Ground Vehicle Dynamic Analysis for Ride Quality

Bor-Tsuen Wang

Professor and Dean
College of Engineering
National Pingtung University of Science and Technology
TEL: 886-8-770-3202 ext. 7017,7981
FAX: 886-8-774-0448
E-mail: wangbt@mail.npust.edu.tw
www: http://140.127.6.133/lab/

www: http://www.npust.edu.tw
E-mail: coe@mail.npust.edu.tw

Outline

- I. Introduction
- II. Procedure for Vehicle Ride Comfort Analysis
- III. Issues to know
  - 3.1 Vehicle Riding Model
  - 3.2 Road Model
  - 3.3 Ride Quality Parameters
  - 3.4 Ride Comfort Criterion
  - 3.5 Solution of Vibration System
- IV. Examples of Vehicle Ride Comfort Analysis
- V. Conclusions

I. Introduction

- Types of vehicles
  - Ground vehicle: supported by ground
  - Aircraft: supported by air
  - Submarine: supported by water
- Ground vehicles
  - Guided vehicles: constrained to move along a fixed path
    - railway vehicle
    - Tracked levitated vehicle
  - Non-Guided vehicles:
    - On-road vehicle: on paved road
    - Off-road vehicle: over unprepared terrain

I. Introduction (Cont'd)

- Possible objectives for vehicle dynamic analysis:
  - obtain physical insight of mathematical model behavior
  - Explore component design concept
  - Evaluate vehicle design
  - Compare with a test on vehicle
  - Perform safety evaluation
  - Conduct accident analysis

I. Introduction (Cont'd)

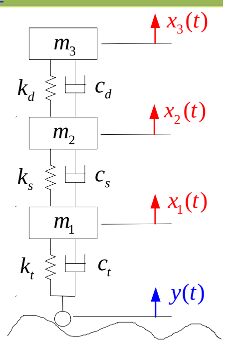
- Major concerns of vehicle dynamic analysis:
  - Performance quality:
    - accelerating, traction, braking, aerodynamics, rolling resistance
  - Handling quality:
    - turning, cornering, steering, directional stability
  - Ride quality:
    - Ride comfort
    - perception of noise, vibration and harshness (NVH) for passengers or cargos

I. Introduction (Cont'd)

- This presentation focuses on:
  - Ground vehicle:
    - non-guided
    - on-road
  - Ride quality -- Ride comfort
    - Perception of vibration for human in the vehicle
- The topics will cover
  - The analytical approach for vehicle ride comfort analysis
  - The solution of vibration system regarding to vehicle dynamic problems
  - The assessment of ride comfort for a simple car model

II. Procedure for Vehicle Ride Comfort Analysis

- 1. Construct Vehicle Riding Model
  - Mathematical modeling
    - Define system model
    - Define system input
      - Road model
    - Define interested output
      - Ride quality parameters
- 2. Solve for system output response
  - (1) modal analysis
  - (2) harmonic response analysis
  - (3) transient response analysis
  - (4) spectrum response analysis
- 3. Compare with ride comfort criterion



III. Issues to know

- 3.1 Vehicle Riding Model
- 3.2 Road Model
- 3.3 Ride Quality Parameters
- 3.4 Ride Comfort Criterion
- 3.5 Solution of Vibration System

3.1 Vehicle Riding Model

- 3.1.1 Examples of Vehicle Riding Models
  - SDOF Quarter Car Model
  - 3 DOFs Quarter Car Model
  - 5 DOFs Half Car Model
  - 7 DOFs Full Car Model
- 3.1.2 Mathematical Modeling
  - 1. Define a multi-body system
  - 2. Define interconnection
  - 3. Define system degree-of-freedom (DOF)
  - 4. Define system input
  - 5. Define interested output

