DETERMINATION OF UNKNOWN IMPACT FORCE ACTING ON ARBITRARY STRUCTURES

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ABSTRACT

This work presents a predictive model for determining the location and amplitude of an unknown impact force acting on arbitrary structures. Both time and frequency domain prediction methods are developed, respectively. The structural modal parameters can be first obtained by theoretical approach or by experimental modal testing. The structural response at time and frequency domains due to an unknown impact force can then be measured and recorded. The predicted response can also be formulated and expressed as functions of amplitude and location of the impact force. The sum of mean square errors between the predicted and measured response is then be defined as the objective function, while the amplitude and the location of the unknown impact force are as design variables. The optimization problem is thereby constructed and can be solved for the amplitude of the impact force. The mode shape information associated to the location of the impact force can also be resolved and compared to the structural mode shapes to obtain the location of the unknown impact force. A simply supported beam is introduced to experimentally verify the force predictive model. Results show that the predictive model is feasible and applicable to arbitrary structures.

NOMENCLATURE

a, $\hat{a}$	structural time domain acceleration for estimated and measured
A, $\hat{A}$	structural frequency domain acceleration for estimated and measured
C	linear homogenous self-adjoint differential for structural damping
$\{D\}_l$	solution of mode shape vector from optimization process
$\{\hat{D}\}_l$	transpose of $[G]_l$
F_j	amplitude of impact force acting at point j
$f(P,t)$	force function
$[G]_l$	the l-th row of modal matrix
L	linear homogeneous self-adjoint differential operators for structural stiffness
MAC_{jl}	modal assurance criterion
$M(P)$	structural mass operator
n	number of modes
N_t	number of points for time domain predictive model
N_ω	number of points for frequency domain predictive model
P	location coordinate of structural physical domain
Q_t	objective function for time domain predictive model
Q_ω	objective function for time domain predictive model
w	structural time domain displacement response
W	structural frequency domain displacement response
δ	unit impulse function or Dirac's delta function
ξ_k	the k-th modal damping ratio
$\phi_{k,i}$	the i-th component of the k-th mode shape vector
$[\Phi]$	structural modal matrix
ω_k	the k-th natural frequency
ω_{d_k}	the k-th damped natural frequency

1 INTRODUCTION

Force prediction is one of inverse engineering problems [1]. Many researchers have dedicated to identify the force content acting on structures [2-4]. It is useful for structural design and failure evaluation to know the force information. In particular, the damage of composite structures subjected to impact forces is not easily visible [5-7]. Previous works on the identification of impact force can be found in many literatures [8-11].

The basic idea of the force identification problem is to determine the system input via the knowledge of system output. The system input can be various forms of forces with time variant characteristics, while several types of sensors can be used to detect the system response.

This work develops a general procedure to identify the impact force acting on arbitrary structures. The predictive model assumes the knowledge of structural acceleration response and the structural modal properties, including natural frequencies, mode shapes and dampings. Either the time and frequency domain method can be applied to determine the location and amplitude of the impact force. A case study of simply supported beam subjected to an impact force is presented to demonstrate the feasibility and validity of the developed algorithm.

2 THEORETICAL ANALYSIS

Consider the equation of motion for an arbitrary structure as follows:

$$L[w(P,t)] + \frac{\partial}{\partial t} C[w(P,t)] + M(P)\frac{\partial^2}{\partial t^2}[w(P,t)] = f(P,t) \tag{1}$$

Assume the impact force acting at $p = p_j$ with amplitude of F_j at $t = 0$. The force function can be expressed as follows:

$$f(P,t) = F_j \delta(t) \delta(P - P_j) \tag{2}$$

where $\delta(P - P_j)$ is the unit impulsive function or Dirac's delta function. Through the force vibration analysis, the system response at $P = p_i$ can be found as follows:

$$w(P_i,t) = \sum_{k=1}^{n} \frac{\phi_k(P_i)\phi_k(P_j)F_j}{\omega_{d_k}} e^{-\xi_k \omega_k t} \sin \omega_{d_k} t \tag{3}$$

