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DESIGN VERIFICATION OF DUAL-TONE CHIME BELL WITH HARMONIC OCTAVE TONE

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Chime bell is one of the traditional Chinese percussion instruments, also known as the dual-tone bell. By striking different positions of the chime, different musical scales of sounds can be produced, i.e. the face-tone sound and the side-tone sound. This work designs and verifies the geometry of the chime, so that the face-tone sounds and the side-tone sounds of the dual-tone chime bell will have the correct standard musical scale frequency, and they each have the overtone frequency of the harmonic octave tone. First, the design variables of the dimensions of the chime are defined to construct the chime model in a parametric manner and apply the modal analysis of finite element analysis (FEA) combined with the optimal design to obtain the geometric dimensions of the chime structure possessing the dual-tone scale with its harmonic octave tone. From natural frequencies and mode shapes of the chime structure obtained through analysis, the fundamental frequencies of the face-tone and side-tone sounds can be obtained, which conform to the corresponding standard musical scale frequencies. And, the dual-tone chime design has harmonic octave tones, respectively. 3D metal printing is then used to create a physical chime structure and perform sound measurements for the analysis of its sound characteristics to verify the accuracy of the numerical model and optimized design. Results show that the optimally designed dual-tone chime structure can produce two tone sound and has the characteristics of harmonic octave tones. Finally, based on this design method, a set of dual-tone chimes with harmonic octave tones was completed. In the future, this design analysis method can be applied to the design and development of different percussion instruments.

Keywords: chime bell, harmonics, percussion instruments, finite element analysis, optimization

1. Introduction

Chimes can be traced back to the Bronze Age and are one of Chinese cultural assets. By striking different positions of the chime, different musical scales of sounds can be produced, i.e. the face-tone sound and the side-tone sound. This work studies the optimization analysis of the structural design of dual-tone chimes and discusses the structural vibration and sound characteristics.

Yan *et al*. [1] analysed the sound characteristics of chimes and found that the elliptical shape allows the bell to produce two tones. The design of the chime has additional load to suppress high-frequency harmonic vibrations. Wu *et al*. [2] conducted sound analysis on chimes and established a computational model for chime sound synthesis. The established model can effectively produce the sound of physical bells. Wang *et al*. [3] explored the five sets of reproduced ancient Chime-bells of Marquis Yi of Zeng, conducted sound measurements, and obtained the

pitch, timbre and attenuation effect of the chime bells. Results showed that the interval difference between the face-tone and side-tone sounds of the chime was a major third or a minor third, while different sets of chime-bells exhibited different pitches, but the basic characteristics of the frequency response were similar. Wang *et al*. [4] discussed the sound characteristics of ancient cymbals percussion instruments. By measuring the sound of percussion, they analysed the pitch, timbre and attenuation rate of ancient cymbals, and found that the scale ratio of the peak frequency sound showed with the harmonics effect.

There is the strong correlation between the sound generation mechanism of musical instruments and structural vibrations. Wang *et al*. [5] explored the sound generation mechanism of smooth surface copper bells, used finite element software to construct a model, and obtained theoretical modal parameters for verification with experiments. In addition, by measuring the sound spectrum of the copper bell, the correlation between the sound frequency and the vibration mode of the copper bell was interpreted. Wang *et al*. [6] explored the correlation between vibration modes and sound characteristics of a miniature bell. They showed the peak frequencies of percussion sound spectrum from the bell can be interpreted by structural vibration modes and realized the sound generation mechanism of the bell.

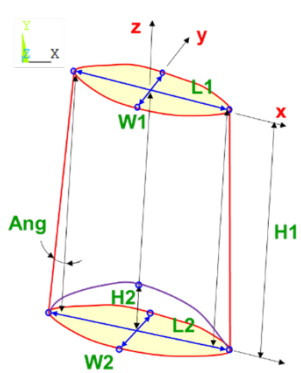
This work designs and verifies the geometry of the chime bell so that the face-tone and the side-tone sounds of the dual-tone chime have the correct standard musical scale frequency, and they each have the overtone frequency of the harmonic octave tone. The chime bells are manufactured using 3D metal printing, and sound measurements are conducted to explore their sound characteristics, thereby validating the reliability of the chime bell design.

