Determination of Mode Shapes from ODS for Beam Structures

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ABSTRACT

Structural modal parameters, including natural frequencies, modal damping ratios and mode shapes, can be obtained via conventional modal testing methods. The major restriction is that the structure must be in static. For the structure subjected to a known harmonic force acting at the known location, the steady state response of the structure can also be harmonic. The operational deflection shape (ODS) of the structure can then be measured in its operating condition. The ODS is known as functions of modal parameters. In particular, natural frequencies and modal damping ratios can be easily obtained from single measurement of structural response. Therefore, this work presents the predictive algorithm to obtain the mode shapes from the ODS for simply supported beam subjected to harmonic excitation. The optimization problem to predict the structural mode shapes will be formulated. The objective function to be minimized is defined as the least square errors between the measured and predicted ODS. The corresponding mode shape components that are used to curve-fit the mode shapes are defined as the design variables. An n-th order polynomial function is adopted to fit the mode shape vector. With the resolution of the optimization problem, the structural mode shapes can then be determined. Numerical examples for an ideal simply supported beam subjected to a harmonic point force are shown to demonstrate the prediction of mode shapes via only one set of ODS available. Results show that the developed predicted algorithm to identify structural mode shapes is feasible and quite efficient in comparison to conventional modal testing method. The developed methodology can be applied to structural modal testing for structures in harmonic operating conditions.

NOMENCLATURE

A_b	beam cross-sectional area
$a_{r,q}$	coefficient of polynomial fitting function
C_b	beam damping coefficient
E_b	beam Young' s Modulus
F_j	amplitude of the *j*-th harmonic force
$F(x,t)$	excitation force function
$\{F(w)\}$	force vector
$H_{ij}(w)$	frequency response function between the *i*-th displacement and the *j*-th force
$[H(w)]$	frequency response function martix
I_b	beam cross-sectional area moment of inertia
L_b	beam length
MAC	modal assurance criterion
m	number of measurement point of ODS and also the number of mode shape components
N	order of the polynomial function
n	number of modes to be determined
p	number of interpolation points for polynomial fit
S_r	sum of the absolute value of mode shape components for *r*-th mode shape
S_{dr}	sum of the absolute value of derivative of *r*-th mode shape component
$w(x,t)$	beam lateral displacement
$X_i(w_s)$	*i*-th component of operational deflection shape for harmonic excitation
$\|X_i(w_s)\|$	absolute value of operational deflection shape for harmonic excitation
$\{X(w)\}$	displacement response vector
$\{X(w_s)\}$	operational deflection shape for harmonic excitation
$\{\hat{X}(w_s)\}$	experimental operational deflection shape for harmonic excitation
$y_{r,s}$	design variable of *s*-th component of the *r*-th mode shape
x_r	*r*-th viscous damping ratio
r_b	beam density
Φ	objective function
$f_{r,j}$	*j*-th component of the *r*-th normalized modal vector
$f_r(x)$	*r*-th mode shape function
$\{f\}_r$	*r*-th normalized modal vector
w_r	*r*-th undamped natural frequency
w_s	excitation frequency

1. INTRODUCTION

In performing conventional experimental modal analysis

(EMA) or testing, structures are generally required to be in static. With the use of actuators, such as shakers or impact hammers, to excite structure and the use of sensors, such as accelerometers, to measure structure response, the frequency response function (FRF) can then be measured. At least a row or a column of FRF matrix should be obtained so that the general curve-fitting process can be applied to extract structural modal parameters [1]. If the test structure is in its operating condition, such as a rotor system in rotating condition, the conventional EMA technique can not be applied directly.

For the restriction of conventional modal testing, the structural modal identification in operational condition arises many interests. James *et al.* [2] proposed the natural excitation technique (NexT) to extract modal parameters from operating structures. Hermans and Auweraer [3] developed the modal identification method by using output-only response and discussed its use for real structures, such as bridges, aircrafts and vehicles. Hermans and Hermans [4] also introduced structural model identification during normal operating conditions.

The ODS is not only useful for machinery diagnosis [5] or damage detection [6] but also applicable to identify excitation force [7]. Wang [8] had a first attempt to extract mode shapes from ODS for MDOF systems. Hu [9] further modified the prediction model by adding realistic constraints and had experimentally validated the modal data for MDOF systems. This paper will adopt the basic idea from Wang [8] and Hu [9] to develop the mode shape prediction model from ODS for beam structures.

