APPLICATIONS OF GENETIC ALGORITHMS TO THE OPTIMUM DESIGN OF ACTIVE CONTROL SYSTEM

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ABSTRACT

This paper presents the application of Genetic Algorithms (GAs) to determine the optimal placement of sensors and actuators in LMS feedforward control systems. A case study of optimal placement of multiple piezoelectric actuators and accelerometer sensors for active structural vibration control is demonstrated. The results show that the optimally located actuators and sensors perform better vibration control than arbitrarily selected ones. This work leads to a design methology for active control systems.

INTRODUCTION

The active control of structural vibration and sound radiation by adaptive feedforward algorithm has become an interesting topic. The application of feedforward control algorithms has been successfully implemented in power flow and vibration control [1,2] and sound radiation control [3]. The newly developed compact distributed actuators and sensors, such as piezoceramic patches and PVDF film, are highly integrated to structures to perform active system control. The location of piezoelectric actuators and PVDF film sensors has been shown critical to the effectiveness of system control [4,5]. The problem of optimum design of active control system is, therefore, to determine the optimal placement of actuators and sensors.

Previous works [6,7] have shown the influence of sensor and actuator location on the performance of an active control system. Many works have treated the optimal selection of number and location of actuators and sensors individually [8,9]. Only little literature considers the optimal placement of actuators and sensors simultaneously [10,11]. Nevertheless, those works exclusively pertain to feedback control systems, and primarily focus on the traditional point force actuators or accelerometer sensors. Wang et al. [12] successfully applied the successive quadratic programming algorithm to determine the optimal location of piezoelectric actuators for plate sound radiation control; however, the computing effort is highly demanded.

The genetic algorithm (GA) a search procedure based on the mechanics of natural selection and natural genetics is increasingly applied to wide areas of optimization problems [13,14]. GA is so attentive because of their simplicity, robust and easy implementation. This paper briefly discusses the GA. A simply-supported beam, as shown in Figure 1, is considered as the plant subjected to a harmonically excited point force disturbance. The beam lateral vibration is controlled by active piezoelectric actuators with the use of accelerometers as error sensors in conjunction with the use of LMS feedforward control algorithm. The GA is applied to determine the optimal placement of piezoelectric actuators and accelerometer sensors simultaneously. The results show that the optimally placed actuators and sensors achieve more efficient vibration control than arbitrarily chosen ones.

GENETIC ALGORITHM

The genetic algorithm is derived based on Darwin's theory of "Survival of the fittest" [13]. The GA is a search procedure for general optimization problems and is numerically simple involving nothing more than random number generation, bit manipulation and string exchange. The objective function is usually transformed to a fitness function

$$F(x) = base\ value - f(x) \tag{1}$$

The GA is to maximize the fitness function, F(x), in contrast to minimize the objective function, f(x), in comparison to the traditional optimization procedures. The design variables, x, are encoded as a binary digit string which is analogous to chromosome in a biological system. When multiple design variables are desired, all design variables are concatenated to one single string. In the begin with, the GA randomly generates a population of strings by successive coin flips. The generation process is then succeeded to produce the new generation of population by performing three basic operators of GA: reproduction, crossover, and mutation. Strings can be decoded to obtain the real values of the design variables in order to calculate the fitness values. The design constraints are treated by the exterior penalty function method such that a constrained problem is transformed to an unconstrained problem.

