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APPLICATION OF MODAL ANALYSIS TO DAMAGE DETECTION IN COMPOSITE LAMINATES

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ABSTRACT

A nondestructive detection of damage in composite laminates by using modal analysis is investigated in this paper. Continued fiber-reinforced composite AS4/PEEK was used to fabricate a symmetrical laminate plate and a surface crack was created in one side of the laminate plate. The results of modal testing are presented for the application of modal analysis to the laminate plate before and after damage. Changes in mode shapes, mode shape slopes and strain energies were used to calculate the damage index for indicating the damage location. Differential quadrature method (DQM) was introduced to solve the problem of partial derivatives function in strain energy formula. A 3-D finite element model was created for comparison with the experimental results. The model accurately predicted the dynamic responses. It was found that damage index using strain energy method provides a more promising result than other methods in locating the damage.

Keywords: modal analysis, composite laminate, surface crack, strain energy, DQM

INTRODUCTION

Many nondestructive testing techniques have been well developed to detect the damage in composite materials, such as scanning acoustic microscopy, radiography and ultrasonics. However, these techniques are time-consuming, costly, and impractical for large components and structures. Thus, the methods of modal analysis have been increasingly adopted in the detection of damage in composite materials due to their flexibility of measurement and relatively low cost. The basic idea of these methods is to use the information of modal parameters, such as frequency, mode shape and damping ratio, to access the structural damage.

Cawley and Adams [1,2] simply used the frequency shifts for different modes to detect the damage in composite structures. Tracy and Pardoen [3] found that the natural frequencies of a composite beam were affected by the size and location of delamination damage. Shen and Grady [4] found that local delamination does not have a noticeable effect on global mode shape of vibration of composite beams, but delamination does cause the irregularity of mode shape. Pandey et al. [5] also show that the irregular of mode shape is significant for relatively large damage. However, Saravanos and Hopkins [6] indicated that small delamination may not be detectable by monitoring global modal characteristic of the composite beam. Ratcliffe and Bagaria [7] converted the displacement eigenvector to curvature mode shape and extracted the reductions in stiffness from the curvature. The advantage of this approach is that damage location can be predicted without a baseline of the undamaged structure. Zou et al. [8] provided a thorough review in vibration-based techniques and indicated that the above methods were unable to detect very small damage and require large data storage capacity for comparisons.

Cornwell et al. [9] utilized the measured mode shapes to calculate the strain energy of a plate-like structure, and established a damage index to locate the damage in the structure. The method only requires the mode shapes of the structure before and after damage and the modes do not need to be mass normalized making it very advantageous when using ambient excitation. Nevertheless, the challenge of the method lies in the accuracy of measured modes. Large amount of data points are required for further analysis to predict the damage location. To solve this problem, Wang and Tsao [10] adopted the differential quadrature method (DQM) to rapidly obtain the accurate solution of strain energy and successfully predicted the edge-crack location in a steel plate. It was reported that the original DQM was first used in structural mechanics problems by Bert et al. [11], and it was able to rapidly compute accurate solutions of partial differential equations by using only a few grid points in the respective solution domains.

The objective of this study is to investigate the application of modal analysis method to detect the surface crack damages in composite laminates. The changes of mode shape, mode shape slope and strain energy obtained from the experiments were used to define the damage indices, which were used to locate the surface crack damage. DQM was employed to obtain the modal quantities using the measured mode shapes. This method provides effective solution by using only few grid data point of experiment. Finite element analysis was also performed for comparison with the experimental results.

DAMAGE INDEX BY USING MODAL ANALYSIS

Three approaches to define damage index were investigated in this study. The mode shapes of composite laminate obtained from the modal analysis were adopted to calculate the damage index. Considering a laminate as shown in Figure 1, we divided the laminate into $N_x \times N_y$ sub-region and denoted the location of each point by (x_i, y_j). The first approach is to examine the mode shape change of laminate before and after damage. The difference of the k^{th} mode shape in location (x_i, y_j) is given by