One can also obtain the acceleration response as follows:

$$a(P_i,t) = \sum_{k=1}^{n} \frac{\varphi_k(P_i)\phi_k(P_j)F_j}{\omega_{d_k}} e^{-\xi_k \omega_k t}\Big[\left(2\xi_k^2\omega_k^2 - \omega_k^2\right)\sin \omega_{d_k} t - 2\xi_k \omega_k \omega_{d_k} \cos \omega_{d_k} t\Big] \tag{4}$$

Taking Fourier Transform upon the above equation, one can get acceleration frequency response as follows:

$$A_i(\omega) = \sum_{k=1}^{n} \frac{\phi_k(P_i)\phi_k(P_j)F_j}{\left(\omega_k^2 - \omega^2\right) + i\left(2\xi_k \omega_k \omega\right)}\left(-\omega_k^2 - i2\xi_k \omega_k \omega\right) \tag{5}$$

From Equations (4) and (5), one can observe that the response is functions of system modal properties as well as the amplitude of the impact force.

Prediction Method I: Time Domain Method

For a proportional, viscous damping structure subject to an impact force, the structural acceleration response at $p = p_i$ can be measured and denoted as $\hat{a}_i(t)$. The corresponding theoretical acceleration response can be approximated and obtained from Equation (4)

$a_i(t) =$

$$a(P_i,t) = \sum_{k=1}^{n} \frac{\phi_{k,i}\phi_{k,j}F_j}{\omega_{d_k}} e^{-\xi_k \omega_k t}\Big[\left(2\xi_k^2\omega_k^2 - \omega_k^2\right)\sin \omega_{d_k} t - 2\xi_k \omega_k \omega_{d_k} \cos \omega_{d_k} t\Big] \tag{6}$$

where $\varphi_{k,i}$ denotes the i-th component of the k-th mode shape vector. $\phi_{k,i}$ and $\varphi_k(P_i)$ have the same physical meaning, except that the forth is obtained from experiment, and the latter is for theoretical solution. In Equation (6), $\phi_{k,i}$, ω_k, ξ_k and ω_{d_k}, which are known as structural modal properties, can be determined through structural modal testing or theoretical modal analysis. F_j and p_j, which are the amplitude and location of the impact force respectively, are unknown and to be determined.

In order to determine the impact force amplitude and its location, an optimization problem can be defined as follows:

Objective function: $Q_i = \sum_{t=1}^{N}\left[a_i(t_t) - \hat{a}_i(t_t)\right]^2$ (7)

Design variables: F_j, $\phi_{k,j}$, $k = 1,2,\ldots,n$ (8)

where Q_i is defined as the sum of square value of the error between the theoretically estimated $a_i(t)$ and the experimentally measured $\hat{a}_i(t)$ in N time steps. The objective is to find F_j and $\phi_{k,j}$ such that the objective function is zero or minimum. Therefore, the amplitude of the impact force F_j and a set of modal vector $\{D\}_j$ can be obtained as follows:

$$\{D\}_j = \left[\phi_{1,j} \quad \phi_{2,j} \quad \cdots \quad \phi_{n,j}\right]^T \tag{9}$$

$\{D\}_j$ is the vector containing the component at location p_j of all mode shape vectors. The modal matrix of the structure can be expressed as follows:

$$[\Phi] = \left[\{\phi\}_1 \quad \{\phi\}_2 \quad \cdots \quad \{\phi\}_n\right] = \begin{bmatrix} \phi_{1,1} & \phi_{2,1} & \cdots & \phi_{n,1} \\ \phi_{1,2} & \phi_{2,2} & \cdots & \phi_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1,m} & \phi_{2,m} & \cdots & \phi_{n,m} \end{bmatrix} = \begin{bmatrix} \{G\}_1 \\ \{G\}_2 \\ \vdots \\ \{G\}_m \end{bmatrix} \tag{10}$$

where

$$\{G\}_l = \left[\phi_{1,l} \quad \phi_{2,l} \quad \cdots \quad \phi_{n,l}\right] = \{\hat{D}\}_l^T, \quad l = 1,2,\cdots,m \tag{11}$$

n is the number of modes, and m is the number of divisions in structures.

