PRELIMINARY STUDY OF HOLLOW PIPE WITH AN UNDERCUT AS PERCUSSION INSTRUMENT

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This work investigates the possibility of using steel hollow pipe with an undercut to make a new type of percussion instrument. From traditional bamboo percussion instruments found in Taiwan or elsewhere, the bamboo geometry is a hollow tube structure generally with the undercut. This work formulates a basic geometry model with specific dimension variables for the hollow pipe with an undercut in the middle. A stainless steel pipe is first chosen to identify its materials properties and used to manufacture the special pipe percussion instrument. A trial study on determining the dimension of the pipe and its undercut is performed to obtain the initial pipe geometry design, in particular for Note A5 with the pitch frequency 880 Hz. Both theoretical modal analysis (TMA) and experimental modal analysis (EMA) are conducted on the pipe, respectively. The obtained structural modal parameters reveal very good agreement between experiments and analysis. The percussion sound of the pipe with the undercut is also studied by spectral analysis. Results show the fundamental frequency of the pipe matches the pitch frequency of Note A5 very well. There are several overtone frequencies over the fundamental frequency having integer ratios. This is known as harmonic sound characteristic that makes the percussion sound of the pipe in harmony and pleasantness. The optimum design for different pitch frequencies of musical notes can be developed accordingly base on the geometry model and under investigation to design a new type of percussion instrument for the hollow pipe with undercut.

1. Introduction

Percussion instruments, such as the xylophone, marimba or metallophone, are consisting of a series wooden bars or metal plates. The percussion sound for the uniform bar or plate may reveal inharmonic due to the transverse vibration modes which frequencies are not harmonically related. Ordutla-Bustaman [1] developed the non-uniform beam model to optimally design the xylophone bar with the undercut so that the frequency ratios of sound radiated modes 2 and 3 are harmonically related with the fundamental one (mode 1). He also presented seven pairs of optimal undercut parameters that will result in the harmonic sound effects. Bretos *et al*. [2] presented the use of finite element analysis (FEA) to analyze the effect of wooden material properties on natural frequency of xylophone bar with the undercut. They adopted the orthotropic material model in FEA and found

Young's modulus and shear modulus along the grain of the wood material being the most crucial parameters as well as the density.

Other than the wooden bar design for achieving the harmonic sound of xylophone, there are similar works in finding the harmonic sound for new types of percussion instruments. Wang and Chien [3] designed a special shape of uniform plate that can produce the harmonic sound effect. Their concept is to use the same thickness of plate but with shape design to achieve the harmonically related modes, in particular for those sound radiated modes. McLachlan [4] adopted FEA to design and manufacture the folded octagonal gong that can produce up to the first five overtones tuned to within 5% of the harmonic sound. McLachlan [5] created the parametric finite element model to design the bell that contains up to seven partial frequencies in the harmonic series.

Rossing and his colleagues [6,7] had the review on the study of percussion instruments. Actually, various types of percussion instruments [8-14] have been drawn many attentions to characterize their vibration properties and acoustic response. Some ancient instruments may also contain special wisdom on the structural design, such as Chinese bell with the semi-tone design for different struck locations [11,12], and Chinese and Korean stone chimes with nearly harmonic sound effect [13,14]. There is a type of hollow pipe shape of bamboo percussion instrument with undercut found in Taiwan [15] or maybe elsewhere. The intention of this work originally comes from this instrument and is to mimic the structure to develop the percussion instrument in a scientific way.

This work shows the procedure to develop the percussion instrument of hollow pipe with the undercut. First, the selection of the pipe for making the instrument is studied to identify the material properties that can be adopted for design analysis. Second, the parametric geometry model is constructed according to the basic shape of hollow pipe with the undercut and performed FEA for structural modal analysis to obtain natural frequencies and corresponding mode shapes. As one knows that the percussion sound pitch frequency as well as overtone frequencies are related to structural modal parameters, the preliminary study is to obtain the initial geometry design of the hollow pipe with the undercut that can produce the correct pitch frequency for some musical note. In this work, Note A5 which pitch frequency is 880Hz is demonstrated to discuss both structural vibration characteristics and percussion sound spectrum and show the feasibility of using the hollow pipe with the undercut to make a new type of percussion instrument.