$$d_{k,ij} = \phi_{k,ij} - \phi^*_{k,ij} \tag{1}$$

where $\phi_{k,ij}$ and $\phi^*_{k,ij}$ refer to the transverse displacement of the k^{th} mode shapes in the location (x_i, y_j) of the laminate before and after damage. $\phi_{k,ij}$ and $\phi^*_{k,ij}$ can be obtained from either analytical or experimental data. Thus, the damage index in location (x_i, y_j) is defined by the summation of all the measured mode shape differences, i.e.,

$$D_{d,ij} = \sum_{k=1}^{m} d_{k,ij} \tag{2}$$

The second approach is to examine mode shape slope change of laminate before and after damage. The differences of the k^{th} mode shape slope in x direction and y direction in location (x_i, y_j) are given by

$$(\theta_{k,ij})_x = \frac{\partial d_{k,ij}}{\partial x} \tag{3}$$

$$(\theta_{k,ij})_y = \frac{\partial d_{k,ij}}{\partial y} \tag{4}$$

Thus, the damage index in location (x_i, y_j) is defined by the summation of all the measured mode shape slope differences, i.e.,

$$D_{\theta,ij} = \sum_{k=1}^{m} \left[(\theta_{k,ij})_x + (\theta_{k,ij})_y \right] \tag{5}$$

The third approach is to examine strain energy change of laminate before and after damage. Considering a symmetrical composite laminate, we have the strain energy during elastic deformation as

$$U = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4\left(D_{16}\frac{\partial^2 w}{\partial x^2} + D_{26}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x \partial y} + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right] dxdy \tag{6}$$

where w is the transverse displacement of the laminate, D_{ij} are the bending stiffnesses of the laminate. For a particular mode shape ϕ_k , the energy associated with the mode shape is expressed as

$$U_k = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 \phi_k}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2 \phi_k}{\partial x^2}\frac{\partial^2 \phi_k}{\partial y^2} + 4\left(D_{16}\frac{\partial^2 \phi_k}{\partial x^2} + D_{26}\frac{\partial^2 \phi_k}{\partial y^2}\right)\frac{\partial^2 \phi_k}{\partial x \partial y} + 4D_{66}\left(\frac{\partial^2 \phi_k}{\partial x \partial y}\right)^2 \right] dxdy \tag{7}$$

Cornwell *et al*. [9] suggested that if the damage is located at a single sub-region then change in the bending stiffness of the sub-region can be used to indicate the damage location. Thus, the energy associated with sub-region (i,j) for the k^{th} mode is given by

$$U_{k,ij} = \frac{1}{2}\int_{y_j}^{y_{j+1}}\int_{x_i}^{x_{i+1}} \left[D_{11}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 \phi_k}{\partial y^2}\right)^2 + 2D_{12}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)\left(\frac{\partial^2 \phi_k}{\partial y^2}\right) + 4\left(D_{16}\frac{\partial^2 \phi_k}{\partial x^2} + D_{26}\frac{\partial^2 \phi_k}{\partial y^2}\right)\frac{\partial^2 \phi_k}{\partial x \partial y} + 4D_{66}\left(\frac{\partial^2 \phi_k}{\partial x \partial y}\right)^2 \right] dxdy \tag{8}$$

A fractional energy is defined as

$$F_{k,ij} = \frac{U_{k,ij}}{U_k} \tag{9}$$

and, we have

$$\sum_{j=1}^{N_y}\sum_{i=1}^{N_x} F_{k,ij} = 1 \tag{10}$$

Similarly, the quantities U^*_k and $U^*_{k,ij}$ represent the total strain energy and the sub-regional strain energy of the k^{th} mode shapes ϕ^*_k for damaged laminate. Thus, a fractional energy of damage laminate is given by

$$F^*_{k,ij} = \frac{U^*_{k,ij}}{U^*_k} \tag{11}$$

Certainly, we have

$$\sum_{j=1}^{N_y}\sum_{i=1}^{N_x} F^*_{k,ij} = 1 \tag{12}$$

Considering all the measured modes, m, in the calculation, we defined the damage index of sub-region (i,j) as

$$D_{S,ij} = \frac{\sum_{k=1}^{m} F^*_{k,ij}}{\sum_{k=1}^{m} F_{k,ij}} \tag{13}$$

It is noted that the partial differential terms in strain energy formula are difficult to be calculated. An alternative numerical method, differential quadrature method (DQM) was therefore introduced to solve problems in structural mechanics field.