This work assumes the beam structure subjected to a known harmonic force excitation. The beam will also appear harmonic response. The ODS of the beam can then be measured and as the input data to the prediction model. The developed mode shape prediction model can identify the structural mode shapes. This work thus enhances the modal parameter extraction technique for structure under harmonic operating condition and can be extended to operational rotor system for extracting system mode shapes.

2. THEORETICAL ANALYSIS

2.1 Modal analysis

Consider a beam neglecting shear deformation and rotary inertia. The equation of motion for lateral vibration can be expressed as follows:

$$E_b I_b \frac{\partial^4 w(x,t)}{\partial x^4} + C_b \frac{\partial w(x,t)}{\partial t} + \rho_b A_b \frac{\partial^2 w(x,t)}{\partial t^2} = F(x,t) \quad (1)$$

For the simply supported beam as shown in Fig. 1, the boundary condition can be shown:

$$w(0,t) = 0 \quad , \quad E_b I_b \left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=0} = 0$$

$$w(L_b,t) = 0 \quad , \quad E_b I_b \left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=L_b} = 0 \quad (2)$$

Natural frequencies of the simply supported beam can be obtained as follows [10]:

$$\omega_r = (\alpha_r L_b)^2 \sqrt{\frac{E_b I_b}{\rho_b A_b L_b^4}} = (\alpha_r)^2 \sqrt{\frac{E_b I_b}{\rho_b A_b}} \quad (3)$$

where

$$\alpha_r L_b = r\pi \ , \ r = 1,2,\ldots,n$$

The normalized mode shapes can be expressed:

$$\phi_r(x) = \sqrt{\frac{2}{\rho_b A_b L_b}} (\sin \alpha_r x) \quad (4)$$

The orthonormality relations of mode shapes can be derived as follows [11]:

$$\int_0^{L_b} E_b I_b \phi_s(x) \frac{d^4 \phi_r(x)}{dx^4} dx = \begin{cases} \omega_r^2 \ , s = r \\ 0 \quad , s \neq r \end{cases} \quad (5)$$

$$\int_0^{L_b} C_b \phi_s(x) \phi_r(x) dx = \begin{cases} 2\xi_r \omega_r \ , s = r \\ 0 \qquad , s \neq r \end{cases} \quad (6)$$

$$\int_0^{L_b} \rho_b A_b \phi_s(x) \phi_r(x) dx = \begin{cases} 1 , s = r \\ 0 , s \neq r \end{cases} \quad (7)$$

2.2 Harmonic analysis

For the beam subject to the harmonic force acting at $x = x_j$, the force function can be written:

$$F(x,t) = F_j \delta(x - x_j) e^{i\omega_s t} \quad (8)$$

The beam response can also be harmonic and determined

$$w(x,t) = \sum_{r=1}^{\infty} \phi_r(x) Q_r e^{i\omega_s t} \quad (9)$$

where

$$Q_r = \frac{F_j \phi_r(x_j)}{(\omega_r^2 - \omega_s^2) + i(2\xi_r \omega_r \omega_s)} \quad (10)$$

The beam response at $x = x_i$ can then be derived

$$w(x_i,t) = \sum_{r=1}^{\infty} \phi_r(x_i) Q_r e^{i\omega_s t} = e^{i\omega_s t} \sum_{r=1}^{\infty} \frac{F_j \phi_r(x_j) \phi_r(x_i)}{(\omega_r^2 - \omega_s^2) + i(2\xi_r \omega_r \omega_s)}$$

$$= X_i(\omega_s) e^{i\omega_s t} \quad (11)$$

where $X_i(\omega_s)$ is the harmonic response of the beam at $x = x_i$. If there are *m* observation points, the ODS vector can be defined as follows:

$$\{X(\omega_s)\} = \begin{Bmatrix} X_1(\omega_s) \\ X_2(\omega_s) \\ \vdots \\ X_m(\omega_s) \end{Bmatrix} = \begin{Bmatrix} |X_1(\omega_s)| e^{i\phi_1(\omega_s)} \\ |X_2(\omega_s)| e^{i\phi_2(\omega_s)} \\ \vdots \\ |X_m(\omega_s)| e^{i\phi_m(\omega_s)} \end{Bmatrix} \quad (12)$$

