis, in particular for sound pressure mode shapes, which is not easily available from testing. This work provides a systematic approach for those techniques required to explore the sound and vibration of tuning fork and can be applied to other musical instruments as well.

*Keywords: percussion instrument, SPR, model verification, vibroacoustic analysis*

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## 1. Introduction

Percussion instruments are a type of musical instrument. In order to effectively explore the vibration and sound-related properties of percussion instruments, a systematic experimental and numerical method must be established. Since the generation of musical sound is dominated by the vibration of the structure,

when the structure resonates, the vibration will be transmitted to the external air, causing the external air to vibrate and produce sound.

Through model verification (MV), a finite element model equivalent to the real structure can be obtained. In order to explore the structural vibration characteristics of tuning forks, Wang et al. [1] conducted the MV on a tuning fork of the A4 scale. The main idea for MV is that modal parameters from analysis and experiment can be comparable to each other.

After obtaining the equivalent finite element model, the vibroacoustic coupling analysis can be performed to explore its sound characteristics. Wang et al. [2] aimed to understand the sound characteristics of chime bell. They obtained an equivalent finite element model of chime bell through MV and explored the sound radiation of chime bell with different striking positions and the effect of with and without “Mei” through vibroacoustic analysis. In order to explore the influence of f-holes on the sound characteristics of violins, Wang et al. [3] conducted vibroacoustic analysis on violins with different f-hole sizes. It was found that at low frequencies, the size of the f-holes is proportional to the natural frequency of the violin cavity mode.

The frequency response function (FRF) and vibration mode shape of the structure can be obtained by experimental modal analysis (EMA), while the sound pressure mode shape of air field can be captured by an acoustic camera. By performing harmonic response analysis of acoustic-structure interaction, the sound pressure spectrum and the operational deflection shape (ODS) of the corresponding mode can be obtained. Wang et al. [4] performed EMA on a bronze bell and simultaneously used an acoustic camera to capture the sound pressure distributions in the spatial domain. The correlation between structural vibration characteristics and sound pressure mode shapes is explored, and the feasibility of using acoustic cameras to explore the modal characteristics of sound fields is shown.

An acoustic camera is composed of an array microphone and a camera. The array microphone consists of multiple precision microphones that can convert the time-domain sound pressure signals measured by multiple precision microphones into spatial-domain signals. Grubesa et al. [5] wanted to find out the optimal number and array shape of microphones for an acoustic camera. They used miniature microphones as array microphones and tested various conditions, including 12, 24, and 48 microphones and circular and square arrays. They finally concluded that a square array with 24 microphones was the best. Carneiro and Berry [6] proposed an algorithm for moving a spherical array microphone based on the acoustic imaging structure from motion (AISFM) method, and verified it experimentally, successfully locating the position, amplitude, and direction of multiple sound sources in three-dimensional space.

This paper aims to illustrate a systematic approach to studying percussion instruments, including both experimental and numerical methods. The source-path-response (SPR) system approach will take the sound and vibration of a tuning fork as an example. The vibration and acoustic response of musical instruments can be explored based on the proposed five-test and five-analysis methods, respectively. Based on experimental tests and numerical analyses, through the case study of tuning forks, four stages of tuning fork research are proposed. In the first stage, the correlation between sound and vibration can be explained by measuring the sound and vibration response. Then, in the second stage, if structural modifications or design modifications (DM) are required, the constructed finite element model needs to be verified, that is, model verification (MV). Furthermore, in the third stage, pure structural analysis can be performed, including modal analysis and harmonic analysis, to obtain the structural modal parameters and frequency response function (FRF), respectively. Finally, the fourth stage is to explore the structure-borne sound radiation through vibroacoustic analysis, especially the sound pressure mode shapes that are difficult to obtain through testing, unless with the available acoustic camera. This work provides a systematic approach in both analytical and experimental for studying the sound and vibration of tuning forks.

## 2. SPR block diagram for percussion instrument

Traditionally, people refer the SPR as Source-Path-Receiver. Source is the excitation source of vibration or noise/sound, and Path is the transfer path between Source and Receiver. Although most people refer R as the Receiver, but here we can reasonably refer R as the Response. Therefore, SPR is referred to Source-Path-Response here after.

If we expand on the SPR, Regarding the “SPR” system block diagram as shown in Figure 1. The SPR system block diagram of vibration-sound transmission has the path that is split into two, namely the Structural Path and the Air (Flow) Path [7]. There will also be two Responses, namely Response for Vibration (**R-V**) and Response for Noise (**R-N**) or Response for sound (**R-S**).