DIFFERENTIAL QUADRATURE METHOD

The basic idea of the DQM is to approximate the partial derivatives of a function $f(x_i, y_j)$ with respect to a spatial variable at any discrete point as the weighted linear sum of the function values at all the discrete points chosen in the solution domain of spatial variable [12]. This can be expressed mathematically as

$$f_x^{(n)}(x_i, y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} f(x_r, y_j) \tag{14}$$

$$f_y^{(m)}(x_i, y_j) = \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_i, y_s) \tag{15}$$

$$f_{xy}^{(n+m)}(x_i, y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_r, y_s) \tag{16}$$

where $i = 1,2,\ldots,N_x$ and $j = 1,2,\ldots,N_y$ are the grid points in the solution domain having $N_x \times N_y$ discrete number of points. $C_{ir}^{(n)}$ and $\overline{C}_{js}^{(m)}$ are the weighting coefficients associated with the n^{th} order and the m^{th} order partial derivatives of $f(x_i, y_j)$ with respect to x and y at the discrete point (x_i, y_j) and $n=1, 2,\ldots,N_x-1$, $m=1, 2,\ldots,N_y-1$. The weighting coefficients can be obtained using the following recurrence formulae

$$C_{ir}^{(n)} = n\left(C_{ii}^{(n-1)} C_{ir}^{(1)} - \frac{C_{ir}^{(n-1)}}{x_i - x_r} \right) \tag{17}$$

$$\overline{C}_{js}^{(m)} = n\left(\overline{C}_{jj}^{(m-1)} \overline{C}_{js}^{(1)} - \frac{\overline{C}_{js}^{(n-1)}}{y_j - y_s} \right) \tag{18}$$

where $i,r=1,2,\ldots Nx$ but $r \neq i$; $n= 2,3,\ldots,Nx-1$; also $j,s = 1,2,\ldots,N_y$ but $s \neq j$; $m= 2,3, \ldots,N_y-1$. The weighting coefficients when $r=i$ and $s=j$ are given as

$$C_{ii}^{(n)} = -\sum_{r=1, r\neq i}^{N_x} C_{ir}^{(n)}\,; \quad i = 1,2,\ldots,N_x, \ \text{and} \ \ n = 1,2,\ldots,N_x - 1 \tag{19}$$

$$\overline{C}_{jj}^{(m)} = -\sum_{s=1, s\neq j}^{N_y} \overline{C}_{js}^{(m)}\,; \quad j = 1,2,\ldots,N_y, \ \text{and} \ \ m = 1,2,\ldots,N_y - 1 \tag{20}$$

$$C_{ir}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_r) M^{(1)}(x_r)}\,; \quad i,r = 1,2,\ldots,N_x, \text{but } r \neq i \tag{21}$$

$$\overline{C}_{js}^{(1)} = \frac{P^{(1)}(y_j)}{(y_j - y_s) P^{(1)}(y_s)}\,; \quad j,s = 1,2,\ldots,N_y, \text{but } j \neq s \tag{22}$$

For equations (21) and (22), $M^{(1)}$ and $P^{(1)}$ are denoted by the following expressions

$$M^{(1)}(x_i) = \prod_{r=1, r\neq i}^{N_x} (x_i - x_r) \tag{23}$$

$$P^{(1)}(y_j) = \prod_{s=1, s\neq j}^{N_y} (y_j - y_s) \tag{24}$$

The above equations are applied to calculate the strain energy once the k^{th} mode shape $\phi_{k,ij} = f(x_i, y_j)$ is obtained from the experiment.

EXPERIMENT

Carbon/epoxy composite AS4/PEEK was used for this study. A sixteen-ply composite panel with 0-degree fiber orientation, $[0]_{16}$, was stacked and cured in a hot-press machine. After curing, the panel was cut to a 209×126×2.3 mm^3 laminate using a diamond saw.

Modal testing was conducted to obtain the dynamic response. A laminate plate was marked to 13×13 parallel grid points, and vertically hanged by two cotton strings to simulate a free-free condition. The laminate plate was excited throughout all grid points by using an impulse hammer with a force transducer, and the dynamic responses were measured by an accelerometer fixing to the corner. The test setup is shown in Figure 2.

