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VIBROACOUSTIC ANALYSIS ON CHIME BELL FOR MODAL PROPERTIES AND SOUND SPECTRUM

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Chime bell, a Chinese ancient type of percussion instrument, can produce two tones sound for different striking locations. The two-tone sound is namely the face-tone sound and the side-tone sound base on striking the central location and the aside location, respectively, at the bottom of chime bell. It is of great interest to explore vibration modes and percussion sound spectrum for the chime bell. This work aims to perform vibroacoustic analysis on a miniature of chime bell. The mini chime bell is with oval shape cross section along vertical direction. There are so-called Mei, a series of small convex dots around the top surface of bell. One of the goals is to examine the effect of Mei on structural vibration modes and its influence on sound spectrum. The chime bell is assumed freely suspended and stroke to produce the percussion sound. The structure-air coupling model is therefore constructed to perform modal analysis to obtain structural vibration modes. The ideal striking force is then applied to examine structural vibration spectrum as well as percussion sound spectrum. Results show those peak frequencies of percussion sound strongly correlate to vibration modes of chime bell. The stroked location is crucial to excite different vibration modes and generate different sound characteristics. This is the physical mechanism for the two-tone chime bell. The face-tone excitation can induce the odd modes and generate those peak sound responses, while the side-tone excitation produces those peak sound responses related to the even modes of vibration.

Keywords: chime bell, vibroacoustic analysis, vibration mode, sound spectrum

1. Introduction

Chime bells are praised as the king of oriental musical instruments. Due to their special structure, there is a musical scale difference of minor third between the face-tone and the side-tone sound. Through vibroacoustic analysis of miniature chimes, we can observe the modal properties, and also use the sound spectrum to examine the correlation between modal response and percussion sound. Wu *et al.* [1] conducted sound analysis on a set of chimes and constructed a synthesis model to create a bell-like sound corresponding to the frequency of the physical chime sound. Pan [2] indicated that ancient Chinese musical bells differ from church bells and oriental temple bells in their almond-shaped cross-section, so that the special structure results in one musical bell having two tones, namely the face-tone and the side-tone sound. Wang *et al*. [3] explored the five sets of reproduced ancient Chime-bells of Marquis Yi of Zeng, conducted sound measurement, and obtained the pitch, timbre and attenuation effect of the chime bells.

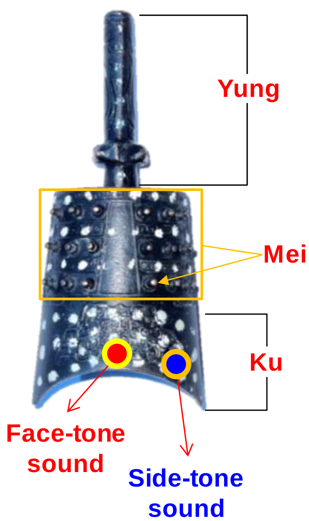
Results showed that the interval difference between the face-tone and side-tone sounds of the chime was a major third or a minor third, while different sets of chime-bells exhibited different pitches, but the basic characteristics of the frequency response were similar. Wang *et al*. [4] carried out model verification and sound spectrum measurement on a miniature bell to obtain the correlation between vibration modes and sound characteristics, and to understand the sound generation mechanism of the bell through the structural vibration modes. Rossing [5] showed vibration modes of wineglasses by means of holographic interferometry and examined the difference of sound spectra from striking and rubbing as well as the fine tuning of pitch frequency of wineglasses by adding water or wine. Wang *et al.* [6] obtained the vibration characteristics and sound spectrum of three types of copper bells through experimental modal analysis (EMA) and sound measurement experiments, and discussed the relationship between the sound characteristics of copper bells and the natural frequencies of structural vibration modes. Results show the circular ring modes of the copper bell are the main sounding modes, and the smaller modal damping ratio results in the longer existing sound. Wang *et al.* [7] performed EMA on the chime bell to obtain its vibration modes and measured percussion sound spectra for both face-tone and side-tone. The pitch frequencies of face-tone and side-tone excitation correspond to the first and second modal frequencies, while those higher vibration modes consist of the overtone frequencies, representing the tonality of chime bell.

McLachlan *et al*. [8] obtained the correlation between the geometric shape and vibration characteristics of the bell through finite element analysis (FEA). The bell models are tuned by using gradient projection method for shape optimization and designed to have up to seven partial frequencies in harmonics. Nakanishi *et al.* [9] used FEA to obtain the mode shape of a Buddhist temple bell, and compared the mode shapes with the sound characteristics. It was observed that a thicker cross-section at the bottom of the bell has a significant impact on the modal frequencies. The "Doza" where the bell is struck is fabricated on one side of the bell with small thickness. The asymmetrical geometry effect can result in beating sound effect. Wang and Yu [10] carried out model verification on the crotale, a step round plate with two different thickness made of copper. The obtained theoretical and experimental modal parameters reveal good agreement and confirm the equivalence between the finite element model and the actual structure. Wang *et al.* [11] carried out EMA on the violin structure without the effect of string tension and applied FEA to the complex composite structure, so as to observe the modal characteristics of the overall violin structural model. Vibration modes of violin can be shown as global modes and local component modes, such as the top plate or back plate. Wang *et al.* [12] conducted acoustic-vibration coupling analysis on the semi-circular pipe structure and air-structure coupling system. Modal analysis is to obtain the theoretical natural frequencies and mode shapes, and harmonic response analysis is to get frequency response functions (FRFs). There shows a reasonable agreement with that of the actual structure. In particular, the sound pressure mode shape in the sound field can be predicted and correlated to the sound spectrum response. Vibroacoustic analysis for percussion instruments such as the bell [13] and the crotale [14] is of interest to realize the sound radiation characteristics. The structure-only and air-structure coupling systems can be considered. By observing the modal characteristics and sound spectrum of the structure, one can understand the sound pressure level and sound directivity of the sound radiation of the structure. Yan *et al.* [15] experimentally measured sound spectrum for the chime bell with and without "Mei", a series of small convex dots around the top surface of bell, that is not only for decoration but may also damp out higher frequency modes response.