2. Optimum design of dual-tone chime bell with harmonics

This section mainly explains the finite element analysis process of dual-tone chimes to have the harmonic octave tones. This work uses ANSYS commercial software to conduct the finite element analysis of the chime bell, and applies modal analysis to obtain the theoretical modal parameters of the chime bell, including the natural frequencies and the mode shapes. Optimal analysis is also employed to obtain the geometric dimensions of the chime structure possessing the dual-tone scale with its harmonic octave tone.

2.1 Finite element analysis

The geometric shape of the chime bell is shown in Figure 1. The dimension parameters are defined as follows: L1 is the length of the top ellipse; W1 is the width of the top ellipse; H1 is the height of the chime bell; Ang is the tilt angle of two sides to be ≥10 degrees; L2 is the length of the bottom ellipse; W2 is the width of the bottom ellipse; H2 is the height from the bottom of the chime bell to the cutting arc. The initial geometry dimensions of the chime bell are given in Table 1.



(1) L1: Length of the top ellipse.
(2) W1: Width of the top ellipse.
(3) H1: Height of the chime bell.
(4) Ang: Tilt angle of two sides (≥10∘)
(5) L2: Length of the bottom ellipse.
L2 = L1 + 2*H1*sin(Ang)
(6) W2: Width of the bottom ellipse.
W2 = W1 + 2*H1*sin(Ang)
(7) H2: Height height from the bottom of the chime bell to the cutting arc.

Figure 1: Dimensions definition of the chime bell.

The chime bell is assumed to be made of isotropic homogeneous material, complying with Hooke's law. Through validation analysis using 3D printed stainless steel plate [7], the modified Young's modulus E is determined to be 177 GPa, Poisson's ratio ν is 0.30, and the density ρ is 7652 kg/m³. Figure 3 illustrates the finite element model of the chime bell. The elements are meshed using quadratic tetrahedral elements (Solid 187), with a total of 32,846 elements and 65,086 nodes. Simulation with fully free boundaries (free-free), hence no displacement constraints need to be set. In addition, modal analysis does not require setting any load conditions, resulting in the theoretical modal parameters of the chime bell, including the natural frequencies and the mode shapes.

Table 1: Initial geometry dimensions of the chime bell.

Parameter	Value (mm)	Parameter	Value (mm)
L1	105	L2	131
W1	45	W2	71
H1	150	H2	22
Angle	≥10∘	Thickness(t)	4

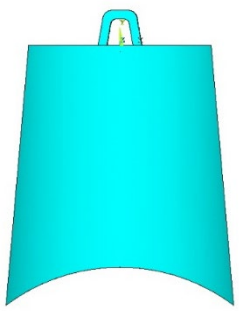


Figure 2: The geometry model of chime bell.

Figure 3: Finite element model of chime bell.

2.2 Optimum design analysis

The general optimization analysis process is typically divided into three main components: design variables, objective function, and constraints. Next, the chime optimization design process is explained:

(1) Design variables (DV): Based on the above bell shape design concept, the design variables include the top ellipse length (L1), top ellipse width (W1), chime height (H1), inclination angle (Ang), bottom ellipse length (L2), bottom ellipse width (W2) and chime undercut arc height (H2).

(2) Objective function (OBJ): The fundamental frequency of the face-tone sound of the chime bell is defined as f_{face}; the fundamental frequency of the side-tone sound is defined as f_{side}. The first natural frequency of Mode (1,1) of the structural mode obtained through CAE analysis is defined as $f_{(1,1)}$; the second natural frequency of Mode (2,1) of the obtained structural mode is defined as $f_{(2,1)}$. The natural frequency error of each structural mode is defined as follows:

$$\varepsilon_1 = \varepsilon_{(1,1)} = \frac{f_{(1,1)} - f_{face}}{f_{face}} \tag{1}$$

$$\varepsilon_2 = \varepsilon_{(2,1)} = \frac{f_{(2,1)} - f_{side}}{f_{side}} \tag{2}$$

The objective function can then be defined as the root-mean-square (rms) of the frequency errors.

$$\varepsilon_{rms} = \sqrt{\frac{\sum_{i=1}^{N} \varepsilon_i^2}{N}} \tag{3}$$

(3) Constraints (SV): The frequency error between the fundamental frequency of the face-tone sound and the side-tone sound of the chime and the standard scale is limited to ±0.34% to obtain the correct musical tone.