Siglab, Model 20-40, was used to record the frequency response functions (FRFs) between measured acceleration and impact force. ME'Scope, a software for the general purpose curve fitting, was used to extract modal parameters, i.e., natural frequencies, damping ratios and mode shapes from the FRFs. In particular, the transverse displacements of mode shape of laminate before and after damage were determined, respectively.

Two types of damage, surface crack and penetrated crack, were created in the undamaged laminate plate. Firstly, a 35 mm long and 1 mm deep surface crack perpendicular to the fiber direction was created by using a laser cutting machine. Secondly, a penetrated crack at the same location was created by cutting the surface crack to pass through the laminate. Consequently, modal parameters of the laminate before and after damage were obtained from the same plate for the further analysis.

FINITE ELEMENT MODEL

A finite element analysis (FEA) was performed to determine the modal parameters of the composite laminate. ANSYS, a FEA software was used for this study. Eight-node solid elements were adopted to model the 209×126×2.3 mm^3, 0-degree and sixteen-ply laminate plate [13]. A convergence study was performed to obtain a 40×30×8 mesh, which was optimal to solve the normal mode problem. To simulate a 35 mm long and 1 mm deep surface crack, we replaced the nodes at location of crack by separate nodes as shown in Figure 3.

The mechanical properties of composite material (E_1=140.3GPa, E_2=E_3=9.4GPa, G_{12}=G_{13}=5.4GPa, ν_{12}=ν_{13}=0.253, ρ=1485kg/m^3) were obtained from the quasi-static tensile tests, and then were entered into ANSYS. A normal mode analysis was performed to obtain the natural frequencies up to 1 KHz and their corresponding mode shapes. Since the finite element model is to simulate the real case in experiment, the mass effect of accelerometer should be taken into consideration. A mass element with 0.002 *kg* was assigned to the laminate plate model.

RESULTS

The first six natural frequencies and mode shapes of the laminate plate before and after damage obtained are shown in Table 1 and Table 2. Damage^1 refers to the surface crack and damage^2 refers to the penetrated crack. Apparently, the damage of surface crack decreases the natural frequency of laminate plate. It is reasonable that laminate plate will lose some degree of bending stiffness due to the damage.

The first six measured modes were used to calculate the damage index in this study. In surface crack damage, Figures 4, 5 and 6 show the damage indices obtained from finite element analysis using the changes of mode shape, mode shape slope and strain energy, respectively. Damage index using mode shape changes slightly indicates the location of surface crack as shown in Figure 4. However, in Figure 5, damage index using mode shape slope changes seem not clear and hide in many pseudo morphs. Damage index using strain energy changes as shown in Figure 6 clearly indicate the surface crack location. Moreover, damage indices become clearer when crack was to penetrate the laminate plate as shown in Figures 7, 8 and 9.

In experimental results, Figures 10-15, damage indices obtained from modal testing using mode shape changes and mode shape slope changes, are unable to indicate the location of surface crack and even penetrated crack. Once again, damage indices using strain energy changes successfully locate the surface crack and penetrated crack as shown in Figures 12 and 15.

CONCLUSIONS

Damage index using modal analysis methods to detect a surface crack in composite laminate is investigated in this paper. This method only requires a few mode shapes of the structure before and after damage. The changes in mode shapes, mode shape slopes and strain energies were used to calculate the damage index. Results from both finite element analysis and modal testing show that damage indices using the strain energy changes of laminate plate before and after damage successfully indicated the damage locations. The application of DQM to solve the partial derivatives function provides us an accurate approach to calculate strain energy by using only a few grid points in laminate plate. Further research interests lie in the implementation of sensitivity of results using various sensors. Future work will focus on a similar study for various types of damage.