MAC (Modal Assurance Criterion) is usually used to evaluate the correlation between the theoretical and experimental mode shape vectors. Here, MAC_l is used to determine the correlation between $\{D\}_j$ and $\{\hat{D}\}_l$ and defined as follows:

$$MAC_l = MAC\left(\{D\}_j, \{\hat{D}\}_l\right), \quad l = 1,2,\cdots,m \tag{12}$$

When MAC_l is close to 1, both $\{D\}_j$ and $\{\hat{D}\}_l$ have very good correlation, i.e., $p_l = p_j$. Therefore, the location of impact force can be determined at $P = p_l$.

Prediction Method II: Frequency Domain Method

From Equation (5), the acceleration frequency response at location $p = p_i$ can be approximated as follows:

$$A_i(\omega) = A_{P_i}(\omega) = \sum_{k=1}^{n} \frac{\phi_{k,i}\phi_{k,j}F_j}{\left(\omega_k^2 - \omega^2\right) + i\left(2\xi_k \omega_k \omega\right)}\left(-\omega_k^2 - i2\xi_k \omega_k \omega\right) \tag{13}$$

Similar to the derivation of the time domain method, the optimization problem can be defined as follows:

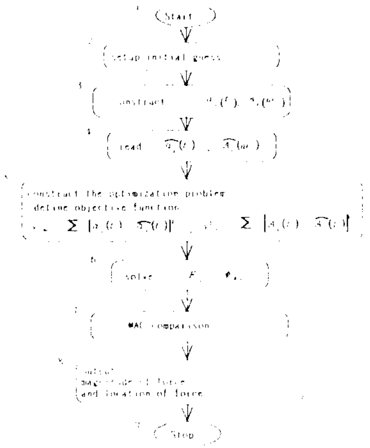


Figure 1: Program flow chart

Objective function: $Q_m = \sum_{i=1}^{N_\omega}\left[A_i(\omega_i) - \hat{A}_i(\omega_i)\right]^2$ (14)

Design variables: F_j , $\phi_{k,j}$, $k = 1,2,...,n$ (15)

where Q_m is defined as the sum of the square value of the error between $A_i(\omega)$ and $\hat{A}_i(\omega)$ in the N_ω frequency points. After the resolution of the optimization problem, the amplitude of the impact force F_j and a set of modal vector $\{\phi\}$ can be obtained. Similar to the definition of Equation (12), MAC_j can be obtained to determine the location of the impact force.

3 FORCE PREDICTION PROGRAM

The prediction program is implemented with MS-FORTRAN PowerStation incorporated with Visual Numerics IMSL Math Library for optimization. The program flowchart is shown in Figure 1. The program has three options based on the selection of theoretical response of $a_i(t)$ or $A_i(\omega)$ and the experimental response of $\hat{a}_i(t)$ and $\hat{A}_i(\omega)$. The purpose of the three kinds of combination application is as follows:

Option I: Use theoretical modal properties to estimate $a_i(t)$ or $A_i(\omega)$ incorporated with theoretical response to represent $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$. This approach is mainly for the validation of the developed methods.

Option II: Use theoretical modal properties to estimate $a_i(t)$ or $A_i(\omega)$ incorporated with experimental response to represent $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$. This approach can be applied to complex structures that are not easy or feasible to experimentally determine the modal properties of the structure.

Option III: Use experimental modal properties to estimate $a_i(t)$ or $A_i(\omega)$ incorporated with experimental response to represent $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$. This approach can be suitable for the case that the experimental modal properties can be available or that the theoretical modal analysis of structures is not possible.