FORMULATION AND SOLUTION OF OPTIMIZATION PROBLEM

As shown in Figure 1, a simply-supported beam is considered as the plant subjected to a harmonically excited point force disturbance. The beam lateral vibration is controlled by active piezoelectric actuators and accelerometer sensors in conjunction with the use of LMS feedforward control algorithm. The size of the i-th piezoelectric actuator is assumed to be fixed, i.e., C_{x_i} is constant. The applied voltages to the i-th piezoelectric actuator, Vi, can be calculated from linear quadratic optimal control theory (LQOCT) which will be discussed further later. Therefore, the design variables can be identified as:

$$\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_{N_c}, x_{a_1}, x_{a_2}, \ldots, x_{a_{N_a}} \tag{2}$$

where $\overline{x}_i$ and x_{a_i} are the location of piezoelectric actuators and accelerometers as illustrated in Figure 1; N_c and N_a are the number of actuators and sensors. The objective function can then be defined as follow:

$$\Phi_y = \int_0^L |\ddot{y}_t|^2 dx \tag{3}$$

where Φ_y is the vibrational energy density; y_t is the total beam response [15]. The design constraints can be described as follow:

(1) to maintain piezoelectric actuators inside of the beam:

$$\bar{x}_i - C_{x_i} \geq 0$$
$$\bar{x}_i + C_{x_i} \leq L \quad (4)$$

(2) to avoid overlapping between piezoelectric actuators:

$$\bar{x}_{i+1} - \bar{x}_i - C_{x_i} > 0 \quad (5)$$

(3) to specify the working range of piezoelectric actuators:

$$|V_i| \leq 150 \ (volt\ p\text{-}p) \quad (6)$$

(4) to avoid overlapping between accelerometer sensors:

$$x_{a_{i+1}} - x_{a_i} > 0 \quad (7)$$

The optimization problem is now well defined; therefore, the GA can be employed to find the optimal configuration.

SOLUTION STRATEGY

Linear quadratic optimal control theory

The linear quadratic optimal control theory (LQOCT) is applied to simulate the LMS feedforward control algorithm. The control voltages to be applied to piezoelectric actuators are calculated by minimizing the cost function which is defined as the least mean square sum of the accelerations measured by the accelerometer sensors. The complete derivation of LQOCT applied to the beam vibration control is shown in [15] and omitted here for brevity.

The genetic algorithm is implemented as FORTRAN code incorporated with the subprogram calculating the structural response to solve the optimal location of piezoelectric actuators and accelerometer sensors. Figure 2 shows the program flowchart for genetic algorithm. The procedure starts with setting up some GA constants, including number of maximum generation, population size and the probabilities of crossover and mutation. The program is then proceeded as illustrated in Figure 2.

NUMERICAL EXAMPLES

Numerical examples presented are based on a steel beam with length of 380mm, width of 40mm and thickness of 2mm. The beam natural frequencies are tabulated in Table 1. The disturbance is assumed to be a point force with magnitude of F=0.1N and located at 67mm. The size of piezoelectric actuators is fixed at 63.5mm, and the other properties are shown in Table 2. The applied voltages to piezoelectric actuators are obtained from LQOCT. The locations of piezoelectric actuators and accelerometers are to be determined.

In order to perform the genetic algorithm, the initial population size is assumed to be 10, and the maximum generation 60. The crossover and mutation probabilities are 0.6 and 0.033 respectively. Each design variable is coded as a ten digit string such that the location accuracy is about 0.4mm. A typical historical track of the best-of-generation (maximum) and generation average results is shown in Figure 3. The maximum and average of generation is oscillating and gradually increased. Although the GA will not converge the solution, one can obtain a nearly optimal solution with the least effort.