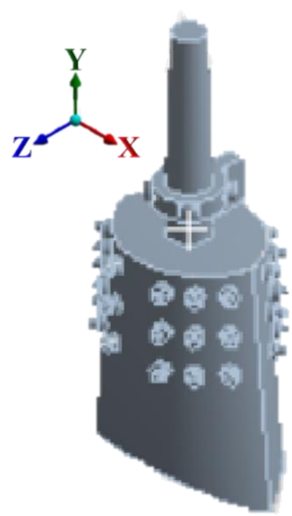
This work aims to perform vibroacoustic analysis on a miniature of chime bell. The finite element model of air-structure coupling is constructed to perform modal analysis, so as to obtain structural natural frequencies as well as corresponding displacement and sound pressure mode shapes. Assuming that the chime is a free-hanging boundary, both face-tone and side-tone excitation are, respectively, applied to get the percussion sound spectrum. The structural vibration spectrum and the sound response spectrum are obtained and shown for the relationship with structural vibration modes and acoustic modes. Section 2 shows the basic structure of chime bell. Section 3 presents the structural vibroacoustic coupling analysis on the chime bell, and Section 4 examines the effect of “Mei” on the vibrational modal response and percussion sound.

2. Structure for the miniature chime bell

Chime bells can be traced back to the Bronze Age and are traditional Chinese percussion instruments. It has a clear and bright sound when struck, and is mainly used as an ancient instrument for sacrifices, celebrations and banquets. A complete set of chimes constitutes the range, from the lowest note to the highest note, spanning five octaves. That a single chime bell can produce two tones by striking different locations is the most distinctive design of the chime bell. Fig. 1 shows the actual structure of the miniature chime. For its striking under free suspension, the Y axis is defined as the axial direction, and the Z direction is defined as the striking direction. The chime bell can be divided into three parts. There are square holes in the “Yung” column, which can be used for hanging chime bells. Along the section perpendicular to the Z axis, there are a series of convex dots called "Mei" allocated on the top surface. The lower part of the chime bell is called “Ku”, the drum, where is the striking position of the chime. As shown in Fig. 1(a), the chime bell is designed for two striking positions, i.e. the face-tone and side-tone sound which sound characteristics are related to structural vibration modes. This work will explore the difference of face-tone and side-tone sound by examining the structural vibration modes as well as acoustic modes.



(a) Real structure



(b) CAD model

Figure 1: Picture of miniature chime bell.

3. Vibroacoustic Analysis on the Chime Bell

In this section, the vibroacoustic analysis of the air-structure coupling system for the chime bell are carried out. Modal analysis is to obtain the theoretical modal parameters, including natural frequencies and mode shapes, that is used to explore the correlation of percussion sound spectrum from the chime bell. The harmonic response analysis for the face-tone or side-tone excitation can obtain the FRFs corresponding to the actual air structure coupling system for both vibration spectrum and sound spectrum.

Fig 2 shows the finite element model of chime bell used to conduct vibroacoustic analysis for the chime bell interaction with air. The bell material is assumed the isotropic and homogeneous with density 956.31 $\mathrm{kg/m^3}$, Young’s modulus 70.4 GPa and Poisson ratio 0.34. The air element of the sphere is surrounding the bell, and the outer surface of the air sphere is an infinite sound field to simulate the actual sound radiation. Air material parameters assume the air sound velocity 346.25 m/s and air density 1.225

$\mathrm{kg/m^3}$. The bell is in free boundary condition, and those air-structure interaction elements are coupling with structural and acoustic degree-of-freedom.

Modal analysis can then be performed to obtain modal parameters of the air-structure coupling system. Other than natural frequencies, the mode shapes can be categorized as the structural displacement mode shapes and the acoustic sound pressure mode shapes. In harmonic response analysis, both face-tone and side-tone excitation are respectively applied as the ideal impact force, so as to generate the FRFs, i.e. the acceleration response $H^{a}_{ai,fj}(f)$ and the percussion sound response $H^{a}_{pk,fj}(f)$. ai denotes the structural acceleration response at i location and pk denotes the sound pressure response at k location, while fj denotes the applied force at j location. As shown in Fig. 2, $j=1$ and $j=2$ indicate the face-tone and side-tone excitation, respectively. The structural vibration FRF $H^{a}_{ai=1,fj=1}(f)$ and $H^{a}_{ai=2,fj=2}(f)$ as well as the sound FRF $H^{a}_{pk=1,fj=1}(f)$ and $H^{a}_{pk=1,fj=2}(f)$ can be obtained accordingly.