2. Identification of Material Properties of Stainless Steel Pipe

Since the objective of this work is to investigate the possibility of using the hollow pipe to manufacture the percussion instrument, several commercially available and cheap pipes are first selected to perform tests on percussion sound. This section shows the procedure to identify the material properties of stainless steel pipe. Model Verification (MV) is conducted on the pipe by performing FE modal analysis and experimental modal analysis (EMA). After quick evaluation, the stainless steel hollow pipe with diameter 30mm and 1mm thickness is chosen. The pipe without undercut is 147mm in length. The density is measured with 7928.14 (kg/m^3), and the presumed isotropic material properties are 189 GPa for Young's Modulus and 0.34 for Poisson ratio.

The FE model is constructed by linear hexahedron elements with eight nodes, SOLID45 in ANSYS. Theoretical modal analysis (TMA) is then carried out to obtain structural natural frequencies and mode shapes. The convergence test for different meshes of the FE model with respect to natural frequencies up to 10 modes about 8,000 Hz is within 1% errors, while the final refined mesh FE model contains 14,553 elements and 29,304 nodes.

EMA is also conducted on the pipe by roving the impact hammer as the exciter through 40 grid points and fixed the microphone near the pipe to measure sound pressure; therefore, the frequency response function (FRF) between the sound pressure and impact force can be determined and used to obtain experimental modal parameters via the curve fitting software, ME'scopeVES.

The first column group in Table 1 reveals mode numbers and experimental natural frequencies. The second column group is from FEA before calibration, and the errors in comparison to those from EMA are also shown. The averaged (AVG) and root mean square (RMS) are -1.78% and 2.19%, respectively. After the calibration of Young's Modulus as 19.3 GPa, the pipe's natural frequencies and errors are summarized in the third column group of Table 1. The AVG and RMS are reduced to -0.64% and 1.44%, respectively. Some selected mode shapes are shown in Figure 1, and the red arrows indicate the nodal line locations of corresponding mode shapes. One can observe the physical meanings of mode shapes between FEA and EMA can match each other very well.

One will notice that there are axisymmetric modes for the hollow pipe structure, such as F-01/F-02 and F-05/F-06 color shaded in Table 1. The FEA natural frequencies for those axisymmetric modes are exact the same for perfect symmetric geometry; however, the EMA natural frequencies reveal slight difference between axisymmetric modes. This indicates some imperfect symmetric geometry for the hollow pipe. Nevertheless, the integration of FEA and EMA to calibrate the analytical FE model is complete base on the very good agreement of modal properties. The analytical approach in adopting the calibrated FE model can then be used to modify the hollow pipe with the undercut for design analysis.

Table 1. Comparison of natural frequencies between FEA and EMA for hollow pipe without undercut.