On-resonance excitation

Figure 3 shows the displacement distribution of the beam subjected to the point force disturbance excited at 33Hz, i.e., near the first resonance mode, under the active control of optimally located actuators and sensors. As expected, the solid line which reveals the first mode shape indicates the beam response due to the disturbance alone. For the case of one piezoelectric actuator and one accelerometer sensor denoted "P1S1", the first mode which is the significant one is well controlled leaving the residual beam response as the third mode. A nodal point is right on the location of the sensor at 109mm because of the effect of LMS control. For the case of P12S12, i.e., two actuators and two sensors are applied, the beam response is further reduced and reveals a more complex shape, because several low order modes can be efficiently controlled by the two actuators. A more complicated residual beam response can be observed for the case of P123S123, since applying more actuators can suppress more modal components contributed to the beam response. This phenomenon can be observed from Table 3 which shows the modal amplitudes of the beam before and after controls. The bolded numbers indicate that the modal amplitudes are reduced. One can see that the first mode is well controlled for all cases. The more actuators and sensors applied, the more modal components controlled. Table 4 also summarizes the reduction of vibration energy and control voltages applied to the piezoelectric actuators as well as the coordinates of actuators and sensors. The symbol '(A)' denotes the cases of arbitrarily located actuators and sensors, while '(O)' denotes the cases of optimally located actuators and sensors corresponding to those in Figure 3. Several observations can be made. Optimally located actuators and sensors generally perform better vibration control than arbitrarily chosen ones in terms of reduction of vibration energy. The more actuators and sensors, the more reduction of vibration energy which agree with the results seen in Figure 3. The control voltages are remained at reasonable small values.

Off-resonance excitation

Figure 4 shows the similar results to Figure 3 except the excitation frequency 200Hz, i.e., between the second and third resonance modes. The solid line represented the beam response due to the disturbance can be seen as the combination of the second and third mode response. The residual response after control again becomes more complex and is generally smaller than before control. Sufficient controls can be achieved. One can see from Table 5 that several modal amplitudes are simultaneously reduced for the cases of P1S1 and P12S12. The control mechanism here is far different from that for the cases of on-resonance excitation in Figure 3. The former is recognized as "modal reconstruction", while the latter is shown as "modal suppression" [17]. It is noted that for the case of (O)P123S123 the first three modal amplitudes are reduced, and the second one is significantly suppressed. The control mechanism is therefore recognized as modal suppression. This can be clarified that modal suppression can also work well for off-resonance excitation. The control mechanism, in fact, is also dependent on the location of actuators and sensors. Table 6 shows the reduction of vibration energy and control voltages respectively for the cases of both optimally and arbitrarily located actuators and sensors. The cases of arbitrarily located ones may result in a negative number of reduction of vibration energy. This means that the control is not effective, and spillover occurs. Appropriately located one actuator and one sensor can achieve better vibration control than three arbitrarily located ones. Again, multiple actuators and sensors perform generally better than the single actuator and sensor.

CONCLUSIONS

This paper presents the application of the genetic algorithm to determine the optimal placement of piezoelectric actuators and accelerometer sensors for structural vibration control in conjunction with the use of LMS feedforward control. Some significant observations may be summarized as follows:

1. The GA is shown a feasible method to determine the optimal placement

of actuators and sensors for active system control.

2. The optimally located piezoelectric actuators and accelerometer sensors perform better vibration control than the arbitrarily located ones.
3. The more actuators and sensors generally provide a larger amount of reduction vibration energy and are seen to achieve better control than the single actuator and sensor.
4. Different control mechanisms in terms of "modal suppression" and "modal reconstruction" for both on- and off-resonance excitation cases are recognized and dependent on the location of actuators and sensors.

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Table 1.Natural frequencies of the simply-supported beam

n-th mode	Natural frequencies (Hz)	n-th mode	natural frequencies (Hz)
1	32.21	6	1159.56
2	128.84	7	1578.29
3	289.89	8	2061.44
4	515.36	9	2609.00
5	805.25	10	3220.99

Table 2.Physical properties of the G-1195 piezoceramic patch [16]

E_a=6.3×10^{10} (N/m^2) ρ_a=7650Kg/m^3

t_a=1.905 (mm) ν_a=0.28

$$d_{31}=d_{32}=166\times10^{-12}\left(\frac{m}{volt}\right)$$

Table 3. Summary of modal amplitudes for on-resonance case, f=33 Hz

mode	Disturbance	P1S1	P2S2	P3S3
1	0	**-68.64**	**-80.68**	**-73.63**
2	-44.95	**-55.75**	**-55.23**	**-75.57**
3	-58.59	-46.70	**-76.11**	**-61.79**
4	-70.57	-51.13	-61.51	-66.90
5	-85.19	-76.56	-57.93	**-95.36**