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Table 1 Natural frequency of the laminate plate before and after damage (by Finite Element Analysis)

		Natural Frequency (Hz)				
Mode No	Mode shape	Before damage	After damage[1]	After damage[2]	$\triangle_1$ (%)	$\triangle_2$ (%)
1	(2,2)	156.5	156.4	156.4	-0.06	-0.06
2	(1,3)	342.1	342.3	342.4	0.06	0.09
3	(2,3)	435.1	434.6	431.2	-0.12	-0.9
4	(3,1)	501.7	495.5	466.6	-1.24	-7
5	(3,2)	578.3	565.4	545.6	-2.23	-5.65
6	(3,3)	831.9	827.8	822	-0.49	-1.19

Table 2 Natural frequency of the laminate plate before and after damage (by modal testing)

		Natural Frequency (Hz)				
Mode No	Mode shape	Before damage	After damage[1]	After damage[2]	$\triangle_1$ (%)	$\triangle_2$ (%)
1	(2,2)	180.6	178.8	178.8	-1.0	-1.0
2	(1,3)	349.4	348.8	348.8	-0.2	-0.2
3	(2,3)	439.4	425.6	402.5	-3.1	-8.4
4	(3,1)	481.3	474.4	471.3	-1.4	-2.1
5	(3,2)	548.8	528.1	515	-3.8	-6.2
6	(3,3)	833.8	830	830.6	-0.5	-0.4

Note: damage[1] refers to the surface crack of 1 mm deep
damage[2] refers to the penetrated crack