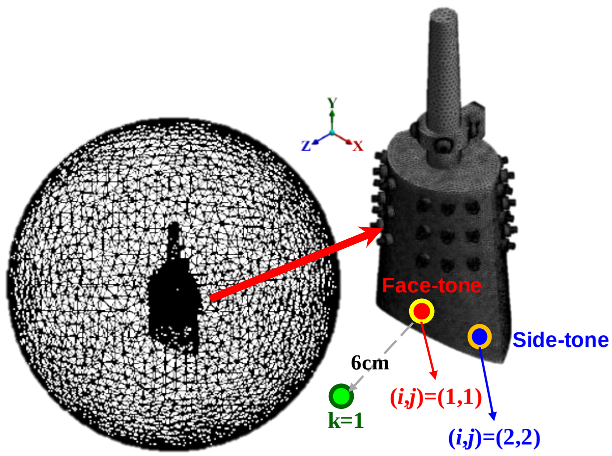


Figure 2: Finite element model of chime bell.

3.1 Modal analysis and structural vibration response

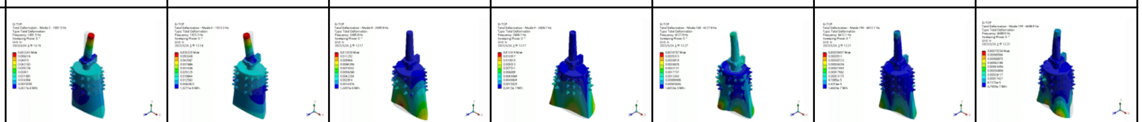
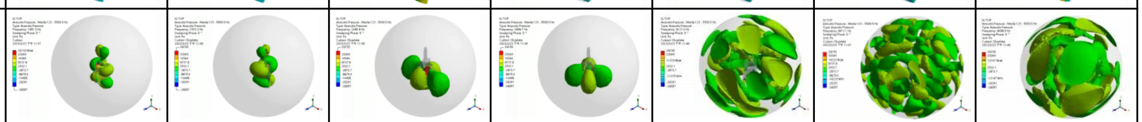
Table 1 shows natural frequencies and mode shapes of the chime bell. In Table 1(a), both displacement mode shapes and sound pressure mode shapes are shown and obtained in air-structure coupling analysis, while those higher modes shown in Table 1(b) are from structure-only analysis. The mode denoted as F_{SA} means structural acoustic coupling analysis. One can observe that those F_{SA} mode number are high solved up to 200 modes. Only those modes related to structural vibration may contribute to percussion sound radiation. Those higher modes not solved in the present work are referred to the structural vibration modes from F-14 to F-20 as shown in Table 1(b). It is of interest to interpret physical meaning of mode shapes as follows:

- From displacement mode shapes, F_{SA}-03 and F_{SA}-04 are characterized as the local mode of "Yung" column and revealed the 1st bending modes in X and Z-direction, simply like a cantilever beam. F-14 and F-15 are the same effect and revealed the 2nd bending modes in X and Z-direction. Below 10000 Hz, there is also the 1st torsional mode in Y-direction for the "Yung" column.
- The most significant and dominating sound radiation modes are those bell circular vibration modes, in which (θ, Y)=(1,1) and (θ, Y)=(2,1) are the first two modes corresponding to the face-tone sound and side-tone sound, respectively. Note that the two natural frequencies will be the fundamental frequencies of the percussion sound, while the other natural frequencies will be the overtone frequencies.
- From sound pressure mode shapes, one can see F_{SA}-03, F_{SA}-04, F_{SA}-08 and F_{SA}-09 having similar pattern like the mono-pole or di-pole characteristics, while F_{SA}-160, F_{SA}-194 and F_{SA}-199 reveal the scattering pattern. These characteristics will be discussed later in sound spectrum analysis.

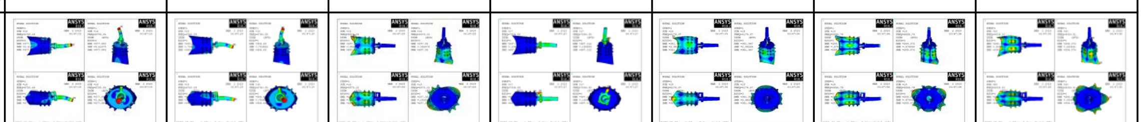
- As mentioned about the face-tone and side-tone excitation, the dual-tone chime bell can be stroked at the middle or near one fourth of circumferential location. The locations may coincide with the nodal line of mode shape or not. This feature makes the chime bell can produce two different fundamental frequencies for the face-tone and side-tone sound. Those modes in circumferential direction with θ=2, i.e. even modes, that cannot be excited for the face-tone sound.
- One can notice that there are modes of (θ,Υ)=(2,2) and (θ,Υ)=(2,1)+. The main difference of "+" is there is the slightly difference of mode shapes near the edge due to the oval shape of cross section of the chime bell.