Table 4. Results for on-resonance excitation, f=33Hz

case	reduction of vibration energy (dB)	control voltage (volt)	location of actuators (mm)	location of accelerometers (mm)
(A)P1S1	27.62	-33.42(P1)	316.8	100.0(S1)
(A)P12S12	44.84	-2.02(P1)	316.8	100.0(S1)
		-16.98(P2)	151.8	255.0(S2)
(A)P123S123	44.41	-1.75(P1)	316.8	100.0(S1)
		-17.32(P2)	151.8	255.0(S2)
		0.72(P3)	33.8	350.0(S3)
(O)P1S1	44.83	-17.93(P1)	155.2	109.6(S1)
(O)P12S12	52.09	-12.00(P1)	99.2	85.8(S1)
		-9.18(P2)	239.6	342.8(S2)
(O)P123S123	59.70	-7.97(P1)	65.7	46.4(S1)
		-6.06(P2)	235.9	134.1(S2)
		-7.88(P3)	149.3	262.6(S3)

Table 5. Summary of modal amplitudes (dB) for off-resonance case, f=200 Hz

mode	Disturbance	P1S1	P12S12	P123S123
1	-9.04	**-11.85**	**-12.47**	**-14.42**
2	0	**-17.56**	**-16.72**	**-71.89**
3	-4.56	**-12.84**	**-32.66**	**-23.52**
4	-20.66	**-38.55**	**-34.30**	-19.38
5	-36.12	-21.14	-49.16	-25.68

Table 6. Results for off-resonance excitation, f=200Hz

case	reduction of vibration energy (dB)	control voltage (volt)	location of actuators (mm)	location of accelerometers (mm)
(A)P1S1	-1.42	4.70(P1)	316.8	100.0(S1)
(A)P12S12	-1.28	4.20(P1)	316.8	100.0(S1)
		-0.61(P2)	151.8	255.0(S2)
(A)P123S123	9.76	-0.32(P1)	316.8	100.0(S1)
		-3.86(P2)	151.8	255.0(S2)
		-8.08(P3)	33.8	350.0(S3)
(O)P1S1	9.78	-6.62(P1)	87.1	197.2(S1)
(O)P12S12	12.40	-1.73(P1)	178.3	21.9(S1)
		-6.33(P2)	80.0	342.5(S2)
(O)P123S123	13.90	-6.01(P1)	63.3	2.6(S1)
		-0.85(P2)	168.0	226.2(S2)
		-4.81(P3)	320.1	226.9(S3)