Mode No.	EMA (Hz)	Mode No.	FEA before calibration		FEA after calibration		Physical meaning of mode shapes
			Natural frequency (Hz)	Frequency error (%)	Natural frequency (Hz)	Frequency error (%)	
E-01	2743.6	F-01	2732.5	-0.40	2764.2	0.75	(θ,z)=(2,1)
E-02	2777.5	F-02	2732.5	-1.62	2764.2	-0.48	(θ,z)=(2,1)
E-03	2793.6	F-03	2772.3	-0.76	2804.4	0.39	(θ,z)=(2,2)
E-04	2825.5	F-04	2772.3	-1.88	2804.4	-0.74	(θ,z)=(2,2)
E-05	4040.6	F-05	3956.0	-2.09	4001.8	-0.96	(θ,z)=(2,3)
E-06	4072.2	F-06	3956.0	-2.85	4001.8	-1.73	(θ,z)=(2,3)
E-07	7279.8	F-07	7036.4	-3.34	7118.0	-2.22	x-bending
E-08	7305.3	F-08	7036.5	-3.68	7118.1	-2.56	y-bending
E-09	7558.8	F-09	7370.6	-2.49	7456.1	-1.36	(θ,z)=(2,4)
E-10	7636.2	F-10	7370.6	-3.48	7456.1	-2.36	(θ,z)=(2,4)
E-11	7719.9	F-11	7714.7	-0.07	7804.1	1.09	(θ,z)=(3,1)
--	--	F-12	7714.9	--	7804.4	--	(θ,z)=(3,1)
E-12	7783.3	F-13	7763.2	-0.26	7853.2	0.90	(θ,z)=(3,2)
E-13	7785.6	F-14	7763.4	-0.29	7853.4	0.87	(θ,z)=(3,2)
AVG	--	--	--	-1.78	--	-0.64	--
RMS	--	--	--	2.19	--	1.44	--

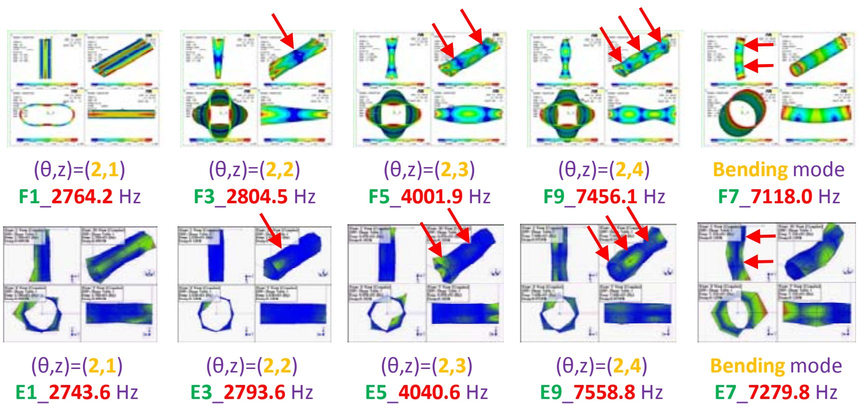


Figure 1. Structural mode shapes for hollow pipe without undercut.

3. Structural Vibration Characteristics of Hollow Pipe with Undercut

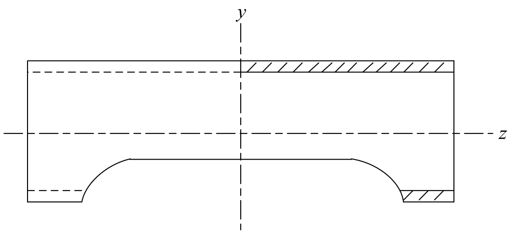
This section shows the initial geometry design analysis of the hollow pipe with the undercut. The parametric geometry model is shown first. The trial-and-error method is performed to obtain a set of geometry parameter to design the pipe with the fundamental frequency 880Hz that is suitable for Note A5. Both FEA and EMA are conducted to obtain structural modal parameters of the hollow pipe with the undercut.

3.1 Parametric geometry model and formulation of optimum design model

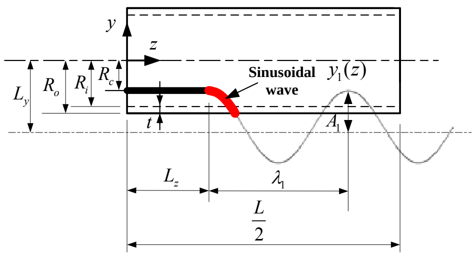
The definition of parametric geometry model is important so as to identify appropriate geometric parameters such that those parameters can be used as design variables for optimum design analysis to tune the pitch frequency. Figure 2(a) depicts the exaggerated geometry of the hollow pipe with the undercut in the middle that is symmetric along *z*-axis. Figure 2(b) shows the half model of the pipe with those geometric parameters defined as follows. The total length of the pipe is *L*; the outer radius of the pipe is R_o; the inner radius is R_i; the pipe thickness is *t*; The undercut is defined by two geometric features. One is the straight line, defined by the length L_z and the radius R_c. The other is the sinusoidal wave as shown the red curve in Figure 2(b) and defined as follows:

(1) $$y_1(z) = A_1 \cos(\frac{2\pi}{\lambda_1} z) .$$

$y_1(z)$ is the cosine wave as depicted in Figure 2(b); A_1 is the amplitude, λ_1 is the wavelength, and *z* is the location along *z*-axis. The design of the undercut with the two features can be varied to obtain wide range of undercuts; in particular the undercut can be manufactured feasibly by laser cut.



(a) basic shape



(b) parametric geometry

Figure 2. Geometry model of hollow pipe with the undercut.

In summary, once the hollow pipe is chosen, the pipe geometry parameters are fixed, including, R_o, R_i, and *t*. For the design of the hollow pipe with the prescribed undercut to fit the specific musical note, the design variables (*X*) can be identified as follows:

(2) $$X = X(R_c, L_z, A_1, \lambda_1, L) .$$

R_c and L_z are for the straight line, and A_1 and λ_1 are for the cosine wave; The total length of the pipe *L* is also to be determined. The geometric constraint is needed and specified accordingly to avoid the conflict between geometric variables.

Since the design objective is to find the structural natural frequencies matching with the pitch frequency of musical note as well as their overtone frequencies, the objective function $F(X)$ can be defined as follows:

(3) $$F(X) = \sum_{r=1}^{N} \sqrt{(\frac{f_r - f_{obj,r}}{f_{obj,r}})^2} .$$

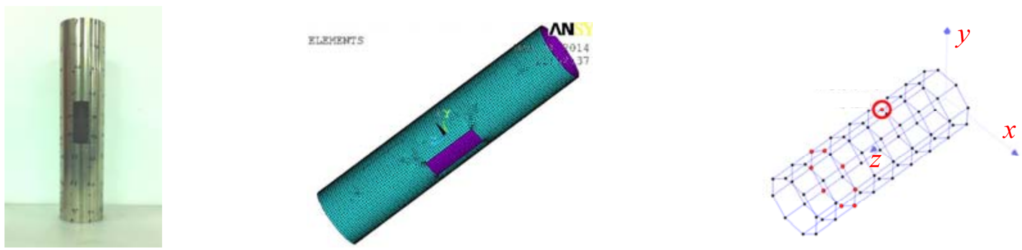
where N is the number of modes to be included; f_r is the r-th structural mode that can radiate percussion sound; $f_{obj,r}$ can be the r-th target overtone frequency as desired. It is noted that r=1 is the fundamental mode which natural frequency should be equal to the pitch frequency. Those modes for $r \geq 2$ can be the overtone frequencies. The desired objective frequencies $f_{obj,r}$ can be selected such as with harmonic series.

3.2 FEA and EMA for Note A5 of the hollow pipe with undercut

Base on the parametric geometry model shown in Figure 2, a series of trial-and-error FEA vibration analysis are performed to find the best fit of hollow pipe with the undercut for Note A5 which pitch frequency is 880Hz.

Figure 3(a) shows the manufactured hollow pipe with the undercut, while Figure 3(b) is the developed FE model. The material properties are set as those obtained in Section 2. Structural modal analysis can be performed on the FE model which geometry parameters as shown in Eq. (2) are modified to find the best fit of fundamental frequency to be 880 Hz. The obtained natural frequencies and mode shapes will be discussed in next Section.

EMA is also carried out similarly as discussed in Section 2, the grid points of the pipe for EMA are shown in Figure 3(c). There are eight points along the circumferential surfaces and nine divisions along the pipe of z-axis. The FRFs will be measured and processed to obtain experimental modal parameters, including natural frequencies, mode shapes, and modal damping ratios.



(a) real structure (b) FE model (c) grid points of pipe for EMA

Figure 3. Hollow pipe with undercut for Note A5.