Table 1: Natural frequencies and mode shapes of chime bell

(a) Chime bell with Mei for Air-Structure Coupling Analysis for Lower Modes

Mode	F_{SA}-03	F_{SA}-04	F_{SA}-08	F_{SA}-09	F_{SA}-160	F_{SA}-194	F_{SA}-199
Natural Frequency (Hz)	**1801.5**	**1923.3**	**2489.8**	**2606.7**	**6127.8**	**6412.1**	**6488.9**
Displacement Mode Shape							
Sound Pressure Mode Shape							
Physical Meaning	**1st-Bending-X**	**1st-Bending-Z**	**(θ,Υ)=(1,1)**	**(θ,Υ)=(2,1)**	**1st-Axil-Y**	**(θ,Υ)=(2,1)+**	**(θ,Υ)=(3,1)**

(b) Chime bell with Mei for Structure-only Analysis for Higher Modes

Mode	F-14	F-15	F-16	F-17	F-18	F-19	F-20
Natural Frequency (Hz)	**6755.64**	**6760.29**	**6979.15**	**7836.55**	**8179.37**	**8888.79**	**9024.91**
Displacement Mode Shape							
Physical Meaning	**2nd-Bending-Z**	**2nd-Bending-X**	**(θ,Υ)=(2,2)**	**1st-Torsion-Y**	**(θ,Υ)=(1,2)+**	**(θ,Υ)=(3,2)**	**(θ,Υ)=(2,2)+**

For the air-structure coupling system, Fig. 3 shows the structural FRF $H^a_{ai=1,fj=1}(f)$ and $H^a_{ai=2,fj=2}(f)$, simply the acceleration spectrum due to the ideal impact force excitation at the face-tone and side-tone location, respectively. Discussions are as follows:

- One can observe that there are anti-resonances between every two resonance peaks. This is the cause that the excitation and response locations are the same, which FRF is referred as the point FRF. This point FRF phenomenon implies the simulation with reasonable results.
- For each resonance peak referred to the natural frequency, the structural displacement mode shapes are depicted on the top of peaks.
- For face-tone excitation, i.e. $H^a_{ai=1,fj=1}(f)$, the highest peak is 2489.8 Hz referred to mode (θ,Υ)=(1,1) and will be the fundamental frequency for the face-tone sound.
- For side-tone excitation, i.e. $H^a_{ai=2,fj=2}(f)$, the highest peak is 2606.7 Hz referred to mode (θ,Υ)=(2,1) and will be the fundamental frequency for the side-tone sound.
- It is noted that mode (θ,Υ)=(1,1) is also excited for side-tone excitation but with the smaller amplitude. However, mode (θ,Υ)=(2,1) is not excited for face-tone excitation, because the excitation location is right on the nodal line of mode (θ,Υ)=(2,1).

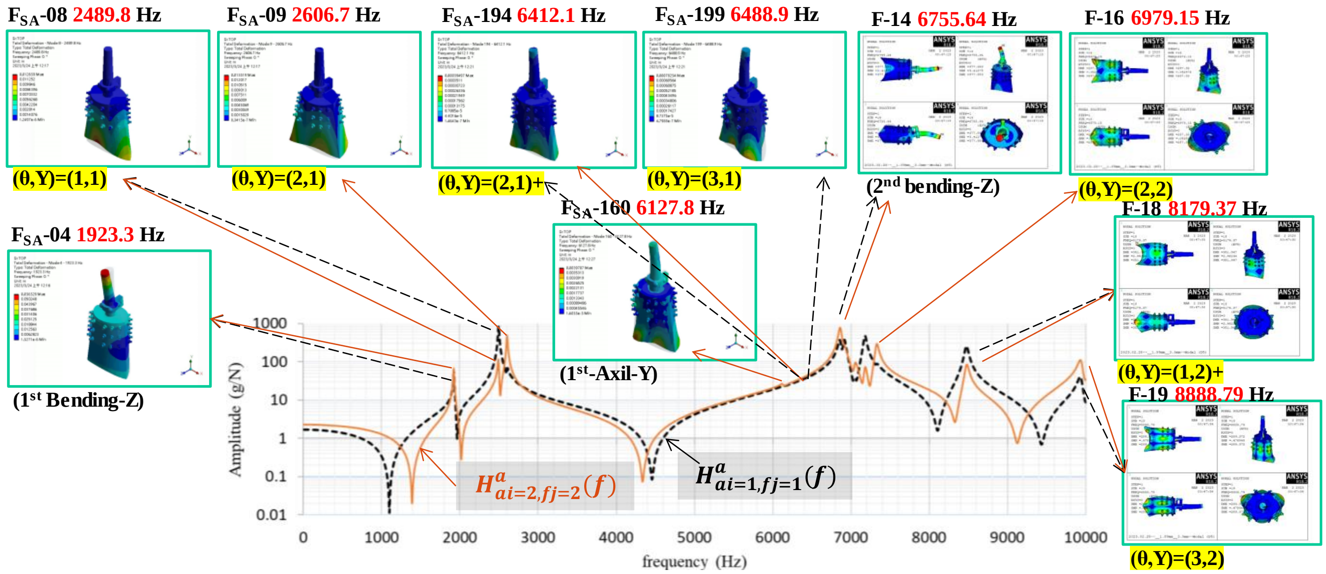


Figure 3: Frequency response function of chime bell and physical meaning of displacement mode shapes.