Figure 2. Number and division of simply supported beam

Table 1: Physical properties of simply supported beam

Material	steel
Length	0.38m
Width	0.04m
Thickness	0.002m
Density	7870 kg/m^3
Young's Modulus	207×10^9 N/m^2
Poisson Ratio	0.292

4 RESULTS AND DISCUSSIONS

This section numerically and experimentally presents the case of a simply supported beam subjected an impact force. The physical properties of the simply supported beam are shown in Table 1. The simply supported beam is divided into 19 divisions and numbered as shown in Figure 2. The developed prediction models for both time and frequency domain methods are applied to show their feasibility and validity in determining the impact force amplitude and location.

4.1 Theoretical Modal Properties with Theoretical Response (Option I)

4.1.1 Effect of force location

Let the accelerometer be located at position 1 to measure the acceleration response, and the impact force be applied at position 5, 7, 10 and 18. Table 2 shows the prediction of force amplitude due to different force locations for division and non-division points, respectively. Symbols i and j denote the measurement point and force location, respectively. The predicted amplitudes of the impact force for both time and frequency domain methods are satisfactory. Figures 3 and 4 show that the predicted force locations are near to the actual locations.

4.1.2 Effect of different force amplitude

For the impact force acting at the same location but different force amplitudes, Table 3 shows that both the time and frequency domain methods provide satisfactory prediction results. Although the actual amplitude is somewhat different, the prediction for the variation of force amplitude is quite correct.

4.1.3 Effect of measurement points

Let the impact force be applied at position 18, different accelerometer locations are tested to study the effect of

Table 2: Prediction of force amplitude for different force locations (Option I)

(i,j) \ F (N)	actual value	Predicted value time method	predicted value freq. method
(1,5)	1.000	0.762	0.763
(1,7)	1.500	1.504	1.058
(1,10)	0.800	0.734	1.253
(1,18)	0.799	0.850	0.976
(3,18.5)	0.380	0.194	0.261
(10,2.5)	0.537	1.575	0.560

Table 3: Prediction of force amplitude for different force amplitude (Option I)

(i,j) \ F (N)	actual value	Predicted value time method	predicted value freq. method
(3,2)	1.470	0.942	1.201
(3,2)	0.581	0.579	0.581
(1,18)	0.350	0.280	0.669
(1,18)	0.799	0.850	0.976

Table 4: Prediction of force amplitude for different measurement points (Option I)

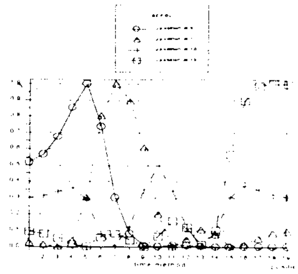
(i,j) \ F (N)	actual value	predicted value time method	predicted value freq. Method
(1,18)	0.799	0.850	0.976
(5,18)	0.883	0.598	1.099
(7,18)	0.560	0.399	1.119
(10,18)	1.110	0.728	1.561

the predictive algorithm. Table 4 shows that the prediction of force amplitude is acceptable, and the results from the frequency domain method are slightly overestimated. Figure 5 shows the prediction of force locations corresponding to the cases of Table 4. Most cases reveal very good location prediction except for the case of measurement point at 10. The position 10 is right at the middle of the beam as known to be the nodal points of even modes. It is reasonable to conclude that the measurement points should be better located away from nodal points.

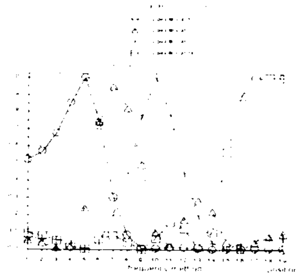
4.2 Theoretical modal properties with experimental response (Option II)

4.2.1 Effect of force location

Table 5 shows the prediction of force amplitude for different force location. The prediction is not quite good, but some predictions are near the actual force amplitude. The discrepancy may be due to the fact that the actual impact force is a triangle shape in time domain as shown in Figure 6. However, in this work the impact force is simulated by an ideal impact force function. Figure 7 corresponding to the cases of Table 6 shows the prediction of force location. One can see that the force applied at position 2 can be well predicted. When the force is applied at position 19, the prediction is not good enough. It can be the cause that the measurement point is at position 3 away from the force location.