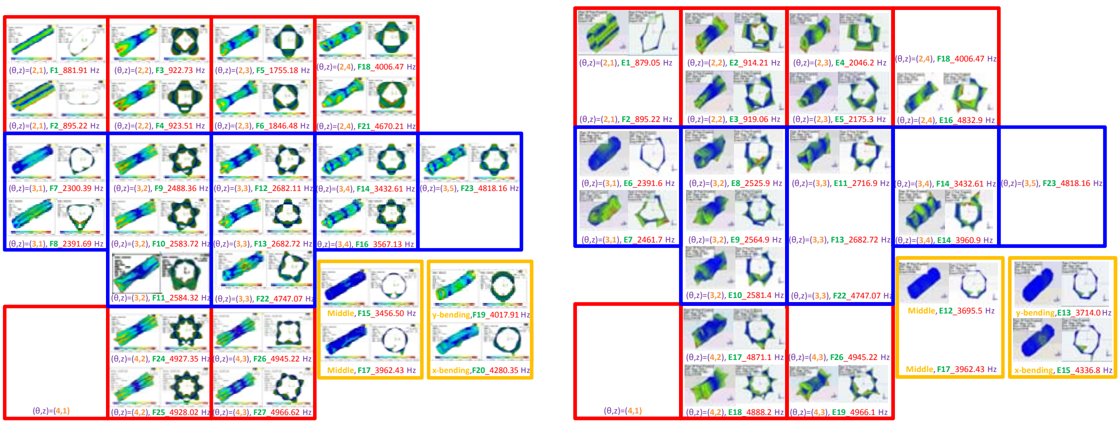
3.3 Vibration characteristics of hollow pipe with undercut

Table 2 shows the comparison of natural frequencies obtained from FEA and EMA for Note A5 of the hollow pipe with the undercut. The first column group refers to FEA results, the second one is for EMA, and the third column shows the frequency errors; The physical meaning of mode shapes and modal damping ratio are also shown for each mode. Figures 4(a) and 4(b) show the corresponding mode shapes for FEA and EMA, respectively. Those axisymmetric modes are grouped in Fig. 4 as well as colour shading in Table 2. Some interesting observations are discussed as follows:

- Natural frequencies between FEA and EMA show very good agreement within 4% errors, except several modes highlighted in red. Both FEA and EMA mode shapes shown in Figs. 4(a) and 4(b), respectively, generally agree very well, although some modes are not determined in EMA and left blank in Fig. 4(b) for comparison purpose.
- The hollow pipe with the undercut is no more perfectly axisymmetric; therefore, those axisymmetric modes are not with the same natural frequency. For the hollow pipe without the undercut, the axisymmetric modes generally contain two modes; however, the pipe with the undercut might have three axisymmetric modes, such as (θ,z)=(3,2) and (θ,z)=(3,3) shaded with the same colour in Table 2. And, interestingly, (θ,z)=(4,1) mode is missing and left blank intentionally in Figs. 4(a) and 4(b), though (θ,z)=(4,2) is present.

Table 2. Comparison of natural frequencies between FEA and EMA for hollow pipe with undercut of Note A5.

