# Damage Detection of Matrix Cracking in Composite Laminates


Huiwen Hu[1], Bor-Tsuen Wang[2], Cheng-hsin Lee[3]
[1]Assistant professor, Department of Vehicle Engineering
[2]Professor, Department of Mechanical Engineering
[3]Graduate student, Department of Vehicle Engineering
National Pingtung University of Science and Technology
1 Hseuh Fu Rd., Neipu, Pingtung 91201, Taiwan
Email : huiwen@mail.npust.edu.tw



**Abstract.** This paper presents a nondestructive detection of matrix cracking in composite laminates using experimental modal analysis and strain energy method. Carbon/epoxy composite AS4/PEEK was used to fabricate laminate $[0_2/90_n/0_2]$. Matrix cracking in 90-degree lamina was created by subjecting to tensile test. Experimental modal analysis (EMA) was performed to obtain the mode shapes of the specimen before and after damage. The measured mode shapes were then used to compute the strain energy using differential quadrature method (DQM). A damage index was defined using the change of strain energy of the specimen before and after damage. Consequently, the damage index successfully located the matrix cracking in the specimen. A pre-study was performed to access this approach by using a 3-D finite element analysis (FEA).



**Keywords:** matrix cracking, composite laminate, modal analysis, strain energy, DQM


## Introduction

Matrix cracking is a long term durability concern of composite structure designers. To nondestructively detect the damage in composite materials, vibration-based methods have been increasingly adopted due to their flexibility in measurement and relatively low cost. The basic idea of these methods is to use the information of modal parameters, such as frequency, mode shape and damping ratio, to access the structural damage.

Cawley and Adams [1] simply used the frequency shifts for different modes to detect the damage in composite structures. Tracy and Pardoen [2] found that the natural frequencies of a composite beam were affected by the size and damage location. Shen and Grady [3] indicated that local delamination does not have a noticeable effect on global mode shape of composite beams, but delamination does cause the irregularity of mode shapes. Zou et al. [4] provided a thorough review in vibration-based techniques and indicated that the above methods were unable to detect very small damage and required large data storage capacity for comparisons. Cornwell et al. [5] utilized the measured mode shapes to calculate the strain energy of a plate-like structure. Fractional strain energy was then used to define a damage index which can locate the damage in structure. The method only requires the mode shapes of the structure before and after damage. Nevertheless, the challenge of the method lies in the accuracy of measured modes. A large amount of data points are required for further analysis to locate the damage. To solve this problem, Hu et al. [6] adopted the DQM to rapidly obtain the accurate solution of strain energy and successfully located damage in a composite laminate plate. It was reported that the original DQM was first used in structural mechanics problems by Bert et al. [7]. This method is able to rapidly compute accurate solutions of partial differential equations by using only a few grid points in the respective solution domains [8].

The objective of this paper is to investigate the damage detection of matrix cracking in composite laminates using experimental modal analysis and strain energy method. Experimental modal analysis was performed to obtain the mode shapes of laminate before and after damaged. The mode shape displacements were then used to calculate the strain energy which was used to define a damage index for locating the damage of matrix cracking.

## Strain energy method and damage index

A plate-like beam as shown in Figure 1 is subdivided into $N_x \times N_y$ sub-region and denoted the location of each point by $(x_i,y_j)$. For laminate plate theory, the strain energy of beam during elastic deformation is given by

$$U=\frac{1}{2}\int_0^b\int_0^a\left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2+2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+4\left(D_{16}\frac{\partial^2 w}{\partial x^2}+D_{26}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x\partial y}+4D_{66}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]dxdy \tag{1}$$

where $w$ is the transverse displacement of the laminate, $D_{ij}$ are the bending stiffnesses of the laminate.