3.2 Harmonic response analysis and sound spectrum

Fig. 4 shows the predicted sound of chime bell for face-tone and side-tone excitation, i.e. $H^a_{pk=1,fj=1}(f)$ and $H^a_{pk=1,fj=2}(f)$, respectively. The observed location for sound spectrum response is at $k = 1$, 6 cm from the bell surface normally as shown in Fig. 2. In Fig. 4, for those resonance peaks, there shows the displacement mode shapes in the dotted-line plots and the sound pressure mode shapes in the solid-line plots. Discussions are as follows:

- For face-tone excitation, i.e. $H^a_{pk=1,fj=1}(f)$, the fundamental frequency is 2489.8 Hz which is the highest peak referred to the natural frequency of mode (θ, Y)=(1,1), and those other peak frequencies will be the overtone frequencies account for the tonality of the sound.
- For face-tone excitation, i.e. $H^a_{pk=1,fj=2}(f)$, the fundamental frequency can be 2606.7 Hz which is the natural frequency of mode (θ, Y)=(2,1). Although the peak at 2606.7 Hz is not the highest, it is because the response location is at $k = 1$. From sound pressure mode shape of (θ, Y)=(2,1), it is a typical dipole radiation pattern such that the peak is not as expected the highest peak. However, the acoustic mode of (θ, Y)=(2,1) is actually excited and can be the fundamental frequency.
- It is of interest to note that there is no peak response corresponding to mode (θ, Y)=(3,1), though the mode is expected with high contribution of sound radiation. One can see that the corresponding sound pressure mode shape is the scattering pattern. Later, the discussion of the effect of "Mei" may explain the phenomenon.

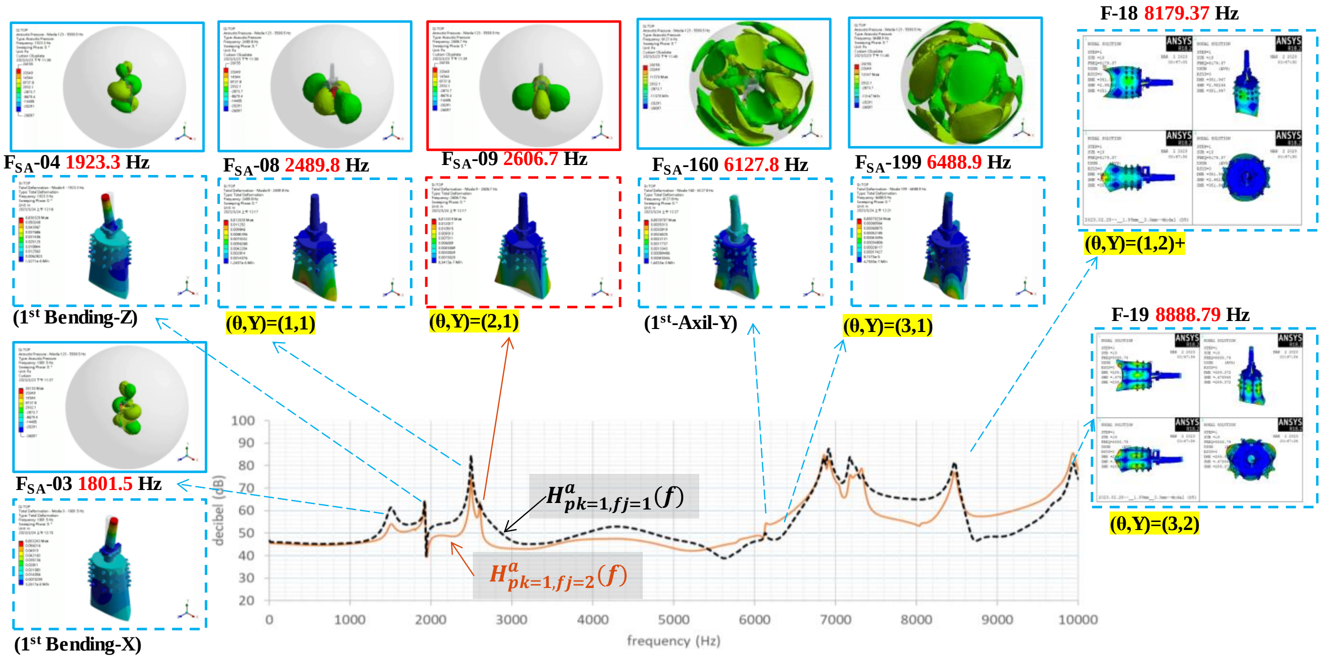
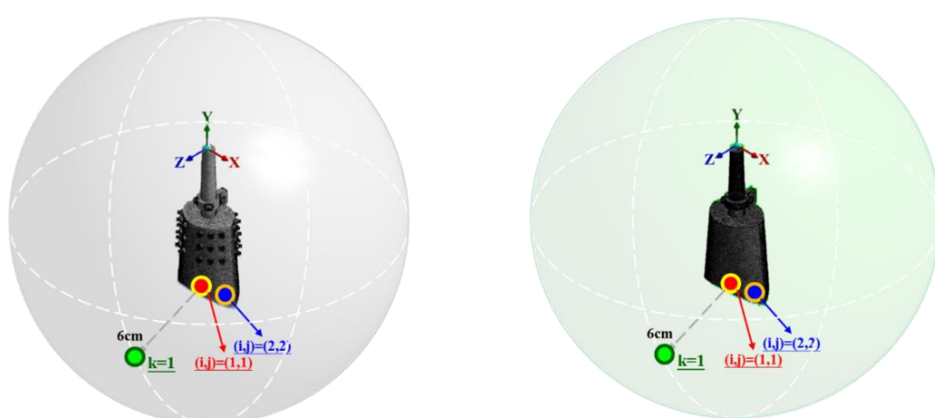