The frequency response function (FRF) between the *j*-th harmonic force and the *i*-th displacement response can also be obtained:

$$H_{ij}(\omega) = \frac{X_i}{F_j} = \sum_{r=1}^{\infty} \frac{\phi_r(x_j) \phi_r(x_i)}{(\omega_r^2 - \omega_s^2) + (2\xi_r \omega_r \omega_s)} \quad (13)$$

The input and output relation can be expressed as follows:

$$\{X(\omega)\} = [H(\omega)]\{F(\omega)\} \quad (14)$$

In conventional structural modal testing, one will experimentally measure a column or a row of the FRF matrix.

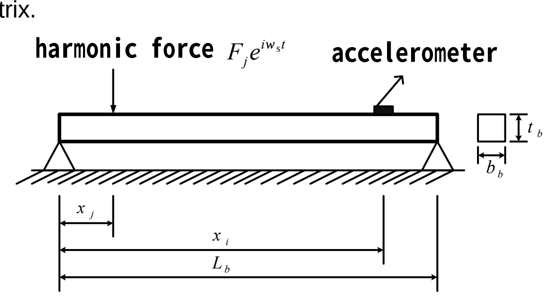


Fig. 1 Simply supported beam model

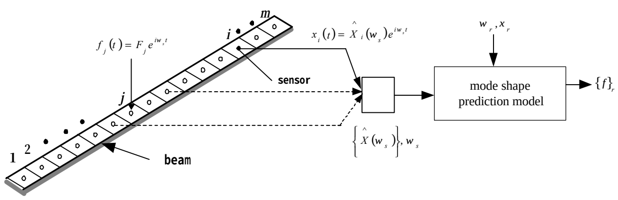


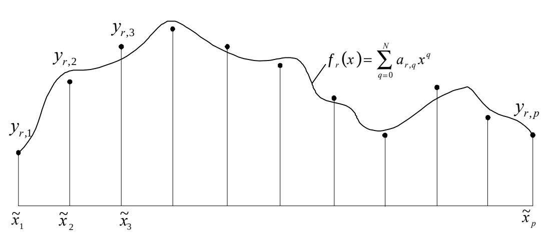
Fig. 2 Idea for mode shape prediction via ODS

Based on the theoretical formulation of FRF such as shown in Equation (14), modal parameter extraction method can then be applied to curve fit the FRF so that natural frequencies, damping ratios and mode shapes can be obtained.

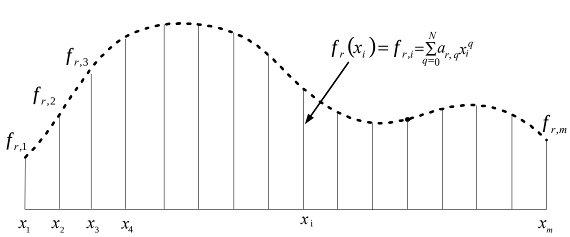
In this work the ODS as defined in Equation (12) are assumed measurable. The harmonic force information is also assumed known in prior. Natural frequencies and damping ratios can be easily obtained from single measurement of FRF. The *i*-th components of ODS vector can be shown as a function of the harmonic force amplitude and the excitation frequency as well as modal parameters. The following section will present the mode shapes prediction model from the ODS.

2.3 Mode shape prediction model from ODS

The general idea for mode shape prediction from the measured ODS is illustrated in Fig. 2. There are *m* grid points in beam. With the assumption that the harmonic point force is applied at the *j*-th location, and the sensor, such as the accelerometer, is located at position i. The ODS vector $\{\hat{X}(w_s)\}_{m\times 1}$ can be measured and as the input data to the prediction model. As mentioned previously, natural frequencies and damping ratios of the beam structure can be easily obtained and also be the input. Through the prediction model, the structural mode shapes can then be determined. In order to develop the mode shape prediction model, the polynomial fit of the mode shape function is first illustrated in Fig. 3(a).