The optimal design of the chimes mainly uses the optimization module of ANSYS software. The Subproblem Approximation Method (SUBP) performs optimization analysis. Finally, the optimal design parameters of the chime that meet the design scale frequency are obtained.

2.3 Design planning of dual-tone chime bells

Table 2 shows a standard musical scale frequency table [8]. Based on this table, the frequency design plan for the dual-tone chime bells with harmonic frequencies is as shown in Table 3. DS-01 corresponds to the frequency of C6; DS-02 corresponds to the frequency of E6; DS-03 corresponds to twice the frequency of E6, and DS-04 corresponds to three times the frequency of C6. A set of dual-tone chime bell design combinations is planned based on the C6 to C7 musical scale, as summarized in Table 4.

Table 2: Standard musical scale frequency table [8].

Note name	-1	0	1	2	3	4	5	6	7	8	9	Frequency ratio
C	8.18	16.35	32.70	65.41	130.81	261.63	523.25	1046.50	2093.00	4186.01	8372.02	1
C♯/E♭	8.66	17.32	34.65	69.30	138.59	277.18	554.37	1108.73	2217.46	4434.92	8869.84	1.059463
D	9.18	18.35	36.71	73.42	146.83	293.66	587.33	1174.66	2349.32	4698.64	9397.27	1.122462
D♯/E♭	9.72	19.45	38.89	77.78	155.56	311.13	622.25	1244.51	2498.64	4978.03	9956.06	1.189207
E	10.30	20.60	41.20	82.41	164.81	329.63	659.26	1318.51	2637.02	5274.04	10548.08	1.259921
F	10.91	21.83	43.65	87.31	174.61	249.23	698.46	1396.91	2793.83	5587.65	11175.30	1.334840
F♯/E♭	11.56	23.12	46.25	92.50	185.00	369.99	739.99	1479.98	2959.96	5919.91	11839.85	1.414214
G	12.25	24.50	49.00	98.00	196.00	292.00	783.99	1567.98	3135.96	6271.93	12543.75	1.498307
G♯/A♭	12.98	25.96	51.91	103.83	207.65	415.30	830.61	1661.22	3322.44	6644.88	13289.75	1.587401
A	13.75	27.50	55.00	110.00	220.00	440.00	880.00	1760.00	3520.00	7040.00	14080.00	1.681793
A♯/B♭	14.57	29.14	58.27	116.54	233.08	466.16	932.33	1864.66	3729.31	7458.62	14917.24	1.781797
B	15.43	30.87	61.74	123.47	246.94	493.88	987.77	1975.53	3951.07	7902.13	15804.27	1.887749
C	16.35	32.70	65.41	130.81	261.63	523.25	1046.50	2093.00	4186.01	8372.02	16744.04	2

Table 3: Design of dual-tone chime bell with harmonic octave tone (C6/E6).

Modes	Standard Scale	Frequency (Hz)
DS-01	C6	1046.50
DS-02	E6	1318.51
DS-03	2×E6 = **E7**	2637.02
DS-04	3×C6 ≅ **G7**	3139.50

Table 4: Design of a set of dual-tone chime bells for C6 to C7 with harmonic octave tone.

No	Standard Scale	No	Standard Scale
1	C6 / E6	5	G6 / B6
2	D6 / F6	6	A6 / C7
3	E6 / G6	7	B6 / D7
4	F6 / A6	8	C7 / E7

3. Measurement analysis of dual-tone chime bells

The chime bells are manufactured using 3D metal printing as shown in Figure 4. For the face-tone sounds and the side-tone sounds, the striking points of the chime in the sound measurement experiment are shown in Figure 4. The frequency response characteristics are obtained by striking the chime bells with a mallet and analysed to explore the relationship between the sound and vibration properties of the chime bells.