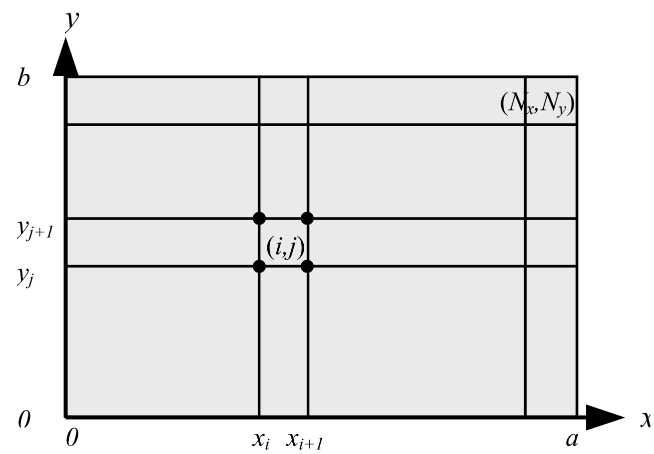

Figure 1 A schematic illustrating a plate

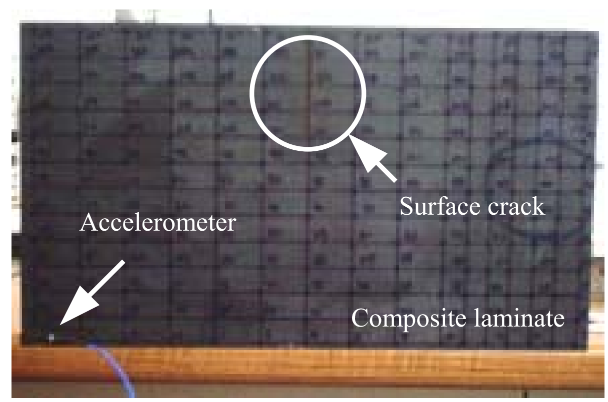

Figure 2 Model testing set-up

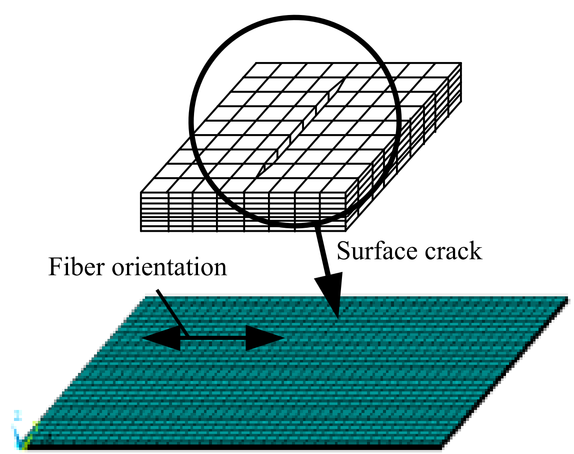

Figure 3 Finite element model of the damaged laminate

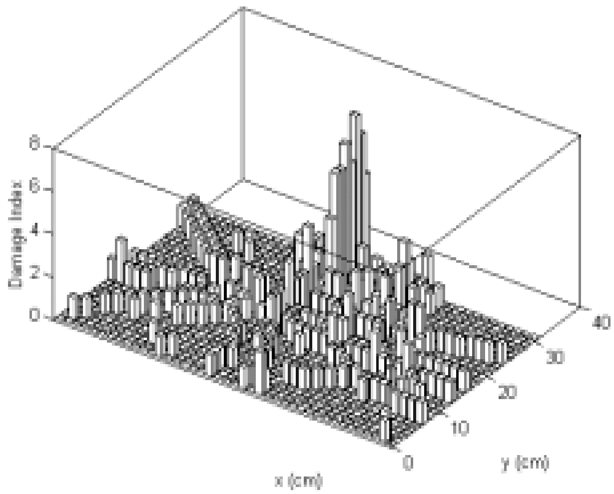

Figure 4 Damage index using mode shape change (FEA for 1 mm deep surface crack)

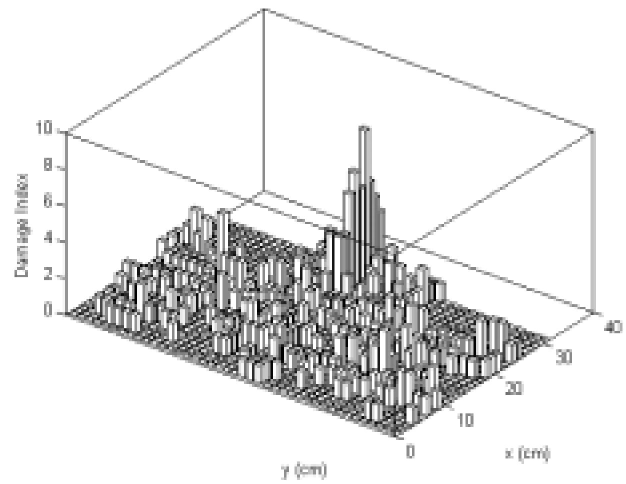

Figure 7 Damage index using mode shape change (FEA for penetrated crack)

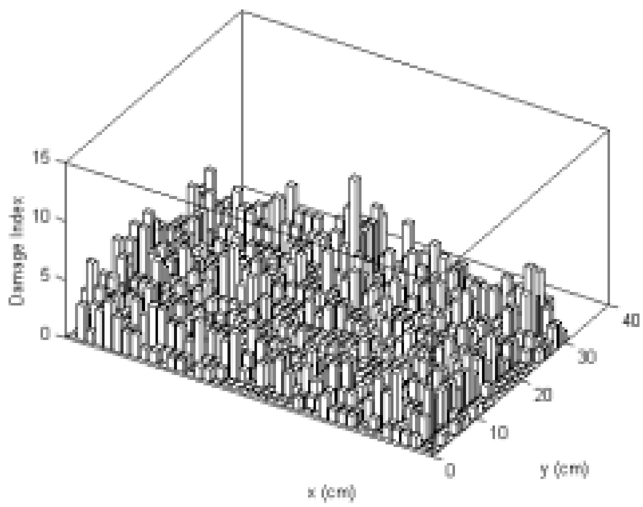

Figure 5 Damage index using mode shape slope change (FEA for 1 mm deep surface crack)

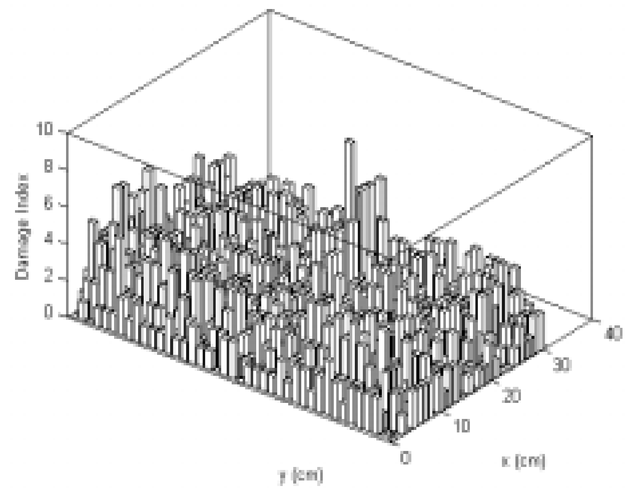

Figure 8 Damage index using mode shape slope change (FEA for penetrated crack)