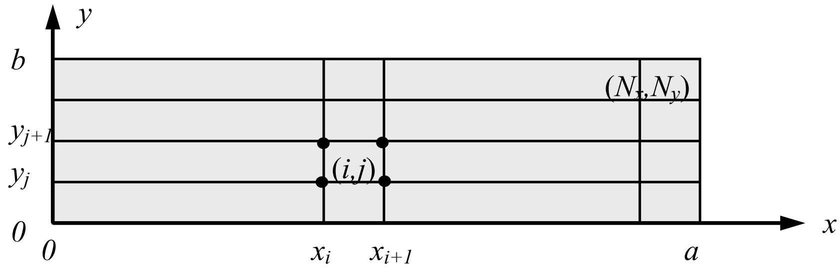


Figure 1 A schematic illustrating of beam

Considering a free-free vibration problem, for a particular normal mode, the total strain energy of the beam associated with the mode shape $\phi_k$ can be expressed as

$$U_k = \frac{1}{2}\int_0^b\int_0^a \left[ D_{11}\left(\frac{\partial^2\phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2\phi_k}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2\phi_k}{\partial x^2}\frac{\partial^2\phi}{\partial y^2} + 4\left(D_{16}\frac{\partial^2\phi_k}{\partial x^2} + D_{26}\frac{\partial^2\phi_k}{\partial y^2}\right)\frac{\partial^2\phi_k}{\partial x\partial y} + 4D_{66}\left(\frac{\partial^2\phi_k}{\partial x\partial y}\right)^2 \right] dxdy \tag{2}$$

Cornwell et al. [5] suggested that if the damage is located at a single sub-region then the change of strain energy in sub-region may become significant. Thus, the energy associated with sub-region ($i,j$) for the $k^{th}$ mode is given by

$$U_{k,ij} = \frac{1}{2}\int_{y_j}^{y_{j+1}}\int_{x_i}^{x_{i+1}} \left[ D_{11}\left(\frac{\partial^2\phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2\phi_k}{\partial y^2}\right)^2 + 2D_{12}\left(\frac{\partial^2\phi_k}{\partial x^2}\right)\left(\frac{\partial^2\phi_k}{\partial y^2}\right) + 4\left(D_{16}\frac{\partial^2\phi_k}{\partial x^2} + D_{26}\frac{\partial^2\phi_k}{\partial y^2}\right)\left(\frac{\partial^2\phi_k}{\partial x\partial y}\right) + 4D_{66}\left(\frac{\partial^2\phi_k}{\partial x\partial y}\right)^2 \right] dxdy \tag{3}$$

Similarly, $U_k^*$ and $U_{k,ij}^*$ represent the total strain energy and sub-regional strain energy of the $k^{th}$ mode shape $\phi_k^*$ for damaged beam. The fractional energies of the beam are defined as

$$F_{k,ij} = \frac{U_{k,ij}}{U_k} \quad \text{and} \quad F_{k,ij}^* = \frac{U_{k,ij}^*}{U_k^*} \tag{4}$$

Considering all modes, $m$, in the calculation, damage index in sub-region ($i,j$) is defined as

$$\beta_{ij} = \frac{\sum_{k=1}^{m} F_{k,ij}^*}{\sum_{k=1}^{m} F_{k,ij}} \tag{5}$$

Equation (5) is used to predict the damage location in composite laminate beam in this study. Since the partial differential terms in strain energy formula are difficult to be calculated, an alternative numerical method, differential quadrature method (DQM) was introduced to solve the problem.