Figure 5 depicts the experimental setup for modal analysis and sound measurement experiments of the chime bells. The chime bells were suspended using nylon ropes to simulate free boundaries during the experiments. A microphone (130E20) was connected to an NI-DAQs (NI-9234) for the sound measurement. Sound signal analysis was conducted using Sound and Vibration Measurement (SVM) software. The Fast Fourier Transform (FFT) and the Short-Time Fourier Transform (STFT) calculations were performed to obtain the sound spectrum. The peak value of the spectrum is to confirm each frequency value of the 3D metal printed chime.

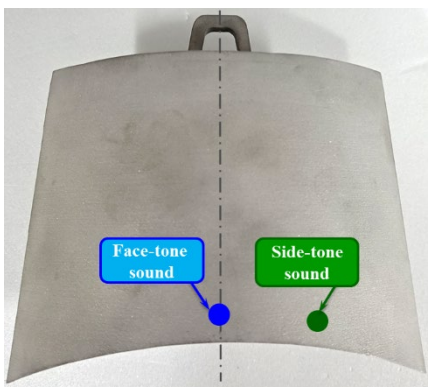


Figure 4: Picture of chime bell measurement position.

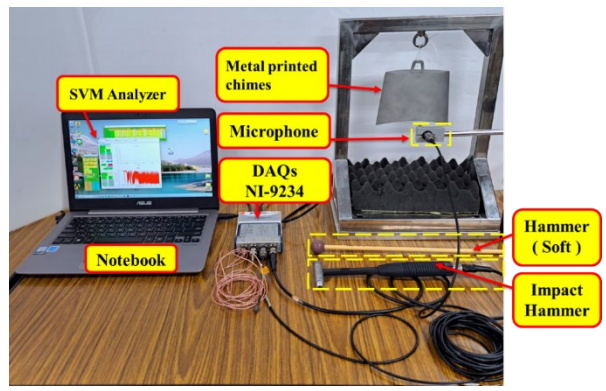
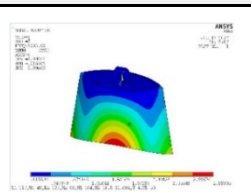
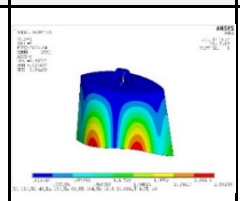
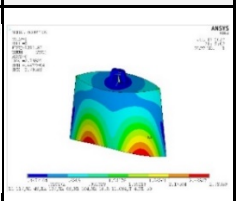
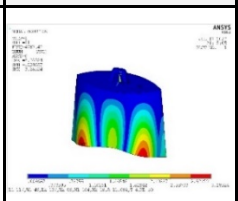
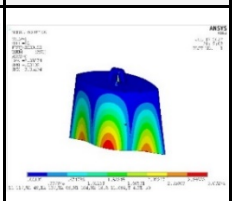
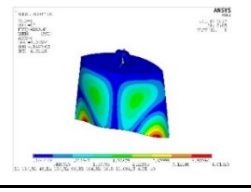
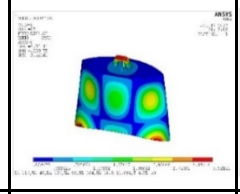
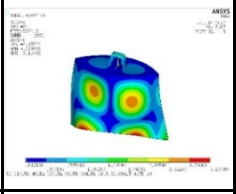
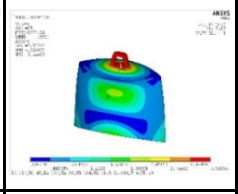
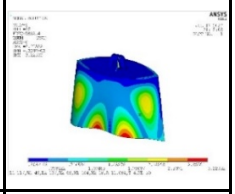


Figure 5: Experimental setup for sound measurement.