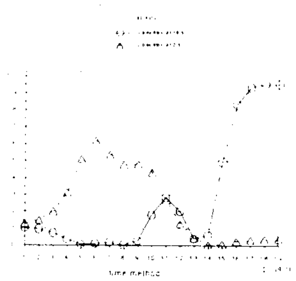


(a) Time domain method

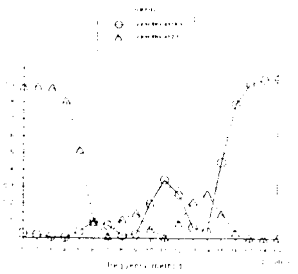


(b) Frequency domain method

Figure 3: Prediction of force location for different force location at division points (Option I)



(a) Time domain method

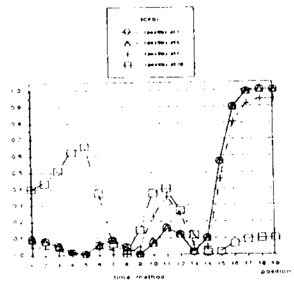


(b) Frequency domain method

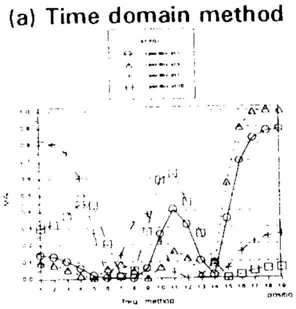
Figure 4: Prediction of force location for different force location at non-division points (Option I)

4.2.2 Effect of different force amplitude

Table 6 shows that for different force amplitudes both time and frequency domain methods can predict the variation of force amplitude, but the exact amplitude value is generally underestimated. As mentioned previously, this may be due to the improper assumption of ideal impact force. The modification of the impact force to an triangle force may improve the prediction algorithm.



(a) Time domain method



(b) Frequency domain method

Figure 5: Prediction of force location for different measurement points (Option I)

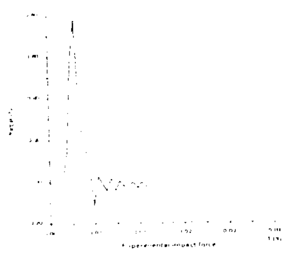


Figure 6: Typical time history of actual impact force

Table 5: Prediction of force amplitude for different force location (Option II)

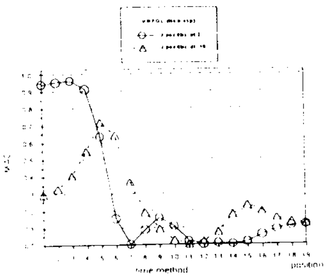
F_i (N) / (i,j)	actual value	predicted value time method	predicted value freq. method
(3,2)	0.581	0.500	0.144
(3,19)	1.600	1.478	0.027
(3,18.5)	0.380	0.675×10^{-6}	0.173
(10,2.5)	0.537	0.142	0.126

Table 6: Prediction of force amplitude for different force amplitudes (Option II)

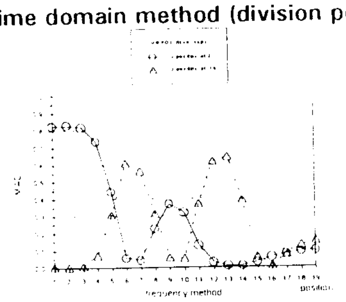
F_i (N) / (i,j)	Actual Value	predicted value time method	predicted value freq. method
(3,2)	1.470	0.301	0.093
(3,2)	0.581	0.500	0.144
(1,18)	0.350	0.349	0.244×10^{-3}
(1,18)	0.799	0.449	0.0134