mode	FEA (HZ)	mode	EMA (HZ)	error (%)	Physical meaning of mode shape	Damping Ratio (%)
F-01	881.9	E-01	879.1	0.32	Global(θ,z)=(2,1)	0.1500
F-02	895.2	--	--	--	Global(θ,z)=(2,1)	--
F-03	922.7	E-02	914.2	0.93	Global(θ,z)=(2,2)	0.1979
F-04	923.5	E-03	919.1	0.48	Global(θ,z)=(2,2)	0.1450
F-05	1755.1	E-04	2046.2	**-14.22**	Global(θ,z)=(2,3)	0.1592
F-06	1846.4	E-05	2175.3	**-15.12**	Global(θ,z)=(2,3)	0.0837
F-07	2300.3	E-06	2391.6	-3.81	Global(θ,z)=(3,1)	0.0971
F-08	2391.6	E-07	2461.7	-2.84	Global(θ,z)=(3,1)	0.6304
F-09	2488.3	E-08	2525.9	-1.49	Global(θ,z)=(3,2)	0.0770
F-10	2583.7	E-09	2564.9	0.73	Global(θ,z)=(3,2)	0.1537
F-11	2584.3	E-10	2581.4	0.11	Global(θ,z)=(3,2)	0.0741
F-12	2682.1	E-11	2716.9	-1.28	Global(θ,z)=(3,3)	0.0728
F-13	2682.7	--	--	--	Global(θ,z)=(3,3)	--
F-14	3432.6	--	--	--	Global(θ,z)=(3,4)	--
F-15	3456.5	E-12	3695.5	**-6.47**	Local, Middle	--
F-16	3567.1	**E-14**	3960.9	**-9.94**	Global(θ,z)=(3,4)	0.1550
F-17	3962.4	--	--	--	Local, Middle	--
F-18	4006.4	--	--	--	Global(θ,z)=(2,4)	--
F-19	4017.9	**E-13**	3714.0	**8.18**	y-bending (z,y)=(3,1)	0.0827
F-20	4280.3	E-15	4336.8	-1.30	x-bending (z,x)=(3,3)	0.0610
F-21	4670.2	E-16	4832.9	-3.37	Global(θ,z)=(2,4)	0.3442
F-22	4747.0	--	--	--	Global(θ,z)=(3,3)	--
F-23	4818.1	--	--	--	Global(θ,z)=(3,5)	--
F-24	4927.3	E-17	4871.1	1.16	Global(z,y)=(4,2)	0.0612
F-25	4928.0	E-18	4888.2	0.82	Global(θ,z)=(4,2)	0.0576
F-26	4945.2	--	--	--	Global(θ,z)=(4,2)	
F-27	4966.6	E-19	4966.1	0.01	Global(θ,z)=(4,3)	0.0949



(a) FEA (b) EMA

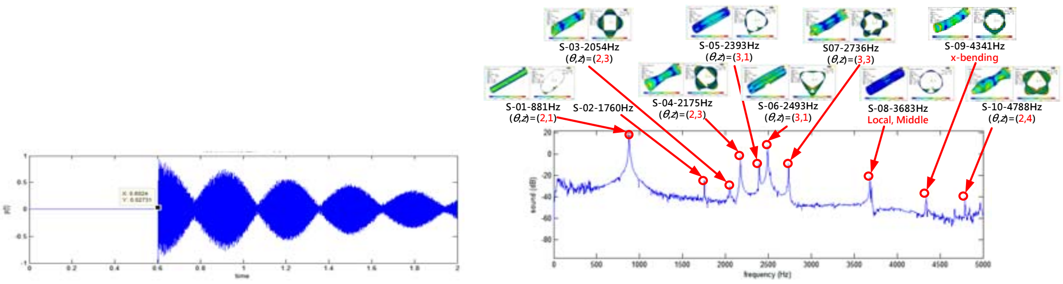
Figure 4. Hollow pipe with undercut for Note A5.

- There are two types of special modal properties regarding to the hollow pipe with the undercut, i.e. the local modes (F-15 & F-17) for middle surface vibration due to the undercut and the bending modes of the hollow pipe in both x-bending (F-20) and y-bending (F-19) vibration, respectively.
- Since the hollow pipe is designed for struck at the centre location on the top of the undercut, those modes with nodal lines near the centre, such as (θ,z)=(2,2), (θ,z)=(3,2) and (θ,z)=(4,2), will not be excited and do not radiate percussion sound in theory.

In summary, the analytical and experimental modal characteristics of the hollow pipe with undercut can be well interpreted and matched very well. The FE model in predicting structural resonance frequencies is validated and can be further applied to design analysis for other musical notes.