(a) Illustration for polynomial fit of mode shape function



(b) Representation of mode shape by polynomial function

Fig. 3 Polynomial interpolation of mode shape function

There are *p* grid points along the beam length. Their coordinates are denoted as $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_p$. Their corresponding components of the *r*-th mode shape are designated as $y_{r,1}, y_{r,2}, ..., y_{r,p}$. The polynomial fitting function for the *r*-th mode shape can be expressed as follows:

$$f_r(x)=\sum_{q=0}^{N} a_{r,q}x^q \tag{15}$$

The coefficient $a_{r,q}$ can be determined by polynomial fit with the known $\tilde{x}_s$, $s=1,2,...,p$. If there are *m* measurement points, and their coordinates are $x_1, x_2, ..., x_m$, then the *i*-th component of the *r*-th mode shape can be interpreted as illustrated in Fig. 3(b). There are several advantages of the adoption of polynomial fitting function to represent the mode shape. The number of interpolation point *p* can be different from the ODS measure point *m*. The order of polynomial function *N* can be easily adjusted for different boundary characteristics. In the following derivation of the mode shape prediction model, the $y_{r,s}$, $r=1,2,...,n$, $s=1,2,...,p$ will be chosen as the design variables. The slight deviation of $y_{r,s}$ from the mode shape components $f_r(\tilde{x}_s)$ will be smoothed. The polynomial fitting function still will be have good interpolation of structural mode shapes.

The purpose of this section is to formulate the optimization problem that can be solved for the mode shapes from ODS. As noted previously, the measured ODS vector is denoted $\{\hat{X}(w_s)\}$. The theoretical ODS vector $\{X(w_s)\}$ has been derived in section 2.2.

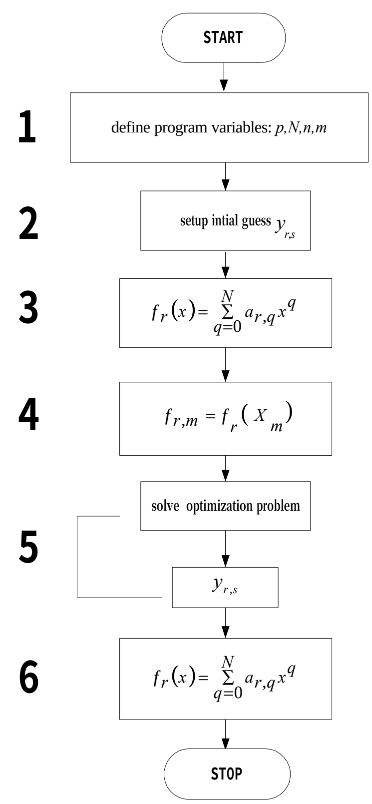


Fig. 4 Mode shape prediction program flow chart

As shown, $\{X(w_s)\}$ are functions of the harmonic force as well as modal parameters. In particular, the mode shape components are unknown and to be determined. Therefore, the optimization problem to predict the mode shape is defined as follows:

Objection function:

$$\Phi = \Phi(y_{r,s}) = \left[MAC\left(\{X(w_s)\}, \{\hat{X}(w_s)\}\right) - 1\right]^2 \tag{16}$$

Design variables: $y_{r,s}$; $r = 1,2,...,n$; $s = 1,2,..., p$

Constraints:

I. $\begin{Bmatrix} MAC(\{f\}_r, \{f\}_s) - 1 \le 0, r = s \\ MAC(\{f\}_r, \{f\}_s) \le 0, r \ne s \end{Bmatrix}$

II. $S_r = \sum_{i=1}^{m} |f_{r,i}|$, $S_1 > S_2 > S_3 > ... > S_n$

III. $S_{dr} = \sum_{i=1}^{m} |(f'_{r,i})|$, $S_{d1} < S_{d2} < S_{d3} < ... < S_{dn}$

The objective function uses the MAC characteristics. If two vectors are proportional to each other, the MAC value will be exactly one. The objection function is based on the minimization of the least square error between the predicted and measured ODS. As shown in Equation (16), the predicted ODS $\{X(w_s)\}$ can be a function of the harmonic force and modal parameters. With the adoption of polynomial fit on mode shape functions as shown in Equation (15). The design variables can be specified as $y_{r,s}$. This approach is different from the previous work [8,9]. They used mode shape components as the design variables for MDOF systems. The polynomial fit will reduce the number of design variables and provide with flexibility to choose the interpolation points.