**Differential quadrature method**

The basic idea of the DQM is to approximate the partial derivatives of a function $f(x_i,y_j)$ with respect to a spatial variable at any discrete point as the weighted linear sum of the function values at all the discrete points chosen in the solution domain of spatial variable. This can be expressed mathematically as

$$f_x^{(n)}(x_i, y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} f(x_r, y_j) \tag{7}$$

$$f_y^{(m)}(x_i, y_j) = \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_i, y_s) \tag{8}$$

$$f_{xy}^{(n+m)}(x_i, y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_r, y_s) \tag{9}$$

where i = 1,2,…,$N_x$ and j =1,2,…,$N_y$ are the grid points in the solution domain having $N_x \times N_y$ discrete number of points. $C_{ir}^{(n)}$ and $\overline{C}_{js}^{(m)}$ are the weighting coefficients associated with the $n^{th}$ order and the $m^{th}$ order partial derivatives of $f(x_i, y_j)$ with respect to $x$ and $y$ at the discrete point ($x_i$, $y_j$) and $n$=1, 2,…,$N_x$-1, $m$=1, 2,…,$N_y$-1. The weighting coefficients can be obtained using the following recurrence formulae

$$C_{ir}^{(n)} = n\left( C_{ii}^{(n-1)} C_{ir}^{(1)} - \frac{C_{ir}^{(n-1)}}{x_i - x_r} \right) \quad \text{and} \quad \overline{C}_{js}^{(m)} = n\left( \overline{C}_{jj}^{(m-1)} \overline{C}_{js}^{(1)} - \frac{\overline{C}_{js}^{(n-1)}}{y_j - y_s} \right) \tag{10}$$

where $i,r$=1,2,…$N_x$ but $r \neq i$ ; $n$= 2,3,…,$N_x$-1; also $j,s$ = 1,2,…,$N_y$ but $s \neq j$; $m$= 2,3, …,$N_y$-1. The weighting coefficients when $r=i$ and $s=j$ are given as

$$C_{ii}^{(n)} = -\sum_{r=1, r\neq i}^{N_x} C_{ir}^{(n)} \; ; i = 1,2,...,N_x, \text{ and } n = 1,2,...,N_x - 1 \tag{11}$$

$$\overline{C}_{jj}^{(m)} = -\sum_{s=1, s\neq j}^{N_y} \overline{C}_{js}^{(m)} \; ; j = 1,2,...,N_y, \text{ and } m = 1,2,...,N_y - 1 \tag{12}$$

$$C_{ir}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_r)M^{(1)}(x_r)} \text{ ; } i,r = 1,2,...,N_x, \text{but} r \neq i \tag{13}$$

$$\overline{C}_{js}^{(1)} = \frac{P^{(1)}(y_j)}{(y_j - y_s)P^{(1)}(y_s)} \text{ ; } j,s = 1,2,...N_y, \text{but} j \neq s \tag{14}$$

where $M^{(1)}(x_i) = \prod_{r=1,r\neq i}^{N_x}(x_i - x_r)$ and $P^{(1)}(y_j) = \prod_{s=1,s\neq j}^{N_y}(y_j - y_s)$. The above equations are applied to calculate the strain energy once the $k^{th}$ mode shape $\phi_{k,ij} = f_k(x_i, y_j)$ is obtained from the experimental results.

**Finite element analysis**

A pre-study was performed by establishing finite element model for composite laminate, $[0_2/90_{12}/0_2]$, with dimension 222×24.7×2.3mm$^3$. ANSYS, a FEA commercial code, was used in this study. Eight-node linear solid element (SOLID46) was used in the modeling. The element provides a layered version allow up to 250 different material layers. A convergence study was performed to obtain a 23×2×16 mesh model, which is sufficient to solve the normal mode problem. A breadth-wide matrix crack with 0.1 mm wide was created throughout the 90-degree laminate by separating the nodes at the elements along the crack.