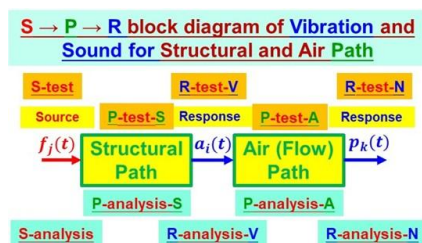
Figure 1: Source-Path-Response (SPR) system block diagram [7].

Referring to Figure 1, taking striking a tuning fork as an example is explained as follows:

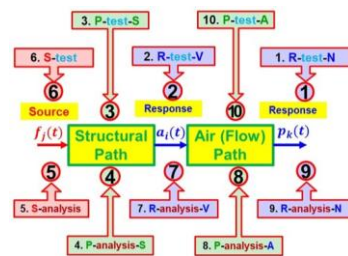
- **Source:** It is the excitation source. An impact hammer or a mallet is shown to strike the tuning fork. The striking force is denoted by  $f_j(t)$  and revealing the impulsive signal.
- **Structural Path:** The tuning fork itself is the structural path. When the tuning fork is struck, the tuning fork structure will vibrate.
- **R-Vibration:** If one touches the tuning fork with his hand, one can feel the vibration response of the tuning fork. The acceleration at any position on the tuning fork can be measured and denoted by  $a_i(t)$ .
- **Air (Flow) Path:** The vibration of the tuning fork will cause fluctuations in the air medium, which transmits the generated sound through the air path and spreads in all directions. This is the so-called sound radiation.
- **R-Sound/Noise:** When the human ear hears the sound of the tuning fork, it is a sound response. If it is an uncomfortable sound, it is a noise. The sound people hear will be the sound pressure, which can be measured by a microphone and denoted by  $p_k(t)$ .

Next, referring to Figure 2(a), there are five-test and five-analysis, respectively [8]:

- **5-Test:** They are **S-test**, **P-test-S**, **R-test-V**, **P-test-A** and **R-test-N** in SPR sequence.
- **5-Analysis:** They are **S-analysis**, **P-analysis-S**, **R-analysis-V**, **P-analysis-A** and **R-analysis-N** in SPR sequence.



(a) 5-test and 5-analysis in SPR [8]



(b) sequence in practical application [9]

Figure 2: Test and analysis for diagnosis steps of applying SPR technical roadmap.

### 3. Experimental and numerical techniques in studying percussion instrument

Refer to the top of Figure 3, which shows the summary of five-test experimental techniques [10], as follows:

- **S-test:** Source-test, which is to measure the striking force  $f_j(t)$ , the purpose is to understand the characteristics of the Source. The ordinary percussion mallet cannot obtain the percussion force. An impact hammer with a force transducer must be used to measure  $f_j(t)$ .
- **P-test-S:** Path test for Structure, which is to perform experimental modal analysis (EMA) on the structure, it is to understand the vibration modes of the structure path. There are two main steps in EMA: (1) Measuring the structure's FRF  $H_{ij}(f)$ . (2) Through curve fitting, the modal parameters of the structure, that is, the vibration mode, can be obtained, including:  $f_r$ ,  $\phi_r$ ,  $\xi_r$ . Among them,  $f_r$  is the natural frequency,  $\phi_r$  is the displacement mode shape, and  $\xi_r$  is the modal damping ratio.
- **R-test-V:** Response test for Vibration, which means measuring the acceleration of vibration  $a_i(t)$ . In the vibration spectrum, one can observe that the frequency corresponding to the peaks is the  $f_r$  natural frequency of the structure. Each  $f_r$  has a corresponding  $\phi_r$  mode shape.
- **P-test-A:** Path test for Air, which is to perform EMA on the air path, it is to understand the acoustic modes of the air path. Advanced acoustic camera equipment is required to perform **P-test-A**. Figure 3 shows the acoustic mode shape of the first vibration mode, and its physical meaning is the sound pressure mode shape. That is, at this frequency, the sound radiation pattern of the tuning fork in the air path.
- **R-test-N:** Response test for Noise, which is to measure the sound pressure time waveform  $p_k(t)$  and noise spectrum  $G_{pp}(f)$  shown in Figure 3. The purpose is to understand the characteristics of the sound.