Constraint I is to confine the determined mode shapes to maintain their orthorgonality relations. Constraint II uses the sum of absolute value of mode shape components for each mode shape to adjust the mode sequence. Constraint III is also applied to adjust the mode sequence by summing the absolute values of derivative of mode shape components. The higher mode results in the higher values of S_{dr}.

After the resolution of the optimization problem, design variables $y_{r,s}$ can be determined. The polynomial fit of mode shape function as shown in Equation (15) can also be solved. The mode shapes can then be interpolated accordingly.

3. NUMERICAL SIMULATION

3.1 Development of prediction program

This section introduces the mode shape prediction program by using MATLAB [12]. Polynomial-fit algorithm polyfit provided by MATLAB is adopted to curve fit the mode shape function with the known $\tilde{x}_s$ and $y_{r,s}$, $s = 1,2,..., p$. In solving the defined optimization problem, a general constrained optimization solver that is also provided by MATLAB is used. The solution flow chart is illustrated in Fig. 4. The major steps of the prediction program is summarized as follows:

1. define program variables: *p* is the number of interpolation points for polynomial fit. *N* is the order of the polynomial function. *n* is the number of modes to be determined. *m* is the number of measurement point of ODS and also the number of mode shape components.
2. setup the initial guess of design variables $y_{r,s}$, $r = 1,2,...,n$; $s = 1,2,..., p$. The number of design variables is $n \times p$.
3. perform polynomial fit to obtain mode shape polynomial function. The coefficient of polynomial $a_{r,q}$ will be determined.
4. determine the mode shape components $f_{r,i} = f_r(x_i)$, at every measurement point $x_1, x_2,..., x_m$.
5. solve the defined optimization problem by substituting the mode shape components obtained in the previous step. An optimum set of design variables $y_{r,s}$ can be resolved.
6. use the optimum design variables $y_{r,s}$ to perform polynomial fit so as to determine the mode shape polynomial function. The mode shape at the measurement points can then be interpolated and predicted.

The factors that affect the optimum solution and the accuracy of the predicted mode shapes are discussed as follows:

1. The number of interpolation points *p* should be proper selected. *p* must be large enough to reveal the mode shape characteristics. The thumb rule is that *p* is at least twice of modes to be predicted.
2. The order of polynomial function *N* should also be selected according to the number of modes to be determined.
3. The proper choice of the initial guess of design variables $y_{r,s}$ will be beneficial to efficiently solve for the optimum. As discussed, this work adopts the polynomial function to fit mode shapes. Therefore, the discrepancy of the guessed initial design variables from the optimum values can be adjusted accordingly. The fitted polynomials also have the ability to smooth the mode shape curve, and so forth the relative deviation of $y_{r,s}$ can be smoothed and minimized. The effect of the prediction error of $y_{r,s}$ on mode shapes will also be reduced.

3.2 Feasible study for simply supported beam

This section will perform the theoretical simulation to extract mode shapes from ODS for a simply supported beam. The beam geometry and material properties are listed in Table 1. The first four natural frequencies are f_1 =32.2509Hz, f_2 =129.0038Hz, f_3 =290.2585Hz, f_4 =516.0151Hz. Their corresponding mode shapes are shown in Fig. 5. Damping ratios are assumed to be 0.01 for all modes.

For verification purpose, the measured ODS $\{\hat{X}(w_s)\}$ is replaced by the theoretical ODS. The mode shape prediction program is tested to validate its feasibility in predicting mode shapes from only ODS available. The program variables are set as *p*=10, *N*=10, *n*=4, and *m*=15.

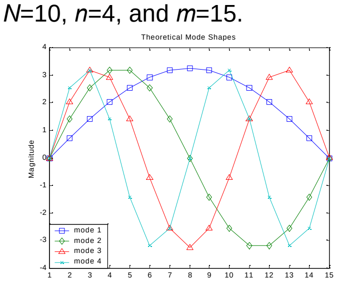


Fig. 5 Theoretical mode shapes of simply supported beam

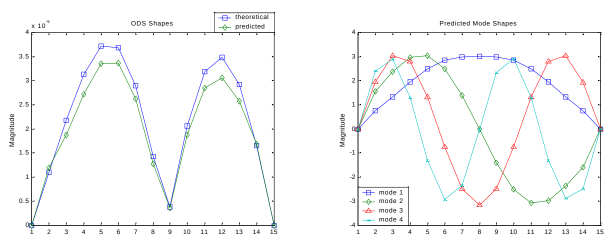
Table 1. Simply supported beam geometry dimension and material properties