Mechanical properties ($E_1$ = 140.3 GPa, $E_2 = E_3$ = 9.4 GPa, $G_{12} = G_{13}$ = 5.4 GPa, $\nu_{12} = \nu_{13}$ = 0.253) for composite beam were entered into ANSYS. These data were obtained from the quasi-static tensile tests of the composite material, AS4/PEEK. Hu et al. [6] found that the effects of out-of-plane shear modulus $G_{23}$ and Poisson's ratio $\nu_{23}$ on the natural frequencies are not critical in thin plate. Thus, the values of $G_{23}$ and $\nu_{23}$ were assumed to be the same as $G_{12}$ and $\nu_{12}$ in this study. Material density was directly measured from the specimen, i.e., ρ=1583kg/m$^3$. A normal mode analysis with completely free boundary condition was performed to obtain the natural frequencies and the associated mode shapes up to 5 kHz. Hu et al. [6] found that mass effect of accelerometer to the natural frequencies of specimen is significant. Thus, a mass element (MASS21) with 0.0015 kg was assigned to fix at the FE model.

**Experimental modal analysis**

Carbon/epoxy composite prepreg AS4/PEEK was used to stack up a laminate, $[0_2/90_{12}/0_2]$ and then cured at a hot-press machine. After curing, the panel was cut to a specimen with dimension $222\times24.7\times2.3$ mm$^3$ and marked with 24×3 parallel grid points. The test specimen was vertically hung by two cotton strings to simulate a completely free boundary condition as shown in Figure 2. Specimen was excited by an impact hammer with a force transducer throughout all grid points. Dynamic responses were measured by an accelerometer fixed at the corner. Siglab, Model 20-40, was used to record the frequency response functions (FRFs) between measured acceleration and impact force. ME'Scope, a software for general purpose curve fitting, was used to extract modal parameters, i.e., natural frequencies, damping ratios and mode shapes, from the FRFs.

Modal testing was conducted on test specimen before damage. After test, specimen was subjected to tensile test to create matrix crack in 90-degree but not in 0-degree laminate. To achieve this, a tiny pre-crack was created at both sides of the grip point 13. Loading was stopped once the loading curve suddenly dropped accompanied a harsh noise. The location of matrix crack can be detected using a X-ray machine, Eresco 200MF, with output voltage 30 KV and exposure time 30 sec.

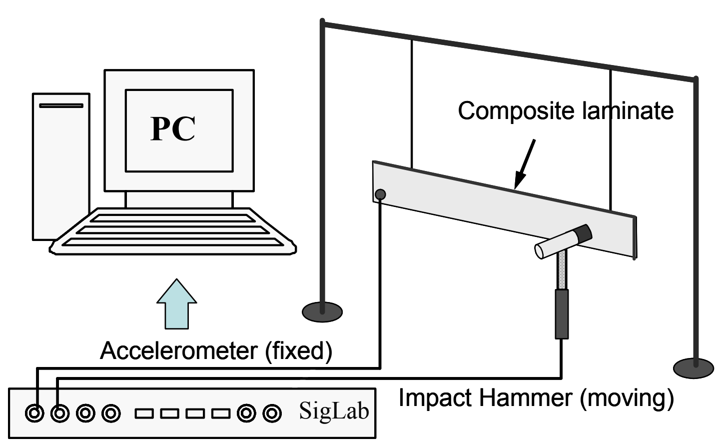


Figure 2 Experimental Set-up

**Results**

A pre-study of finite element analysis was performed to evaluate this approach. The first five mode shapes obtained from normal mode analysis were used to compute the strain energy and damage index of the laminate specimen. Figure 3 show the damage index of laminate for putting a breadth-wide matrix crack at 90-degree laminate near grid point 13. The peak values clearly locate the matrix crack. Similarly, Figures 4 and 5 show the predicting results of two and three matrix cracks in the specimens, respectively. These encouraging outcomes lead to the following experimental results.