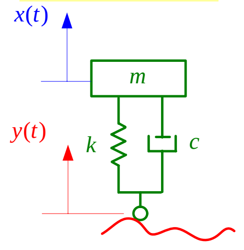
3.1.1 Examples of Vehicle Riding Models

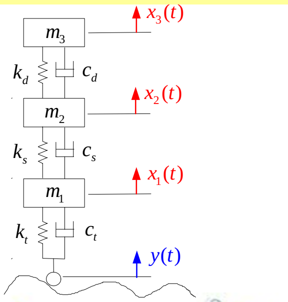
- SDOF Quarter Car Model

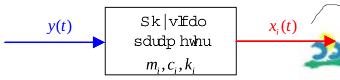
$$m\ddot{x}+c\dot{x}+kx=c\dot{y}+ky$$

- 3 DOFs Quarter Car Model

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\}$$

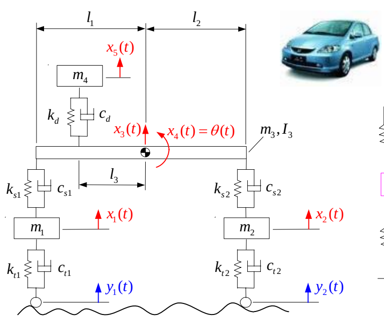


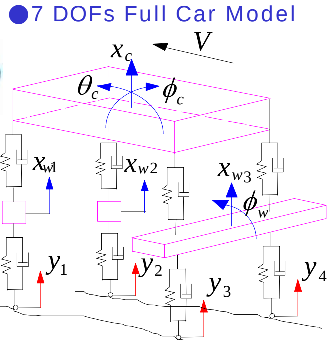




3.1.1 Examples of Vehicle Riding Models

- 5 DOFs Half Car Model
- 7 DOFs Full Car Model

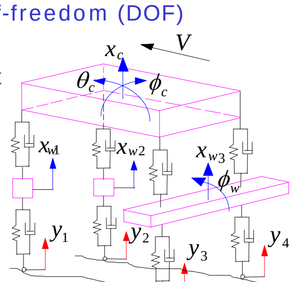




$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\}$$

3.1.2 Mathematical Modeling

- 1. Define a multi-body system
- 2. Define interconnection
- 3. Define system degree-of-freedom (DOF)
- 4. Define system input
- 5. Define interested output
  - Reasonable
  - Correct
  - Adequate



III. Issues to know

- 3.1 Vehicle Riding Model
  - 3.1.1 Examples of Vehicle Riding Models
  - 3.1.2 Mathematical Modeling
- 3.2 Road Model
  - 3.2.1 Irregular Random Profile
  - 3.2.2 Continuous Sinusoidal Profile
  - 3.2.3 Half-Sine Bump
- 3.3 Ride Quality Parameters
- 3.4 Ride Comfort Criterion
- 3.5 Solution of Vibration System

3.2 Road Model
3.2.1 Irregular Random Profile

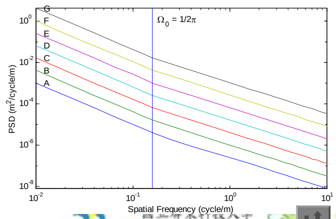
- The ISO road model – PSD function in spatial frequency

$$S_g(\Omega)=S_g(\Omega_0)\left(\frac{\Omega}{\Omega_0}\right)^{-N_1},\ \Omega\le\Omega_0=\frac{1}{2\pi}\ (\frac{\text{cycle}}{\text{m}})\qquad N_1=2.0$$

$$S_g(\Omega)=S_g(\Omega_0)\left(\frac{\Omega}{\Omega_0}\right)^{-N_2},\ \Omega>\Omega_0=\frac{1}{2\pi}\ (\frac{\text{cycle}}{\text{m}})\qquad N_2=1.5$$

Classification of Road Roughness Proposed by ISO (1982)

Degree of Roughness $S_g(\Omega_0)$, 10^{-6} m^2/cycles/m		
Road Class	Range	Geometric Mean
A (Very Good)	<8	4
B (Good)	8-32	16
C (Average)	32-128	64
D (Poor)	128-512	256
E (Very Poor)	512-2048	1024
F	2048-8192	4096
G	8192-32768	16384
H	>32768	



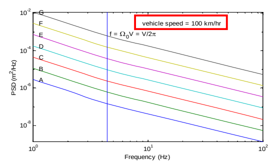
3.2 Road Model
3.2.1 Irregular Random Profile