4.2.3 Effect of measurement points

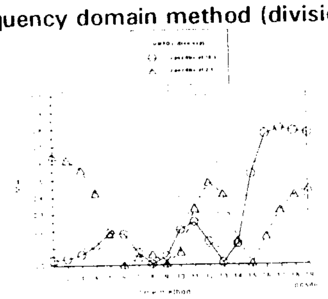
To discuss the effect of measurement points on the prediction model, let the impact force be fixed at position 18. Table 7 shows that the predictive force amplitudes are generally small. Figure 8 corresponding to the cases of Table 7 shows that the force location can generally be well predicted. In summary, the prediction algorithm is again shown for its feasibility especially in determining the force location.



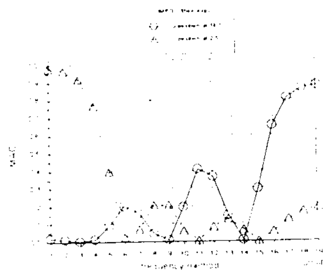
(a) Time domain method (division point)



(b) Frequency domain method (division point)



(c) Time domain method (non-division point)

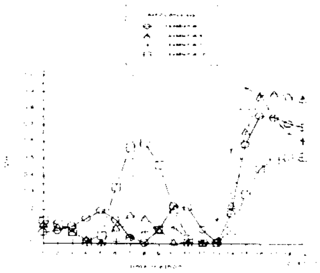


(d) Frequency domain method (non-division point)

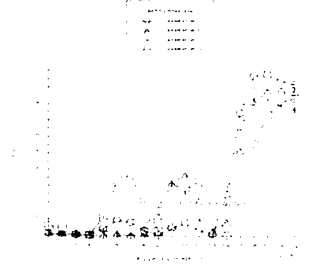
Figure 7: Prediction of force location for different force location (Option II)

Table 7: Prediction of force amplitude for different measurement points (Option II)

F_i (N) / (i,j)	Actual value	predicted value time method	predicted value freq. method
(1,18)	0.799	0.449	0.0134
(5,18)	0.883	0.016	0.048
(7,18)	0.560	0.004	0.316
(10,18)	1.110	0.343×10^{-4}	0.306



(a) Time domain method



(b) Frequency domain method

Figure 8: Prediction of force location for different measurement points (Option II)

4.3 Experimental modal properties with experimental response location (Option III)

4.3.1 Effect of force location

The force location is also considered for both division and non-division points. Table 8 shows that the predictive force amplitudes are generally smaller than the actual values. This again can be due to the improper assumption of the impact force. Figure 9 corresponding to the cases of Table 8 reveals that the force acting at position 2 can be well predicted, and the force acting at position 19 can only be predicted for its possibility due to the measurement point is at position 3 far away from the force location. In summary, the force amplitude may not have good prediction due to the improper assumption of impact force. The location of force can be generally well predicted, if the measurement points can be properly selected.

4.3.2 Effect of different force amplitude

Table 9 shows the results for the prediction of force amplitudes. The prediction is not good enough for force variation but also do not approach to the actual value. To modify the assumption of the impact force for estimating the response is necessary so as to improve the algorithm.

4.3.3 Effect of measurement points

Figure 10 shows the prediction of force location for different measurement points, while the impact force is applied at position 18. Except that the case of measurement point at position 10 is not good, else cases provide a satisfactory prediction of force location. Table 10 shows the prediction of force amplitudes that are generally smaller than the actual values as discussed previously.