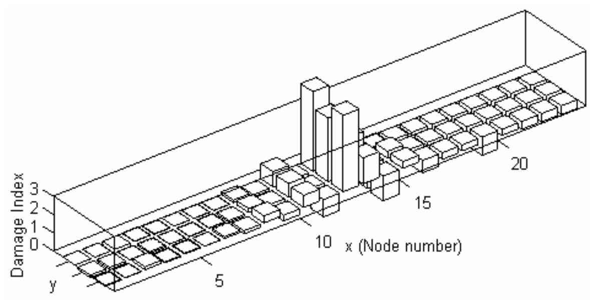


Figure 3 Damage index of laminate for a crack at grid point 13 (FEA)

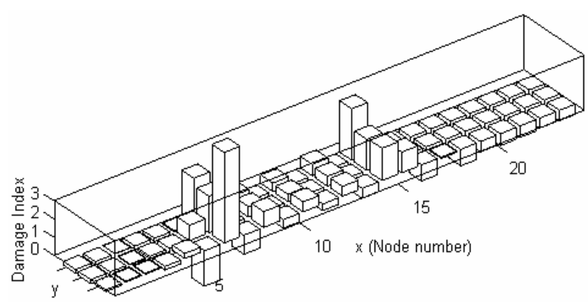


Figure 4 Damage index of laminate for cracks at grid points 6 and 15 (FEA)

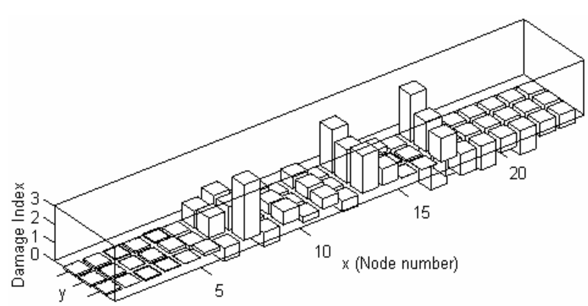


Figure 5 Damage index of laminate for cracks at grid points 7, 14 and 18 (FEA)

In EMA results, the first five natural frequencies of the laminate specimen are listed in Table 1. The test results indicate that matrix cracking significantly decreases the natural frequencies of the test specimen except for mode (4,1). To predict the location of matrix crack, the first five mode shapes were used to compute the strain energy and damage index for the specimen. Figure 6 shows the damage index using EMA results. The peak values clearly locate the matrix crack; however, many peak values also emerged at some other undamaged areas. The deviation in measurement may attribute to these pseudomorphs. Cornwell et al. [5] suggested that damage indices with values greater than two are associated with potential damage locations. Thus, we tried to truncate the peaks of damage index less than two. The improve outcome is obtained in Figure 7. The prediction of matrix crack location was verified by using a X-ray machine as shown in Figure 8. The picture shows that matrix crack is located at grip point 13 which is identical to the prediction.

Table 1 Natural frequencies of the test specimen (EMA)

| | Natural Frequency (Hz) | | |
|---|---|---|---|
| Mode shape | Before damage | After damage | △(%) |
| (3,1) | 302 | 295 | -2.3 |
| (2,2) | 670 | 661 | -1.3 |
| (4,1) | 856 | 861 | 0.6 |
| (3,2) | 1405 | 1397 | -0.6 |
| (5,1) | 1681 | 1659 | -1.3 |

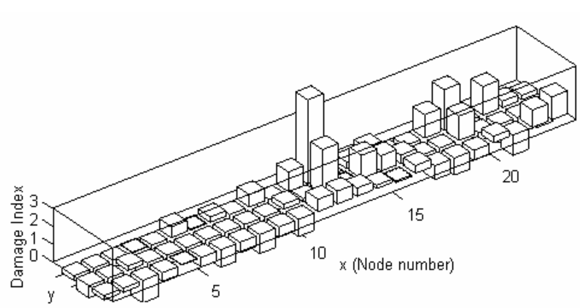


Figure 6 Damage index of laminate for a crack at grid point 13 before truncation (EMA)