Material	Steel
Length (L_b)	0.3m
Width (b_b)	0.04m
Thickness (t_b)	0.002m
Density (r_b)	7870kg/m^3
Young' s Modulus (E_b)	207×10^9 N/m^2

On-resonance excitation case

For the case of f_s =129Hz $\approx f_2$, i.e. near the second mode, the theoretical prediction results are shown in Table 2 and Fig. 6. The discussions are as follows:

1. Table 2(a) shows the comparison between the measured (theoretical) and predicted ODS. The relative errors of predicted are ODS within 13%.
2. Fig. 6(a) depicts both the measured (theoretical) and predicted ODS that appear very good agreement.
3. Fig. 6(b) reveals the predicted first four mode shapes that agree very well with the theoretical mode shapes.
4. Table 2(b) and 2(c) show the MAC and MSF matrices for theoretical and predicted mode shapes, respectively. That the diagonal elements of MAC matrix are very close to 1 indicates very good similarity. That the off-diagonal elements close to zero means the orthonormality between the theoretical and predicted mode shapes. Similar phenomenon can be observed for the MSF matrix. In particular, if the MSF value equals to 1, both vectors are exactly the same. The diagonal terms of MSF matrix reveal to be 1.03~1.07. This implies that the predicted mode shape error is with in 7%.



(a) Theoretical and predicted ODS (b) Predicted mode shapes

Fig. 6 Theoretical prediction of mode shapes for on-resonance excitation, f_s =129Hz $\approx f_2$

Table 2.Theoretical prediction of mode shapes for on-resonance excitation, f_s =129Hz $\approx f_2$

(a) Comparison between theoretical and predicted ODS

NO.	Measured (theoretical) $X\times10^{-4}$	Predicted $\hat{X}\times10^{-4}$	Error (%) $\frac{\lvert\hat{X}\rvert-\lvert X\rvert}{\lvert X\rvert}\times100$
1	0	0	0
2	-0.1095-0.0075i	-0.1189-0.0076i	8.4957
3	-0.2176-0.0133i	-0.1875-0.0117i	-13.7959
4	-0.3126-0.0162i	-0.2720-0.0142i	-13.0099
5	-0.3717-0.0156i	-0.3350-0.0140i	-9.8642
6	-0.3686-0.0119i	-0.3366-0.0109i	-8.6971
7	-0.2893-0.0060i	-0.2632-0.0055i	-9.0324
8	-0.1428+0.0008i	-0.1282+0.0008i	-10.1907
9	0.0381+0.0071i	0.0361+0.0066i	-5.1401
10	0.2066+0.0118i	0.1876+0.0109i	-9.1907
11	0.3190+0.0142i	0.2853+0.0127i	-10.5786
12	0.3485+0.0138i	0.3058+0.0120i	-12.2756
13	0.2914+0.0109i	0.2574+0.0094i	-11.6466
14	0.1647+0.0060i	0.1692+0.0063i	2.7187
15	0	0	0

(b) MAC matrix for theoretical and predicted mode shapes

theoretical / predicted	Mode 1	Mode 2	Mode 3	Mode 4
Mode 1	**0.9994**	0.0000	0.0001	0.0000
Mode 2	0.0000	**0.9983**	0.0000	0.0000
Mode 3	0.0001	0.0000	**0.9998**	0.0000
Mode 4	0.0000	0.0000	0.0000	**0.9997**

(c) MSF matrix for theoretical and predicted mode shapes

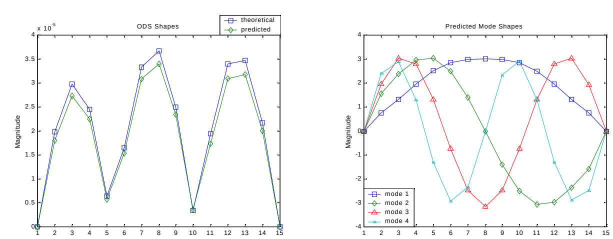
theoretical / predicted	Mode 1	Mode 2	Mode 3	Mode 4
Mode 1	**1.0419**	-0.0002	-0.0125	-0.0001
Mode 2	0.0002	**1.0378**	0.0002	0.0008
Mode 3	0.0127	-0.0003	**1.038**	-0.001
Mode 4	0.0001	-0.0015	0.0009	**1.0799**