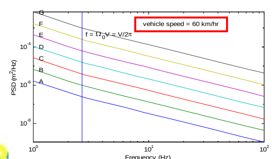
- The road profile PSD function
  - for Road Class A with geometric mean
  - in temporal frequency
  - regarding to vehicle speed Y

$$S_g(f)=\frac{4\times10^{-6}\left(2\pi\frac{f}{V}\right)^{-2}}{V}\left(\frac{\text{m}^2}{\text{Hz}}\right)\qquad \frac{f}{V}\le\frac{1}{2\pi}$$

$$S_g(f)=\frac{4\times10^{-6}\left(2\pi\frac{f}{V}\right)^{-1.5}}{V}\left(\frac{\text{m}^2}{\text{Hz}}\right)\qquad \frac{f}{V}>\frac{1}{2\pi}$$

$$f=\Omega V$$
$$(\text{Hz})=(\frac{\text{cycle}}{\text{m}})(\frac{\text{m}}{\text{s}})$$





3.2 Road Model
3.2.2 Continuous Sinusoidal Profile

- spatial domain equation:

$$y(x)=Y\sin\frac{2\pi}{L}x$$

where

L =wave length (m)

Y =displacement amplitude (m)

V =vehicle speed (m/sec)

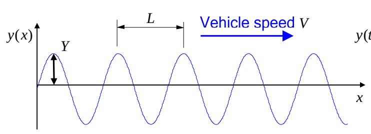
- time domain equation:

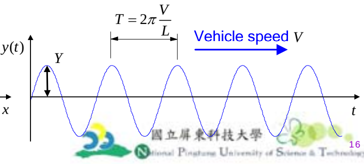
$$y(t)=Y\sin\omega t=Y\sin2\pi ft=Y\sin2\pi\frac{V}{L}t$$

where

$f=\frac{V}{L}$ =excitation frequency (Hz)

$\omega=2\pi f=2\pi\frac{V}{L}$ = excitation frequency (rad/sec)



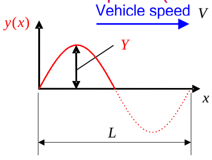


3.2 Road Model
3.2.2 Half-Sine Bump

- spatial domain equation:

$$y(x)=\begin{cases} Y\sin\frac{2\pi}{L}x, 0<x<\frac{L}{2} \\ 0, x\ge\frac{L}{2}\end{cases}$$

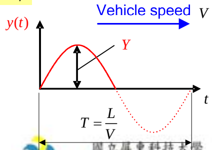
where
L =wave length (m)
Y =displacement amplitude (m)
V =vehicle speed (m/sec)



- time domain equation:

$$y(t)=\begin{cases} Y\sin 2\pi\left(\frac{V}{L}\right)t, 0<t<\frac{L}{2V} \\ 0, t\ge\frac{L}{2V}\end{cases}$$

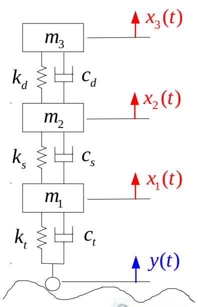
where
$f=\frac{1}{T}=\frac{V}{L}$ = excitation frequency (Hz)
$T=\frac{L}{V}$ = period (sec)



3.3 Ride Quality Parameters

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\}$$

- 1. Driver/passenger acceleration
  $\ddot{x}_3(t)$
- 2. SEAT (Seat Effective Amplitude Transmissibility)
  $$SEAT(\%)=\frac{\text{seat r.m.s.}}{\text{floor r.m.s.}}\times 100=\frac{\ddot{x}_{3,r.m.s.}}{\ddot{x}_{2,r.m.s.}}\times 100$$
- 3. Road holding
  $h(t)=x_1(t)-y(t)$
- 4. Suspension travel
  $s(t)=x_2(t)-x_1(t)$
- 5. Tire force
  $$f_t(t)=k_t h(t)+c_t\dot{h}(t)=k_t[x_1(t)-y(t)]+c_t[\dot{x}_1(t)-\dot{y}(t)]$$
- 6.Suspension force
  $$f_s(t)=k_s s(t)+c_s\dot{s}(t)=k_s[x_1(t)-x_2(t)]+c_s[\dot{x}_1(t)-\dot{x}_2(t)]$$