Table 8: Prediction of force amplitude for different force amplitude (Option III)

F_j (N) / (i,j)	Actual value	predicted value time method	predicted value freq. method
(3,2)	0.581	0.093	0.089
(3,19)	1.600	0.022	0.138×10^{4}
(3,18.5)	0.380	0.170×10^{-3}	0.860×10^{3}
(10,2.5)	0.537	0.468×10^{-5}	0.054

Table 9: Prediction of force amplitude for different force amplitudes (Option III)

F_j (N) / (i,j)	actual value	predicted value time method	predicted value freq. method
(3,2)	1.470	0.147	0.123
(3,2)	0.581	0.093	0.089
(1,18)	0.350	0.370×10^{2}	0.011
(1,18)	0.799	0.260	0.001

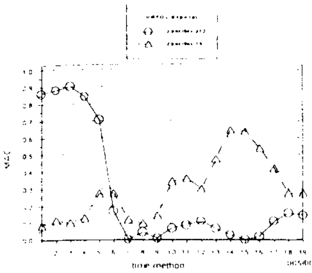
Table 10: Prediction of force amplitude for different measurement points (Option III)

F_j (N) / (i,j)	actual value	predicted value time method	predicted value freq. method
(1,18)	0.799	0.260	0.001
(5,18)	0.883	0.031	0.150×10^{4}
(7,18)	0.560	0.203	0.029
(10,18)	1.110	0.092	0.143

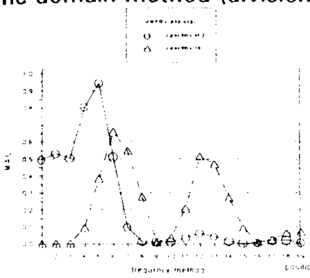
6 CONCLUSIONS AND RECOMMENDATIONS

This work develops a force predictive model for arbitrary structure subjected to an impact force to determine the force amplitude and location. Both time and frequency domain methods are presented. Some conclusions are summarized as follows:

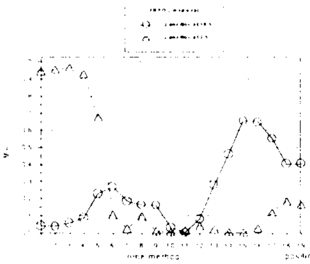
1. The predictive model considers theoretical modal properties incorporated with theoretical response (Option I) to demonstrate the feasibility of the developed algorithm. Results show that the predictive model has satisfactory adaptation to different force amplitudes and locations as well as different measurement locations.
2. When structural modal properties are not available, the theoretical modal properties can also be incorporated with experimental response (Option II) to determine the impact force amplitude and location.
3. Both the system modal properties and system response can be determined from experiments (Option III). This approach is also shown for the feasibility of the developed predictive model.
4. The developed force predictive model has better adaptation in determining the force location than the force amplitude.
5. The actual impact force is a triangular shape in time domain. The modification of impact force function is desired for better force amplitude prediction and under investigation.



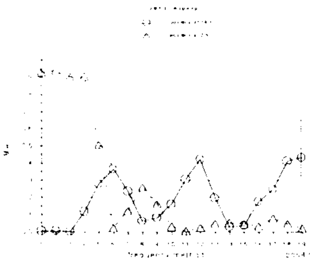
(b) Time domain method (division point)



(b) Frequency domain method (division point)



(c) Time domain method (non-division point)



(d) Frequency domain method (non-division point)
Figure 9: Prediction of force location for different force location (Option III)

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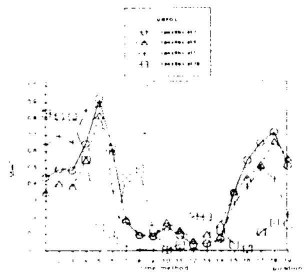
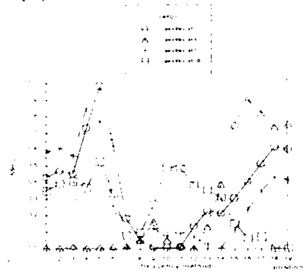


(a) Time domain method



(b) Frequency domain method
Figure 10: Prediction of force location for different measurement points (Option III)

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