Figure 4: Predicted sound spectrum of chime bell for face-tone and side-tone.

4. Effect of Mei on modal response and sound spectrum

As exploring the sound spectrum response of chime bell related to structural vibration modes and the acoustic sound pressure mode shapes, one of the goals in this work is to examine the effect of “Mei” on structural vibration modes and its influence on sound spectrum. Fig. 5(a) and 5(b) respectively show the finite element models of chime bell with and without “Mei”. Both models have the same geometry, material parameters and related setup except the without “Mei” model where all of “Mei” on the chime bell surface are removed.



(a) with “Mei” (b) without “Mei”

Figure 5: Finite element model of chime bell with and without “Mei”.

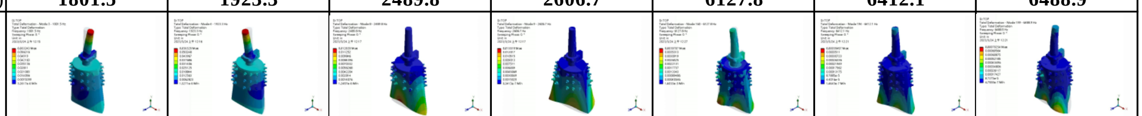
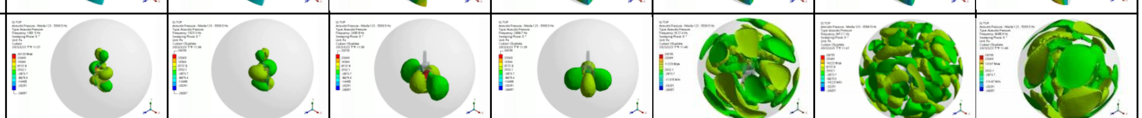
4.1 Comparison of vibration modes for chime bell with and without “Mei”

Tables 2(a) and 2(b) shows the coupled vibration and acoustic modes below 6500 Hz up to the first 200 modes for the chime bell with “Mei” and without “Mei”, respectively. Discussions are as follows:

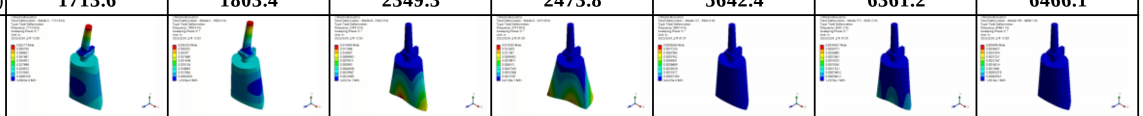
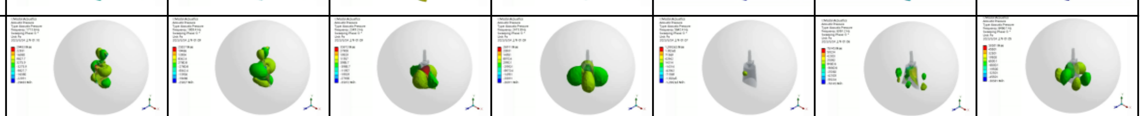
- The natural frequencies for without “Mei” are generally smaller than those for with “Mei”. It is noted that the convergence of finite element model is not well validated; however, the tendency of lowering natural frequency is evidence. This implies that the “Mei” would somewhat increase the overall stiffness of the structure and complies with the discussion of Yan et al. [15].
- For the first four modes shown in Table 2, the displacement and sound pressure mode shapes for both the chime bell with and without “Mei” are consistent with each other.
- It is noted that the higher three modes have the same physical meaning of displacement mode shapes for with and without “Mei”. For the sound pressure mode shapes, those for with “Mei” are in scattering pattern, while those without “Mei” reveal with the same pattern as those for the first modes. In particular, mode (θ, Y)=(2,1)+ and mode (θ, Y)=(3,1) reveal the four-pole and tri-pole sound radiation, respectively. This can explain the suspect of the effect of “Mei” on trendily damping out the higher mode response as Yan et al. [15].
- Besides, in comparison of sound pressure mode shapes, mode (θ, Y)=(2,1) reveals the di-pole pattern in one side or the four-pole around the chime bell, while mode (θ, Y)=(2,1)+ reveals near the six-pole pattern around the chime bell. This is due to different physical meaning of displacement mode shapes. Although these kinds of radiation pattern are different, the circumferential vibration modes of chime bell can contribute to sound radiation dominantly.