III. Issues to know

- ▲ 3.1 Vehicle Riding Model
- ▲ 3.2 Road Model
- ▲ 3.3 Ride Quality Parameters
- 3.4 Ride Comfort Criterion
  - 3.4.1 Overview of ISO 2631-1 (1997)
    - ▲ Evaluation indices
    - ▲ Comfort evaluation procedure
  - 3.4.2 Comfort evaluation in ISO 2631-1 (1985)
- 3.5 Solution of Vibration System

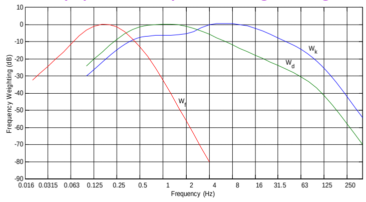
3.4 Ride Comfort Criterion

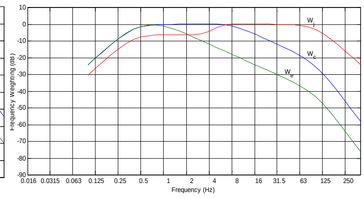
- International Organization for Standardization, 1997, ISO 2631-1. Mechanical Vibration and Shock – Evaluation of Human Exposure to Whole–Body Vibration. Part 1: General Requirements. Geneva.
- International Organization for Standardization, 1985, ISO 2631-1. Evaluation of Human Exposure to Whole–Body Vibration. Part 1: General Requirements. Geneva.
- British Standards Institution BS 6841, 1987, Measurement and Evaluation of Human Exposure To Whole–Body Mechanical Vibration. London.

3.4.1 Overview of ISO 2631-1 (1997)

- The effect of vibration on human comfort is evaluated by using the frequency weighted r.m.s. acceleration.

(a) Principal weightings (b) Additional weightings



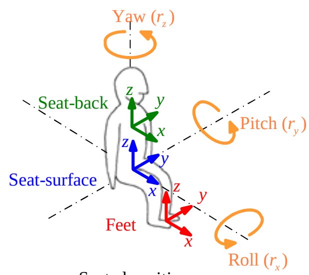


3.4.1 Overview of ISO 2631-1 (1997)

- Frequency-weighting functions recommended for the various directions are as follows.

Comfort evaluation (0.5-80Hz)

Human Body Position	Measurement Location	Axis	Multiplying Factor *k*	Frequency Weighting
Seated	Seat surface	x	1	W_d
		y	1	W_d
		z	1	W_k
		r_x	0.63 m/rad	W_e
		r_y	0.4 m/rad	W_e
		r_z	0.2 m/rad	W_e
	Seat backrest	x	0.8	W_c
		y	0.5	W_d
		z	0.4	W_d
	Feet	x	0.25	W_k
		y	0.25	W_k
		z	0.4	W_k



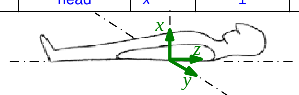
Seated position

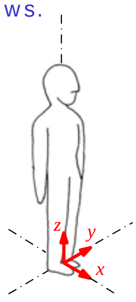
3.4.1 Overview of ISO 2631-1 (1997)

- Frequency-weighting functions recommended for the various directions are as follows.

Comfort evaluation (0.5-80Hz)

Human Body Position	Measurement Location	Axis	Multiplying Factor *k*	Frequency Weighting
Standing	Floor	x	1	W_d
		y	1	W_d
		z	1	W_k
Recumbent	Supporting area (under the pelvis except head)	$x^{vertical}$	1	W_k
		$y^{horizontal}$	1	W_d
		$z^{horizontal}$	1	W_d
	head	$x^{vertical}$	1	W_j


Recumbent position


Standing position

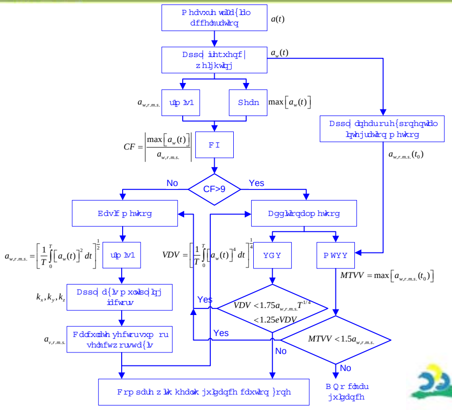
3.4.1 Overview of ISO 2631-1 (1997)

- For comfort evaluation, the r.m.s. value of the frequency weighted acceleration can be compared with the guidance shown below.