Corresponding to the 5 types of test techniques of the experimental approach, there are also 5 types of analysis techniques for the analytical approach. See the bottom of Figure 3, and the explanation is as follows:

- **S-analysis:** Source-analysis, that is, the analysis of the external force of the knocking impact, although advanced collision analysis can be used. However, since the actual  $f_j(t)$  impact force is close to a triangular wave, the ideal impact force can be assumed and the corresponding  $G_{ff}(f)$  external force spectrum will be a flat curve of white noise.
- **P-analysis-S:** Path analysis for Structure, that is, system analysis of the structural path, including: (1) Modal analysis:  $f_r$  natural frequency and  $\phi_r$  mode shape can be analysed. (2) Harmonic response analysis: The frequency response function  $H_{ij}(f)$  corresponding to EMA can be analysed accordingly.
- **R-analysis-V:** Response analysis for Vibration, that is structural vibration response analysis. Since the analysis assumption in **S-analysis** is white noise excitation with an ideal impact force, the harmonic analysis in **P-analysis-S** is equivalent to the spectrum analysis here.
- **P-analysis-A:** Path analysis for Air, that is, the acoustic field analysis of the air path, requires the vibroacoustic coupling analysis, that is, considering the coupling effect between the tuning fork structure and the external air. The following are required: (1) Modal analysis: The  $f_r$  natural frequency and  $\phi_r$  acoustic mode shape of the structure in air coupling condition can be analysed. (2) Harmonic response analysis: The  $G_{pp}(f)$  sound pressure spectrum corresponding to the striking on the tuning fork can be obtained due to the white noise excitation.



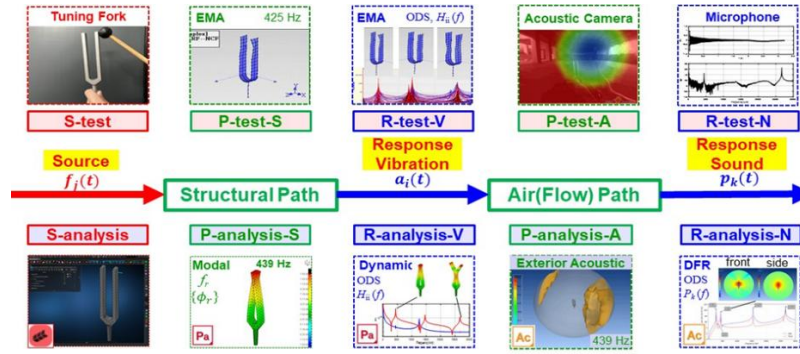


Figure 3: Test and analysis results in SPR technical roadmap for tuning fork [10].

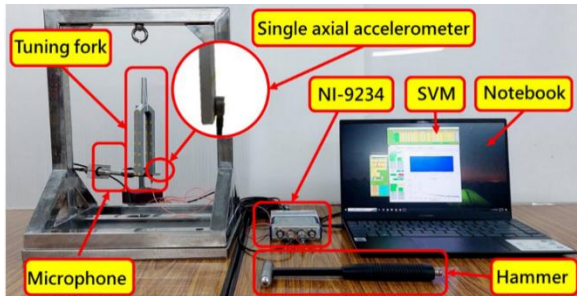
- **R-analysis-N**: Since white noise excitation is assumed, the harmonic analysis in **P-analysis-A** is equivalent to the spectrum response analysis of the sound spectrum  $G_{pp}(f)$ . Each sound frequency has its own specific sound pressure mode shape. When there is no equipment such as an acoustic camera, **R-analysis-N** is a very useful analysis tool and technique that can assist in understanding the pattern of sound radiation for the tuning fork.

#### 4. Case study: tuning fork

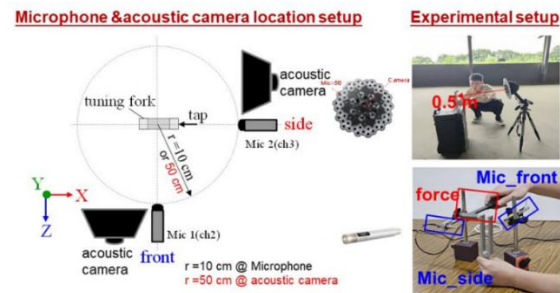
This section will use the SPR systematic approach proposed in Figure 2(b) to analyse the sound and vibration response of a tuning fork structure after being struck based on the proposed 5-test and 5-analysis methods. According to the needs, the four stages of tuning fork research and the corresponding experimental tests and numerical analysis methods are proposed, as follows:

1. The first stage is to do Sound and Vibration measurement and correlation study, and the corresponding experimental techniques are ①**R-test-N** and ②**R-test-V**.
2. The second stage is to perform Model Verification (MV) of FE model, and the corresponding experimental and analysis techniques are ③**P-test-S** and ④**P-analysis-S**.
3. The third stage is to carry out Structure only Analysis for tuning fork, and the corresponding experimental and analysis techniques are ⑤**S-analysis**, ⑥**S-test** and ⑦**R-analysis-V**.
4. The fourth stage is to proceed Vibroacoustic Analysis for tuning fork, and the corresponding experimental and analysis techniques are ⑧**P-analysis-A**, ⑨**R-analysis-N** and ⑩**P-test-A**.