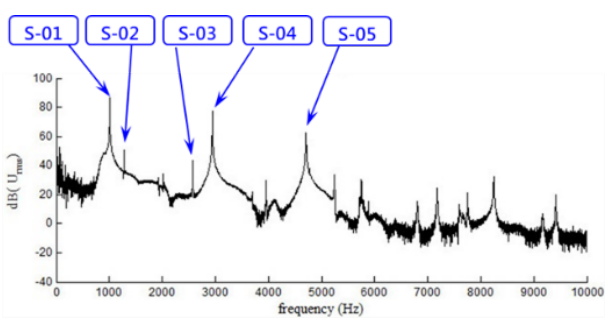
4. Results and discussions

The mode shapes of the chime were obtained through FEA optimization analysis, and the results are shown in Table 5. The physical meanings of the mode shapes are represented by θ and Z respectively. The physical meaning of the mode shape is represented by θ and Z respectively, where θ is the number of nodal region in the circumferential direction and Z is the number of nodal region in the axial direction. Taking the face tone sound F_01 as an example, the number of nodal region in the circumferential direction is 1 and the number of nodal region in the axial direction is 1, so the physical meaning of the modal shape is (θ, Z)=(1, 1). Observing the side tone sound F_02, the number of node region in the circumferential direction is 2, the number of node region in the axial direction is 1, and there is obvious swing on the left and right boundaries, which are defined with a + sign, so the physical meaning of this mode shape is (θ, Z)=(2, 1)+. (3, 1)+ and (4, 1)+ are those with the similar characteristics.

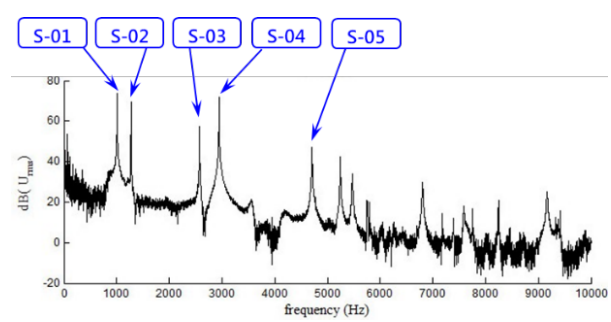
Table 5: Mode shapes of dual-tone chime bell (C6/E6).

Modes	F_01	F_02	F_03	F_04	F_05
Mode Shapes					
Physical Meaning of Mode Shapes (θ, Z)	(1, 1)	(2, 1)+	(2, 1)	(3, 1)+	(3, 1)
Modes	F_06	F_07	F_08	F_09	F_10
Mode Shapes					
Physical Meaning of Mode Shapes (θ, Z)	(4, 1)	(3, 2)	(2, 2)+	(1, 1) Hook	(4, 1)+

The dual-tone chime obtained by 3D metal printing was subjected to sound measurement and analysis to obtain the sound spectrum, as shown in Figure 6. Figure 6(a) shows the frequency response diagram of striking the middle position of the chime (the face-tone sound). S-01 and S-04 have relatively high amplitude responses, but S-02 and S-03 have relatively low amplitude responses. Similarly, Figure 6(b) shows the frequency response diagram of the side tone sound. In addition to the higher amplitude response of S-01 and S-03, S-02 and S-03 also show a higher amplitude response, which means that the designed chime has the harmonic octave tone characteristics.



(a) Sound spectrum of the face-tone sound



(b) Sound spectrum of the side-tone sound

Figure 6: Picture of chime bell measurement position.

The comparison between the peak response frequencies from sound measurement analysis of the dual-tone chime bell and the standard musical scale frequencies are shown in Table 6. From Table 6, it is observed that the deviation of the sounding frequencies S-01 and S-02 of the dual-tone chime bells from the standard musical scale frequencies C6 and E6 are -3.49% and -3.45%, respectively. And, the deviation of the sounding frequency S-03 and S-04 is approximately -2.4% and -6.2%. Furthermore, considering the frequency of C6 as the reference, it is noted that the ratio of DS-02 frequency to DS-01