Approximate indications of likely reactions to various magnitudes of overall vibration total values in public transport as stated in ISO 2631-1 (1997).

Weighted vibration magnitude (sum of three axes)	Likely reaction in public transport
Less than 0.315 m/s²	Not uncomfortable
0.315 m/s² to 0.63 m/s²	A little uncomfortable
0.5 m/s² to 1 m/s²	Fairly uncomfortable
0.8 m/s² to 1.6 m/s²	Uncomfortable
1.25 m/s² to 2.5 m/s²	Very uncomfortable
Greater than 2 m/s²	Extremely uncomfortable

3.4.1 Overview of ISO 2631-1 (1997): Comfort evaluation procedure



Procedure for comfort evaluation and assessment of whole-body vibration according to ISO 2631-1 (1997).

3.4.1 Overview of ISO 2631-1 (1997): Evaluation indices

- Basic evaluation method:
  - The weighted r.m.s. acceleration: $a_{w,r.m.s.}$

$$a_{w,r.m.s.} = \left[\frac{1}{T}\int_0^T \left[a_w(t)\right]^2 dt\right]^{\frac{1}{2}}$$

- The overall weighted acceleration (1/3 octave): $a_{w,r.m.s.}$

$$a_{w,r.m.s.} = \left[\sum_i (W_i a_i)^2\right]^{\frac{1}{2}}$$

- The vibration total value or vector sum of weighted r.m.s. acceleration: $a_{v,r.m.s.}$

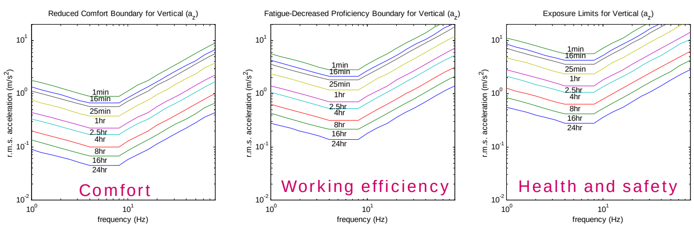
$$a_{v,r.m.s.} = \left[\left(k_x a_{wx,r.m.s.}\right)^2 + \left(k_y a_{wy,r.m.s.}\right)^2 + \left(k_z a_{wz,r.m.s.}\right)^2\right]^{\frac{1}{2}}$$

3.4.2 Comfort evaluation in ISO 2631-1 (1985)

- ISO 2631-1 (1985) with regard to whole-body vibration provides:
  - 1.Reduced Comfort Boundary (RCB)
  - 2.Fatigue-Decrease Proficiency Boundary (FDPB)
  - 3.Exposure Limit Boundary (ELB)
- These boundaries, respectively, specifies preservation of
  - 1.Comfort
  - 2.Working efficiency
  - 3.Health and safety

Exposure limits as a function of frequency and exposure time ISO 2631-1 (1985)

Longitudinal (a_z) acceleration limits

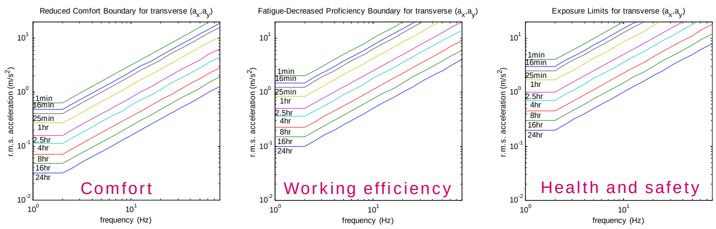


$$RCB = \frac{FDPB}{3.15} \text{ (a reduction of 10 dB)}$$

$$ELB = FDPB \times 2 \text{ (an increase of 6 dB)}$$

Exposure limits as a function of frequency and exposure time ISO 2631-1 (1985)