This work uses a tuning fork with an A4 scale that is commonly used in tuning musical instruments. The A4 note corresponds to Fa in the musical score, and its scale frequency is 440 Hz. Figure 4(a) shows the EMA setup of the experimental measurement instrument, and Figure 4(b) shows the acoustic camera setup.



(a) EMA experimental setup [1]



(b) acoustic camera experimental setup

Figure 4: Experimental equipment setup.

#### 4.1 Sound and vibration measurement and correlation study

To understand the correlation between sound and vibration, firstly, the sound and vibration response measurement of a tuning fork of A4 scale (scale frequency 440 Hz) were performed. The test method can refer to **①R-test-N** and **②R-test-V** methods in the SPR system method shown in Figure 2(b). The setup for both **R-test-N** and **R-test-V** is shown in Figure 4(a).

The right-hand side of Figure 5 shows the measurement results of the sound and vibration response of the A4 scale tuning fork. Results show that when striking the fork arm, the fundamental frequency is about 440 Hz, and its sound pressure level is the highest. After decaying to 2 seconds, only the low frequency modes remain with response. The left-hand side of Figure 5 shows the experimental modal analysis results of the tuning fork vibration response. Observing its modal parameters, including the natural frequency and mode shape, its fundamental frequency is about 440Hz, the mode shape conforms to the physical meaning, and the peaks of the frequency response function clearly correspond.

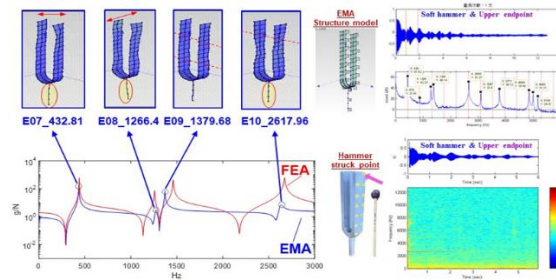


Figure 5: Measurement results of **R-test-N** and **R-test-V** from the struck tuning fork [1].

#### 4.2 Model verification of FE model

When a structure requires design modification (DM), the constructed finite element model needs to be verified first, namely model verification (MV). To perform MV needs the **③P-test-S** and **④P-analysis-S** as shown in Figure 2(b).

Figure 6 is a comparison of the results of EMA and FEA for the A4 scale tuning fork model verification (MV). The vibration mode shapes and frequency response function results of the FEA and EMA using the corrected material parameters are observed respectively. Results show that the mode shapes of the first four modes have a reasonable agreement, and the frequency error percentage is all lower than 3.94%. Confirm that the constructed finite element analysis model is equivalent to the actual structure and the MV is completed.

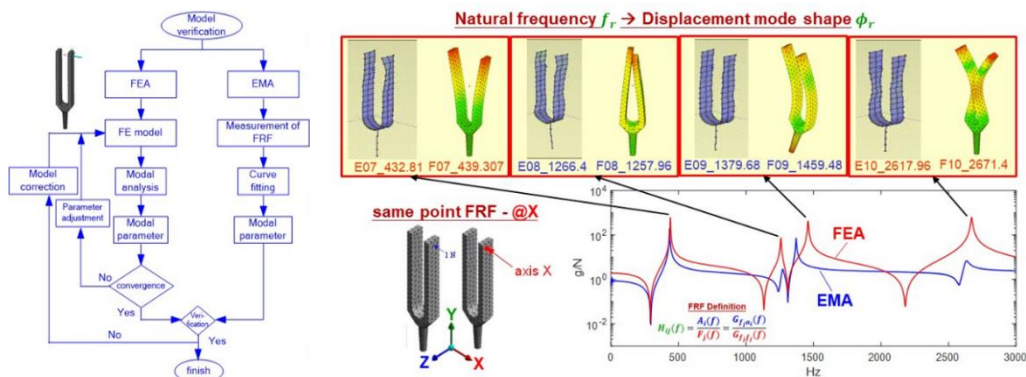


Figure 6: Comparison of EMA and FEA results for model verification of the tuning fork [1].