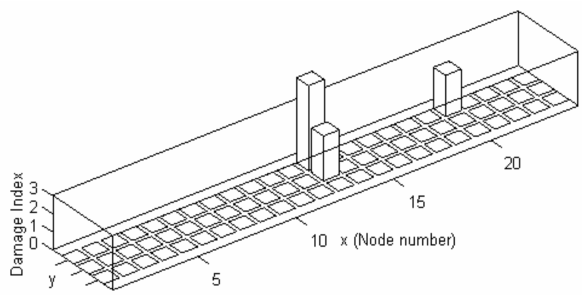


Figure 7 Damage index of laminate for a crack at grid point 13 after truncation (EMA)

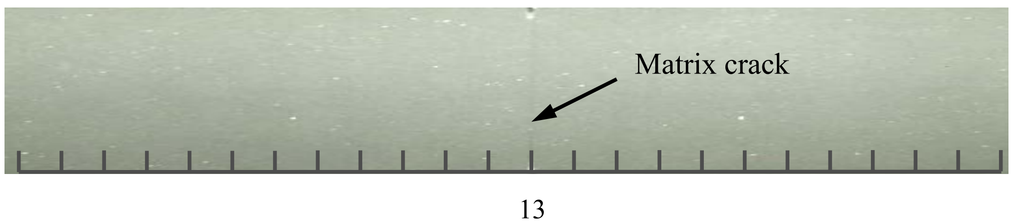


Figure 8 Matrix crack at test specimen (X-ray picture)

**Conclusions**

Damage index using modal analysis and strain energy methods was developed to detect a matrix crack in composite laminate specimen in this paper. This method only requires a few mode shapes of the laminate specimen before and after damage. In fact, the changes of mode shapes before and after damage are almost invisible. However, the irregularities of mode shapes due to matrix cracking become significant from the perspective of strain energy approach. Both FEA and experimental results show that matrix cracks were successfully located by using damage indices. Since the number of measured points was limited, DQM provides us an accurate approach to compute strain energy by using only a few grid points in the test specimen. This nondestructive method provides a reliable, cost-effective approach for damage detection in the utilization of composite structures. Further research interests lie in the application of this method to various composite laminates.

## Acknowledgment

The authors would like to acknowledge the support of Taiwan National Science Council through grant No. NSC-93-2212-E-020-010.

## References

[1] Cawley, P., and Adams, R. D., "A Vibration Technique for Non-destructive Testing of Fiber Composite Structures," *Journal of Composite Materials*, Vol. 13, pp. 161-175, (1979).

[2] Tracy, J. J., and Pardoen, G. C., "Effect of Delamination on the Natural Frequencies of Composite Laminates," *Journal of Composite Materials*, Vol. 23, pp. 1200-1215, (1989).

[3] Shen, M. H. H., and Grady, J. E., "Free Vibration of Delaminated Beams," *AIAA Journal*, Vol. 30, pp. 1361-1370, (1992).

[4] Zou, Y., Tong, L., and Steven, G.P., "Vibration-based Model-dependent Damage (Delamination) Identification and Health Monitoring for Composite Structures－A Review," *Journal of Sound and Vibration*, Vol. 2, pp.357-378, (2000).

[5] Cornwell, P. J., Dobeling, S. W., and Farrar, C. R., "Application of the Strain Energy Damage Detection Method to Plate-like Structures," *Proceedings of the International Model Analysis Conference-IMAC*, pp. 1312-1318, (1997).

[6] Hu, H., Wang, B. T., and Su, J. S., "Application of Modal Analysis to Damage Detection in Composite Laminates," *The 7$^{th}$ ASME Biennial Conference on Engineering System Design and Analysis*, No. 58296, (2004).

[7] Bert, C. W., Jang, S. K., and Striz, A. G., "Two New Approximate Methods for Analyzing Free Vibration of Structural Components," *AIAA Journal*, Vol 26, pp. 612-618, (1988).

[8] Bellman, R. E., Kashef, B. G., and Casti, J., "Differential Quadrature: A Technique for the Rapid Solution of Nonlinear Partial Differential Equation," *Journal of Computational Physics*, Vol 10, pp. 40-52, (1972).