Transverse (a_x, a_y) acceleration limits

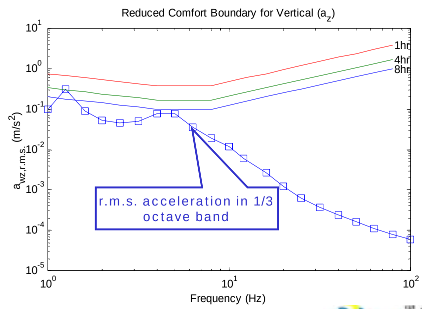


$$RCB = \frac{FDPB}{3.15} \text{ (a reduction of 10 dB)}$$

$$ELB = FDPB \times 2 \text{ (an increase of 6 dB)}$$

3.4.2 Comfort evaluation in ISO 2631-1 (1985)

- ISO 2631-1 (1985) evaluates the effect of vibration in frequency spectrum



III. Issues to know

- ▲ 3.1 Vehicle Riding Model
- ▲ 3.2 Road Model
- ▲ 3.3 Ride Quality Parameters
- ▲ 3.4 Ride Comfort Criterion
- 3.5 Solution of Vibration System
  - 3.5.1 Modal Analysis
  - 3.5.2 Harmonic Response Analysis
  - 3.5.3 Transient Response Analysis
  - 3.5.4 Spectrum Response Analysis

3.5 Solution of Vibration System 3.5.1 Modal Analysis

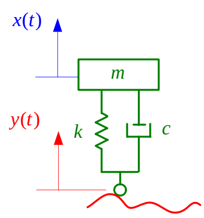
- SDOF quarter car model:
  - System equation:

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

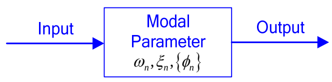
$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 2\xi\omega_n\dot{y} + \omega_n^2 y$$

where

- ▲ $\omega_n = \sqrt{k/m}$
- ▲ $\xi = c/c_c$
- ▲ $c_c = 2\sqrt{mk} = 2m\omega_n$

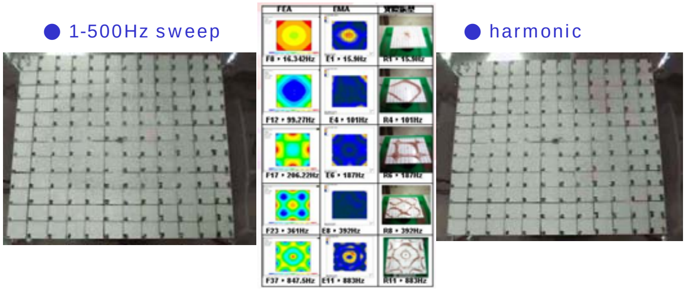


1. Modal Analysis



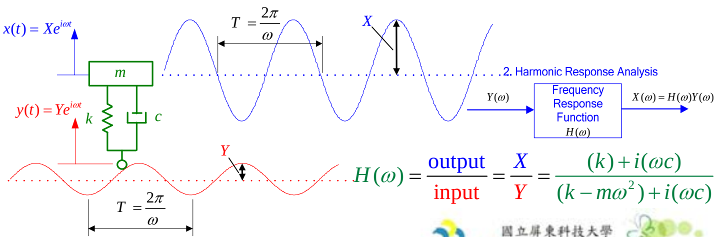
3.5 Solution of Vibration System
3.5.1 Modal Analysis

- Modal characteristics of a plate
  - 1-500Hz sweep
  - harmonic



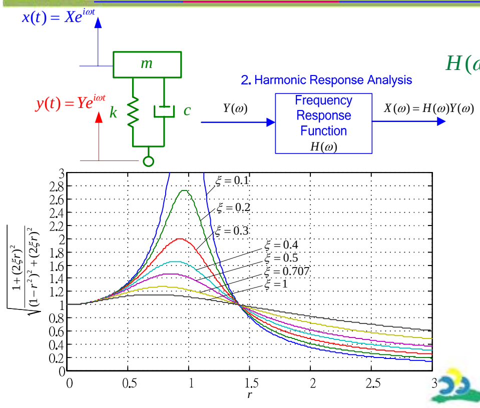
3.5 Solution of Vibration System
3.5.2 Harmonic Response Analysis