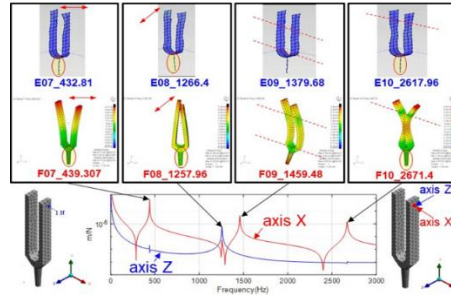


Figure 7: Modal parameters and FRF of structure only analysis for the tuning fork.

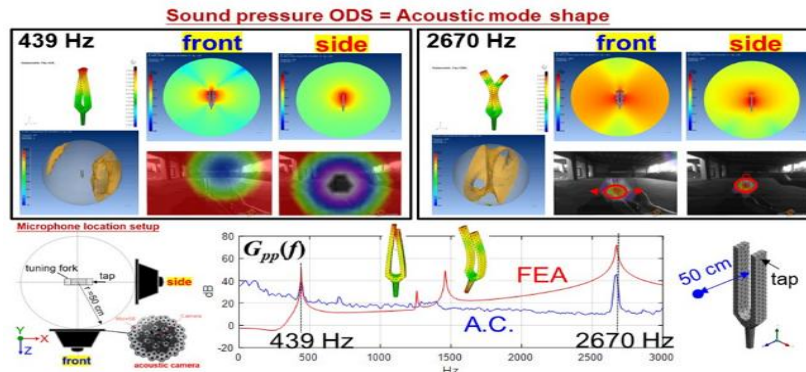


Figure 8: Sound pressure ODS and acoustic mode shape of vibroacoustic analysis for the tuning fork.

### 4.3 Structure only analysis for tuning fork

After MV, one has obtained modal parameters through EMA and FEA, including natural frequency, mode shape and modal damping ratio. In order to understand the vibration response characteristics of the tuning fork structure, pure structural theoretical and experimental analysis can be performed. This involves ⑤S-analysis, ⑥S-test and ⑦R-analysis-V as shown in Figure 2(b).

Figure 7 shows the displacement mode shape corresponding to the structural displacement FRF of the A4 scale tuning fork. The upper part is the modal parameters obtained from EMA, coded as E. The lower part of the black frame is the modal parameters of the modified model from FEA, coded as F. The arrows indicate the vibration direction of the tuning fork mode shapes. The red lines in the mode shape represent the nodal lines, that is, the position that will not vibrate. Below the black box is the FRF of the tuning fork at the same point, that is, the response right at the load input point. There are four modes of the tuning fork below 3000 Hz. The physical meaning of the mode shapes corresponds well to the experimental mode shapes, which confirms that the analytical model is equivalent to the actual structure. The frequency response function and mode shape in the X and Z directions can fully interpret the modal characteristics of the A4-scale tuning fork structure.

### 4.4 Vibroacoustic analysis for tuning fork

The fourth stage is to explore the sound characteristics and sound-generating mechanism of the tuning fork. The analysis and test methods can refer to ⑩P-analysis-A, ⑪R-analysis-N and ⑫P-test-A as shown in Figure 2(b). An acoustic camera is used to obtain the sound pressure distribution and sound spectrum of the tuning fork for ⑫P-test-A.

The upper part of Figure 8 is the analysis and comparison of the experimental ODS of the two corresponding sound spectrum peak frequencies of the A4 scale tuning fork. For 439 Hz, it is the overall

fork arm swing mode, in which the vibration of the tail end of the fork arm is the largest, so the air pressure is mainly concentrated at the tail end of the fork arm. The 2670 Hz structural vibration has a nodal line, so the pressure in the air is also divided into upper and lower parts. The high-pressure characteristic in the lower half of the tuning fork can be observed.

The lower part of Figure 8 shows the SPL spectrum results of the A4 scale tuning fork. There are four obvious peaks in the theoretical analysis, but only two peaks in measurement. The missing two modes are due to little modal effect on sound radiation. For the frequencies of the two peaks between measured and theoretical analysis, they are quite consistent. The amplitude at 2670 Hz in measurement is slightly lower, which is due to the actual striking force. Since the actual force spectrum is not truly white noise, which is a straight flat spectrum, the force amplitude will decrease at higher frequency ranges.

## 5. Conclusions

This paper aims to present a systematic approach to studying percussion instruments, including both experimental and numerical methods. The source-path-response (SPR) system approach takes a tuning fork as an example to explore its vibration and acoustic response based on the proposed five-test and five-analysis techniques. According to the needs, four stages of tuning fork research are illustrated. This work provides a systematic approach for the SPR technical roadmap required in studying the sound and vibration of tuning forks. The proposed systematic approach, including 5-test and 5-analysis to examine the sound and vibration of tuning forks, which could also be applied to other musical instruments or engineering structures as well.

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