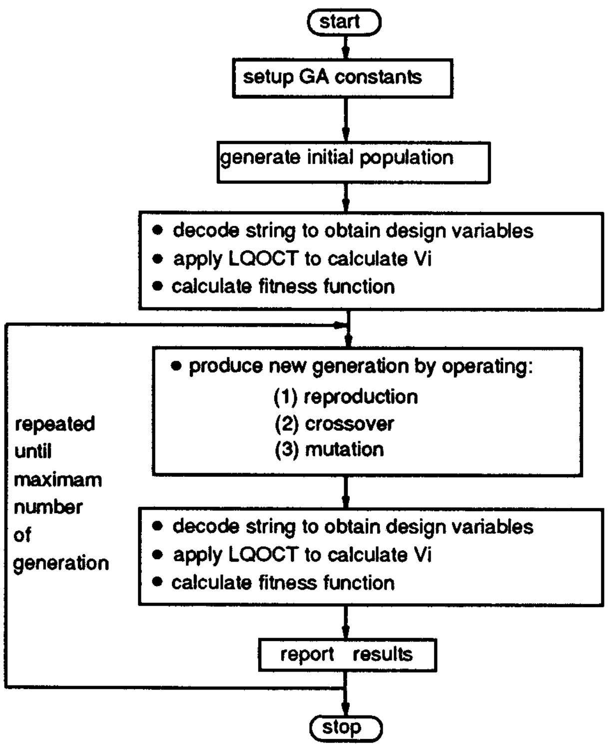

Figure 2. Flowchart of genetic algorithms

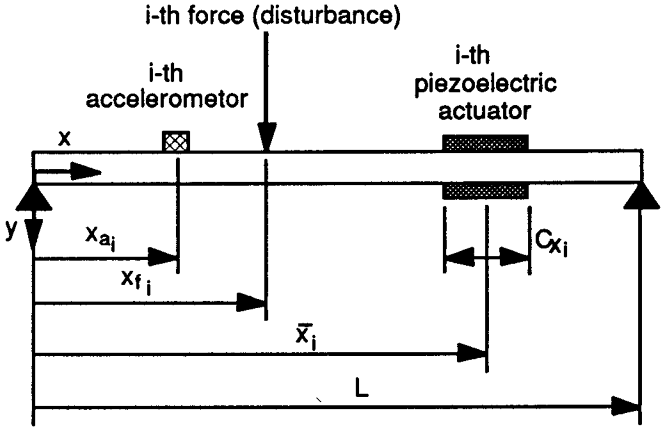

Figure 1. The arrangement and coordinates of simply-supported beam

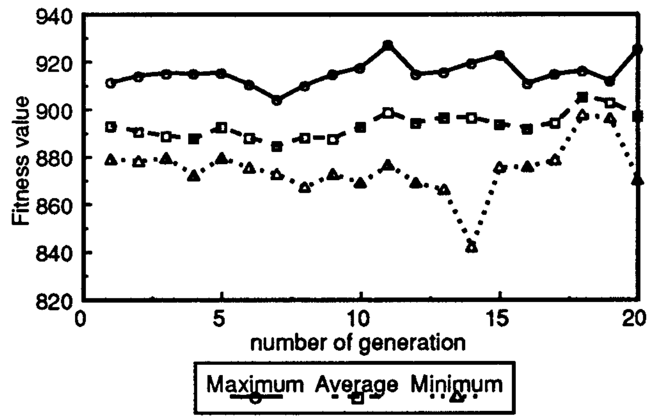

Figure 3. Historical track of the best-of-generation and genteration average

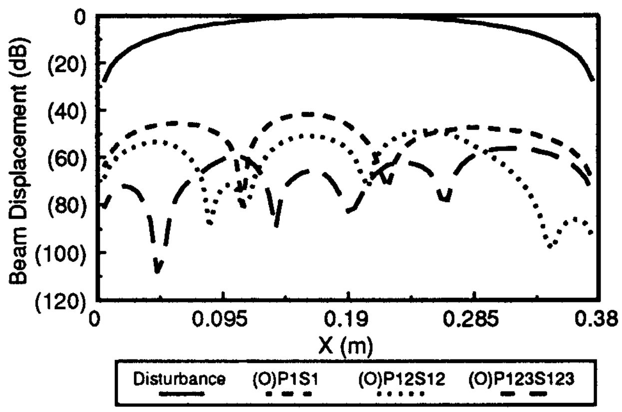

Figure 4. Displacement distribution of the beam for optimally located actuators and sensors, f=33 Hz

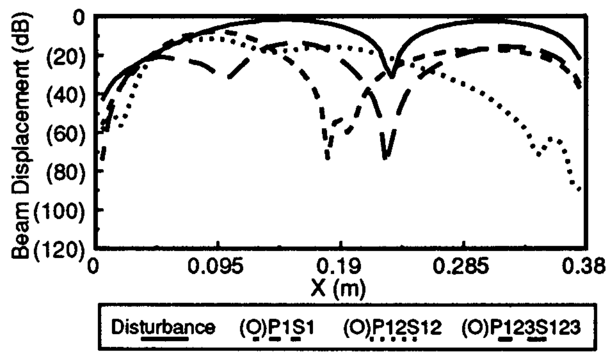

Figure 5. Displacement distribution of the beam for optimally located actuators and sensors, f=220 Hz