Table 2: Comparison of natural frequencies and mode shapes of chime bell with and without “Mei”

(a) Chime bell with **“**Mei**”**

Mode	F_{SA}-03	F_{SA}-04	F_{SA}-08	F_{SA}-09	F_{SA}-160	F_{SA}-194	F_{SA}-199
Natural Frequency (Hz)	1801.5	1923.3	2489.8	2606.7	6127.8	6412.1	6488.9
Displacement Mode Shape							
Sound Pressure Mode Shape							
Physical Meaning	1st-Bending-X	1st-Bending-Z	(θ,Y)=(1,1)	(θ,Y)=(2,1)	1st-Axial-Y	(θ,Y)=(2,1)+	(θ,Y)=(3,1)

(b) Chime bell without **“**Mei**”**

Mode	$F_{SA,wo}$-03	$F_{SA,wo}$-04	$F_{SA,wo}$-08	$F_{SA,wo}$-09	$F_{SA,wo}$-127	$F_{SA,wo}$-177	$F_{SA,wo}$-199
Natural Frequency (Hz)	1713.6	1803.4	2349.3	2473.8	5642.4	6361.2	6466.1
Displacement Mode Shape							
Sound Pressure Mode Shape							
Physical Meaning	1st Bending-X	1st-Bending-Z	(θ,Y)=(1,1)	(θ,Y)=(2,1)	1st-Axial-Y	(θ,Y)=(2,1)+	(θ,Y)=(3,1)

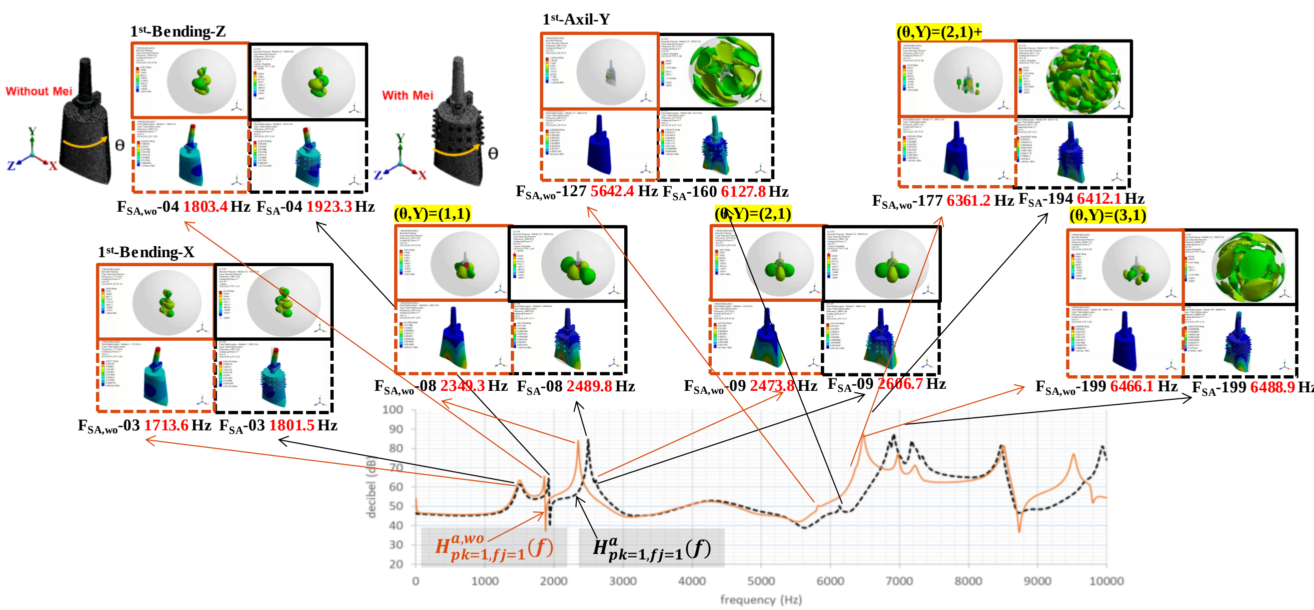
4.2 Comparison of sound spectrum for chime bell with and without “Mei”

Figs. 6(a) and 6(b) shows the predicted sound spectrum of chime bell with and without “Mei” for both face-tone and side-tone sound. $H^{a}_{pk=1,fj=1}(f)$ and $H^{a}_{pk=1,fj=2}(f)$ are the face-tone and side-tone sound spectrum with “Mei”, while $H^{a,wo}_{pk=1,fj=1}(f)$ and $H^{a,wo}_{pk=1,fj=2}(f)$ are the face-tone and side-tone sound spectrum without “Mei”. For those resonance peaks, the displacement mode shapes are shown in the dotted-line plots and the sound pressure mode shapes are shown in the solid-line plots. Discussions are as follows:

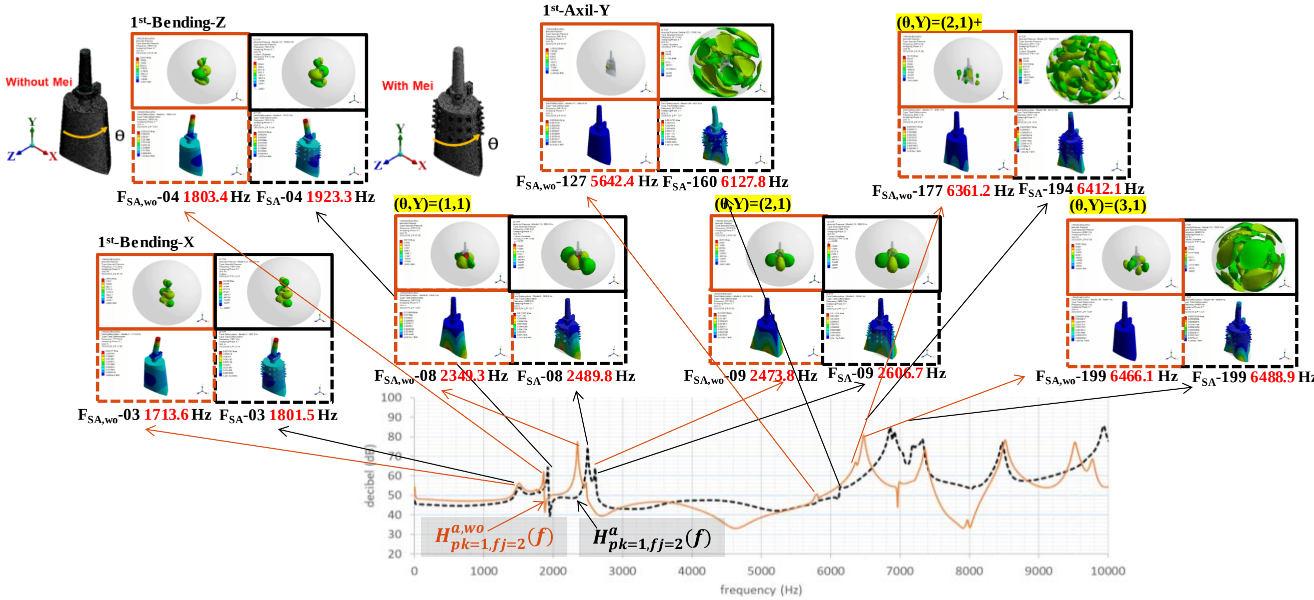
- In general, the chime bell without “Mei” reveal smaller natural frequencies than those of with “Mei”. This can be observed from the slight shift of those peaks to the left in sound spectrum.

- For the face-tone sound, the dominated vibration mode is (θ, Y)=(1,1) and the sound radiation pattern is the mono-pole in one-side or the four-pole around the chime bell.
- For the side-tone sound, the dominated vibration mode will be (θ, Y)=(2,1); however mode (θ, Y)=(1,1) also contributes to sound radiation. It is noted that the sound radiation pattern for mode (θ, Y)=(2,1) is the di-pole in one-side or the four-pole around the chime bell; however, the physical meaning of sound pressure mode shapes of (θ, Y)=(2,1) and (θ, Y)=(1,1) are different.
- The most interesting effect of "Mei" on sound spectrum is modes (θ, Y)=(2,1)+ and (θ, Y)=(3,1). One can see that for the chime bell without "Mei" there appear peaks at the corresponding frequencies, but the chime bell with "Mei" does not. The reason for this phenomenon can be postulated due to the "Mei", the convex dots on the surface of chime bell. The "Mei" tends to damp out the sound response; therefore, there is no peaks for the two modes which sound pressure mode shapes are in the scattering pattern. On the other hand, the chime bell without "Mei" have the peaks in sound spectrum for the two modes which sound pressure mode shapes are di-pole and tri-pole, respectively.

In summary, there are two major effects of "Mei" on the chime bell. First, the natural frequencies will slightly increase for the chime bell with "Mei" than without "Mei" due to the convex dots increasing the overall structural stiffness. Second, the chime bell with "Mei" does not incur the peaks in the percussion sound spectrum for those higher modes, such as (θ, Y)=(2,1)+ and (θ, Y)=(3,1) with the scattering sound radiation pattern, but the chime bell without "Mei" does contribute to the sound radiation and incur those higher mode response.



(a) Face-tone sound



(b) Side-tone sound

Figure 6: Comparison of predicted sound spectrum of chime bell with and without Mei.

5. Conclusions

This work performs vibroacoustic analysis on a miniature of chime bell. The air-structure coupling finite element model is constructed to carry out modal analysis and obtain the system's natural frequencies and corresponding structural displacement mode shapes and acoustic sound pressure mode shapes. Through harmonic response analysis, both face-tone and side-tone excitation are considered to obtain structural vibration spectrum and sound spectrum so as to examine the sound characteristics of the face-tone and side-tone. The peaks of sound spectrum response can be correlated to the displacement mode shapes as well as the sound pressure mode shapes. The chime bell with and without "Mei" are also analysed, respectively, to explore the effect of "Mei", which are those convex dots around the top surface of the chime bell. Results show there are two major effects of "Mei" on the chime bell. One is that the "Mei" increases the overall structure stiffness and so forth slightly increases structural natural frequencies. The other is that the "Mei" does tend to damp out higher mode sound response due to the scattering sound radiation pattern, while the chime bell without "Mei" has the normal sound radiation patterns and reveals sound peaks at higher modes.

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