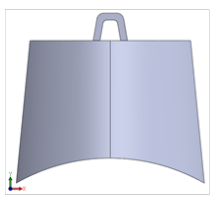
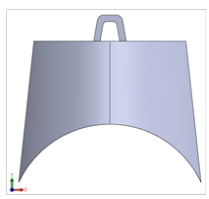
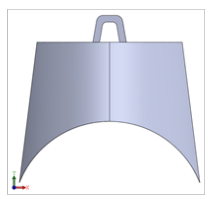
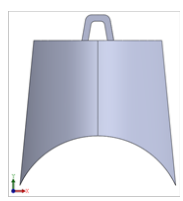
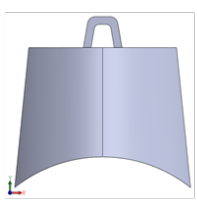
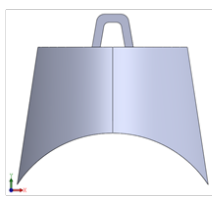
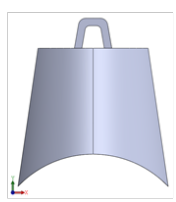
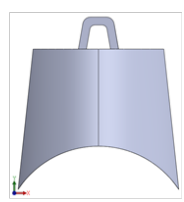
frequency remains at 1.26. This means the dual-tone design with the same amount of deviation and fitting the dual-tone design goals for C6 and E6. For harmonics of face-tone design, the frequency ratio of S-04 and S-01 is 2.92% very closed to the design goal for 3X. Similarly, considering the frequency of E6 as the reference, the frequency of DS-03 is twice that of DS-02, indicating that the designed dual-tone chime bell maintains a consistent level of sound frequency ratio.

As the above discussions in Figure 6 and Table 6, the design concept for the dual-tone chime bell is to have a C6/E6 combination design. Results show the feasibility of design method is potentially achievable. The fabricated chime bell has the close fundamental frequencies for both the face-tone and side-tone sound, and their harmonics with three times and two times can be produced as expected, although there is some discrepancy due to manufacturing deviation. Finally, with the same optimum design methodology, Table 7 shows the conceptual design of a set of the dual-tone chime bells with one octave musical scale from C6/E6 to C7/E7. In the future, this set of chimes can be manufactured using 3D metal printing and played like an percussion instrument with the octave musical sound.

Table 6: Comparison results of dual-tone chime sound frequency and standard frequency.

Standard scale	Modes	(1) Design frequency (Hz)	Modes	(2) Sound frequency (Hz)	Frequency error(%) of (1) compared to (2)	Frequency ratio to **C6** reference	Frequency ratio to **E6** reference
C6	DS-01	1046.50	S-01	1010	-3.49	**1**	–
E6	DS-02	1318.51	S-02	1273	-3.45	1.26	**1**
2×E6	DS-03	2637.02	S-03	2574	-2.39	2.55	**2.02**
3×C6	DS-04	3139.50	S-04	2945	-6.20	**2.92**	2.31

Table 7: Final design of a set of the dual-tone chime bells.

Design Combination	(1) C6 / E6	(2) D6 / F6	(3) E6 / G6	(4) F6 / A6
Appearance				
Design Combination	(5) G6 / B6	(6) A6 / C7	(7) B6 / D7	(8) C7 / E7
Appearance				

5. Conclusions

This paper primarily focuses on the exterior geometric dimension design of dual-tone chime bells with harmonic frequencies. This work applies the optimization design method of finite element analysis to obtain theoretical modal parameters of the chime bells, including natural frequencies and mode shapes. Subsequently, the chime bells designed through analysis are manufactured using 3D metal printing. Finally, sound quality assessment of the dual-tone chime bells is conducted through sound measurement analysis. Based on the comprehensive analysis results, the following conclusions are summarized:

1. Through the frequency spectra obtained from sound measurement analysis of the dual-tone chime bells, it is observed that the frequency response for striking the chime bell at the middle position (Face tones), denoted as S-01, exhibits a relatively higher amplitude response. Additionally, the frequency responses charts for striking the chime bell at the left and right positions (Side tones), denoted as S-02 and S-03, respectively, show significantly higher amplitude responses. This indicates that the design of the chime bells possesses the characteristic of harmonic frequencies.
2. The deviation of the frequencies of the dual-tone chime bells from the standard musical scale frequencies (C6 and E6) is approximately -3.5%. Observing the frequency ratio based on the C6 frequency as the reference, it is noted that the ratio of DS-02 frequency to DS-01 frequency remains at 1.26. Similarly, considering the E6 frequency as the reference, the frequency of DS-03 is twice that of DS-02. This implies that the dual-tone chime bells optimized in design exhibit good sound response characteristics as expected.

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