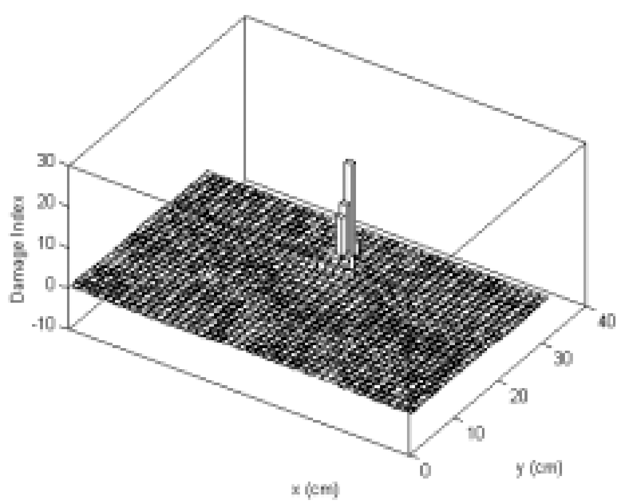

Figure 6 Damage index using strain energy change (FEA for 1 mm deep surface crack)

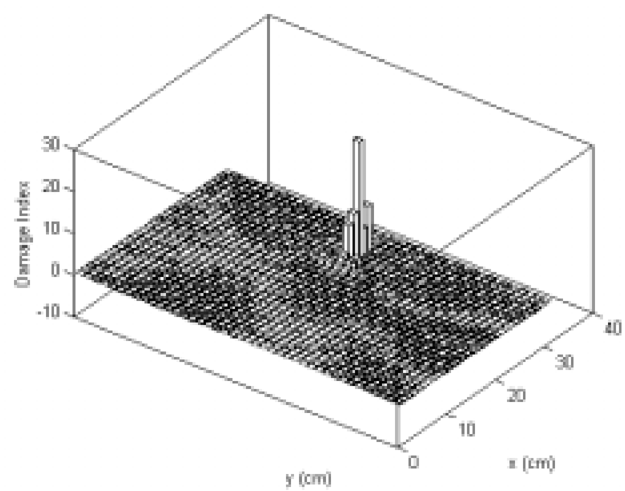

Figure 9 Damage index using strain energy change (FEA for penetrated crack)

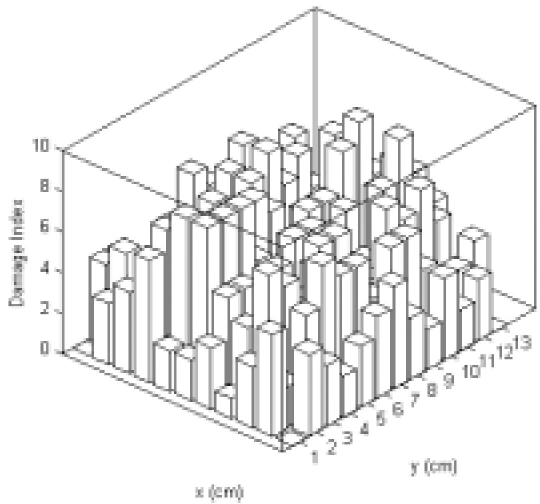

Figure 10 Damage index using mode shape change
(Experiment for 1 mm deep surface crack)

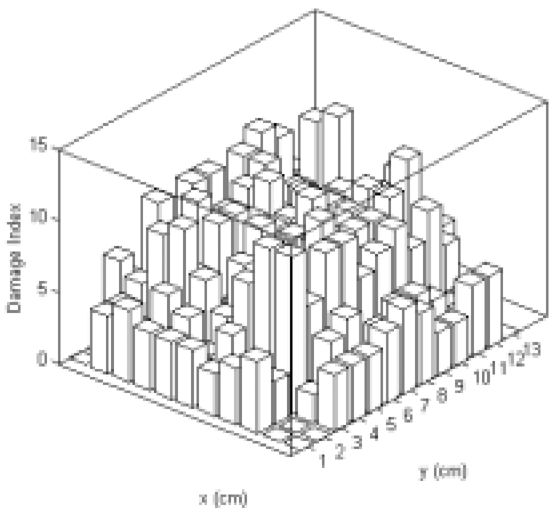

Figure 11 Damage index using mode shape slope change
(Experiment for 1 mm deep surface crack)