Off-resonance excitation case

For the case of $f_2 < f_s$ =215Hz< f_3, i.e. between the second and third modes, Table 3 and Fig. 7 show the prediction results. The discussions are as follows:

1. From Table 3(a), the predicted ODS errors can be observed within 10%.
2. Fig. 7(a) and 7(b) show the predicted ODS and mode shapes, respectively. They appear very good prediction.
3. From Table 3(b) and 3(c), one can observe that the maximum error of predicted mode shapes has no more than 8%.

This section demonstrates the developed mode shape prediction model to be applied to a simply supported beam structure. The mode shapes can be satisfactory predicted from ODS within 8% error for different excitation frequency cases.



(a) Theoretical and predicted ODS (b) Predicted mode shapes

Fig. 7 Theoretical prediction of mode shapes for off-resonance excitation, $f_2 < f_s$ =215Hz < f_3

Table 3.Theoretical prediction of mode shapes for off-resonance excitation, f_2 < f_s =215Hz < f_3

(a) Comparison between theoretical and predicted ODS

NO.	Measured (theoretical) $X \times 10^{-4}$	Predicted $\hat{X} \times 10^{-4}$	Error (%) $\frac{\lvert\hat{X}\rvert - \lvert X\rvert}{\lvert X\rvert} \times 100$
1	0	0	0
2	0.1967-0.0285i	0.1782-0.0264i	-9.3376
3	0.2941-0.0445i	0.2697-0.0406i	-8.2895
4	0.2414-0.0412i	0.2224-0.0376i	-7.9233
5	0.0608-0.0200i	0.0549-0.0181i	-9.6820
6	-0.1648+0.0096i	-0.1535+0.0094i	-6.8285
7	-0.3318+0.0348i	-0.3070+0.0326i	-7.4504
8	-0.3644+0.0447i	-0.3375+0.0416i	-7.3861
9	-0.2479+0.0350i	-0.2319+0.0327i	-6.4566
10	-0.0319+0.0102i	-0.0347+0.0099i	7.5682
11	0.1935-0.0189i	0.1729-0.0169i	-10.6458
12	0.3376-0.0395i	0.3071-0.0361i	-9.0364
13	0.3448-0.0428i	0.3160-0.0392i	-8.3585
14	0.2147-0.0273i	0.1993-0.0249i	-7.1929
15	0	-0.0001+0.0000i	0

(b) MAC matrix for theoretical and predicted mode shapes

theoretical / predicted	Mode 1	Mode 2	Mode 3	Mode 4
Mode 1	**0.9994**	0.0000	0.0001	0.0000
Mode 2	0.0000	**0.9982**	0.0000	0.0000
Mode 3	0.0002	0.0000	**0.9998**	0.0000
Mode 4	0.0000	0.0000	0.0000	**0.9997**

(c) MSF matrix for theoretical and predicted mode shapes

theoretical / predicted	Mode 1	Mode 2	Mode 3	Mode 4
Mode 1	**1.0418**	-0.0002	-0.0126	-0.0002
Mode 2	0.0002	**1.0375**	0.0003	0.0007
Mode 3	0.0128	-0.0003	**1.0381**	-0.0009
Mode 4	0.0002	-0.0013	0.0009	**1.0800**

4. CONCLUSIONS

This work presents the mode shape prediction model from ODS. The structural ODS is defined as the steady state response of the structure subject to a harmonic force. The simply supported beam structure is studied and shown for the feasibility of the developed prediction model numerically. The beam ODS response analysis is presented. The polynomial fit technique is also adopted to interpolate the mode shape function. The optimization problem is formulated to develop the mode shape prediction model. The main idea is to find the mode shapes such that the error between the measured and predicted ODS will be minimized. The numerical simulation is performed and validated for the feasibility in determining mode shapes from ODS. This work will improve the restriction of conventional modal testing that requires the tested structure in static. In particular, for harmonic excitation system, such as a rotor system, the developed methodology can be applied. This work thus enhances the experimental modal analysis technique for the structure in harmonic operating condition.

5. ACKNOWLEDGEMENT

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