4. Percussion Sound Characteristics of Note A5

This section will discuss the perceived percussion sound for Note A5 of the hollow pipe with the specified undercut. Figures 5(a) and 5(b) show both time and frequency domain response, respectively, for the hollow pipe struck at the centre location on the top of the undercut. The sampling frequency is 44.4 kHz, and frequency resolution is 1 Hz. The auto sound spectrum in Fig. 5(b) is obtained by FFT applying exponential window for 20 averages 90% overlap. In Fig. 5(b), the corresponding mode shapes for the peak resonances are also depicted. Only those modes (θ,z) for odd number of z contribute to radiated sound. Figure 5(a) shows a typical beating phenomenon due to S-02=1760Hz, an unidentified structural mode. The radiated percussion sound from the hollow pipe with the undercut can be identified with the fundamental frequency 881Hz that is close to the pitch frequency 880Hz for Note A5. Those overtone frequencies come from higher structural modes as those modes (θ,z) for odd number of z. The well separated overtone modes can reveal a good perceived sound quality. In particular, the overtone frequencies can be adjusted by optimum analysis on the undercut design such as those geometry variables shown in Eq. (2) and under investigation.



(a) Time domain response (b) Frequency domain response

Figure 5. Hollow pipe with the undercut for Note A5.

5. Discussions and Conclusions

This paper shows the systematic procedure to develop a new type of percussion instrument, i.e. the hollow pipe with the specific undercut as layout in Fig. 2. The idea of using the hollow pipe comes from the inspiration of traditional bamboo percussion instrument in Taiwan [15]. The indigenous people also have this similar configuration of percussion instrument. It will be interesting to investigate the sound quality of such instrument and to precisely control the geometry design for better sound quality. This work summarizes the development process and suggests the future implementation. The complete sound waves for different notes can be reviewed at website [16].

We start with the selection of hollow pipe in considering geometry, material, manufacture, availability, cost and perceived sound quality. In preliminary study, several different diameters of hollow pipes made of stainless steel and bronze alloy are considered. For making the final product similar to marimbas or xylophones, the suitable diameter of the pipe should be better about 3-5mm; therefore, the instrument can be in a reasonable length.

Section 2 presents the application of FEA and EMA to perform model verification of the selected hollow pipe, a thin wall cylindrical tube. The FE model for the hollow pipe can be well calibrated by the comparison of modal parameters obtained from FEA and EMA. The material properties can be identified and applied for further analysis. In this stage, the key issue is to evaluate the proper selection of the pipe for the next stage to design and manufacture the hollow pipe with the undercut.

For quick evaluation of a structural geometry to be the percussion instrument, computer aided engineering (CAE) technique is required. FEA vibration analysis to obtain structural natural frequencies and corresponding mode shapes are particularly important. Before the application of CAE software for design analysis or optimum geometry design, the parametric geometry model is needed such as discussed in Section 3.1. In this work, the hollow pipe with the undercut as shown in Fig. 2(b) contains five design variables as defined in Eq. (2). To formulate the design optimization problem, one has to clearly define design variables (DVs), objective function (OBJ) and constraint equations (CEs), respectively. The selection of OBJ is crucial in the design of qualitative percussion instrument. In Eq. (3), not only the fundamental or pitch frequency is considered, but also as many modes as desired to achieve overtone harmony can be included as well.

The main idea of OBJ is base on the least square error method. The sound radiated mode f_r needs to be carefully justified by the structural mode shape. For example about the hollow pipe with the undercut in this work, only those modes (θ,z) for odd number of z can contribute to the radiated sound as discussed in Fig. 5(b).

This work shows the initial design of the hollow pipe with the undercut for Note A5 and discusses its vibration characteristics analytically and experimentally as well as the percussion sound. The perceived sound quality can be well interpreted and related to structural modal vibration. The potential of the hollow pipe with the undercut to make a percussion instrument is demonstrated. Furthermore, the design of different musical notes can be proceeded to complete the design of a whole set of pipes with two or more octaves that can be made for the percussion instrument.

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