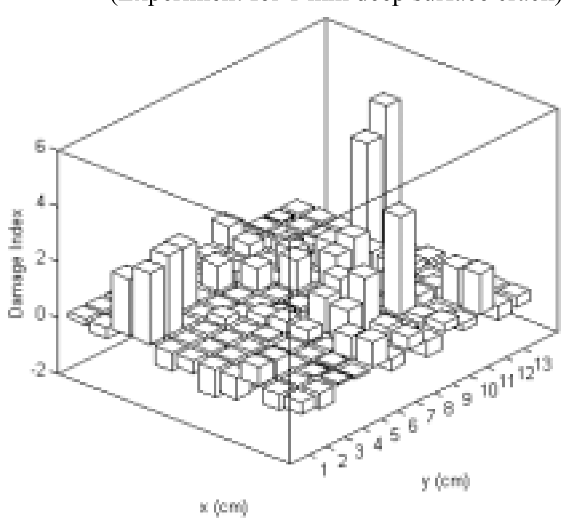

Figure 12 Damage index using strain energy change
(Experiment for 1 mm deep surface crack)

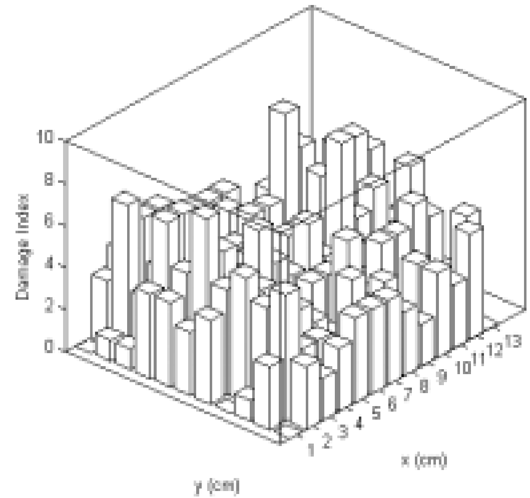

Figure 13 Damage index using mode shape change
(Experiment for penetrated crack)

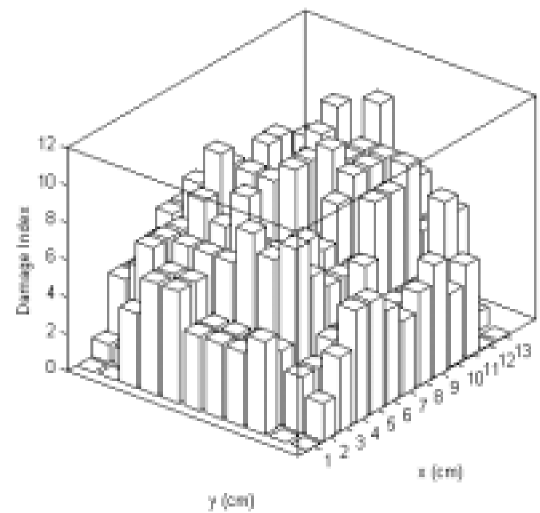

Figure 14 Damage index using mode shape slope change
(Experiment penetrated crack)

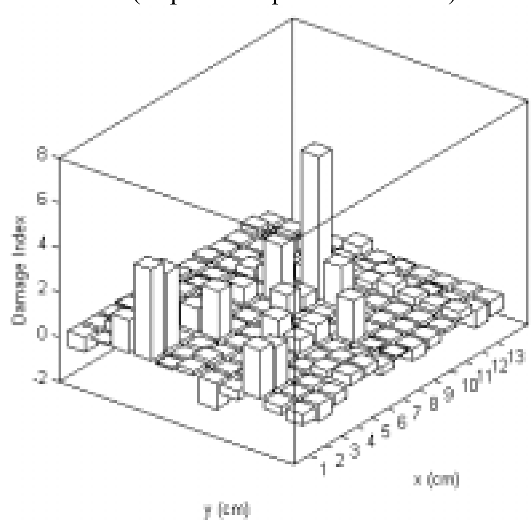

Figure 15 Damage index using strain energy change
(Experiment for penetrated crack)