- Harmonic response analysis is to solve system frequency response functions (FRFs)
  - ▲ that can be defined as the output response divided by the input excitation due to harmonic or sinusoidal excitation.



$$H(\omega)=\frac{\text{output}}{\text{input}}=\frac{X}{Y}=\frac{(k)+i(\omega c)}{(k-m\omega^2)+i(\omega c)}$$

3.5 Solution of Vibration System
3.5.2 Harmonic Response Analysis



$$H(\omega)=\frac{X}{Y}$$
$$=\frac{k+i\omega c}{(k-m\omega^2)+i(\omega c)}$$
$$=\frac{\omega_n^2+i(2\xi\omega_n\omega)}{(\omega_n^2-\omega^2)+i(2\xi\omega_n\omega)}$$
$$=\frac{1+i2\xi r}{(1-r^2)+i(2\xi r)}$$
$$r=\frac{\omega}{\omega_n}$$

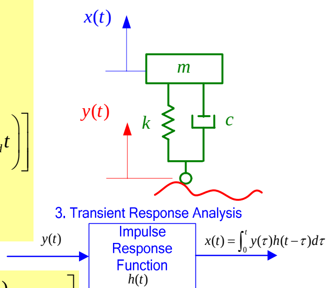
3.5 Solution of Vibration System
3.5.3 Transient Response Analysis

- For the known displacement input $y(t)$, the system output response can be determined by:

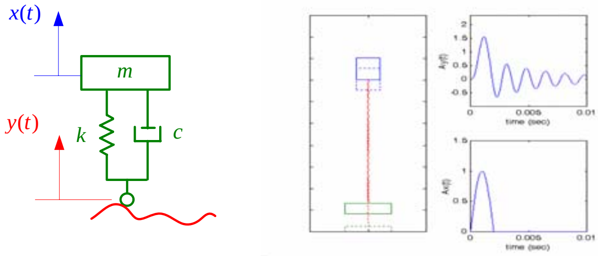
$$x(t)=\int_0^t y(\tau)h(t-\tau)d\tau$$
$$-2\xi\omega_n y_0\left(\frac{1}{\omega_d}e^{-\xi\omega_n t}\sin\omega_d t\right)$$
$$+x_0\left[e^{-\xi\omega_n t}\left(\cos\omega_d t+\frac{\xi\omega_n}{\omega_d}\sin\omega_d t\right)\right]$$
$$+v_0\left(\frac{1}{\omega_d}e^{-\xi\omega_n t}\sin\omega_d t\right)$$

where

$$h(t)=e^{-\xi\omega_n t}\left[2\xi\omega_n\cos\omega_d t+\frac{\omega_n^2(1-2\xi^2)}{\omega_d}\sin\omega_d t\right]$$

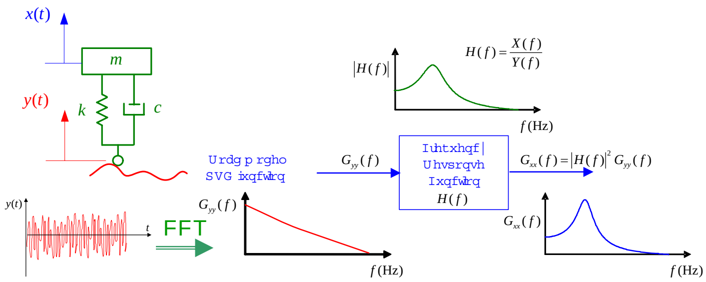


3.5 Solution of Vibration System
3.5.3 Transient Response Analysis



3.5 Solution of Vibration System
3.5.4 Spectrum Response Analysis

- Solve for frequency domain response
  - Due to random input



III. Issues to know

- 3.1 Vehicle Riding Model
- 3.2 Road Model
- 3.3 Ride Quality Parameters
- 3.4 Ride Comfort Criterion
- 3.5 Solution of Vibration System
  - 3.5.1 Modal Analysis
  - 3.5.2 Harmonic Response Analysis
  - 3.5.3 Transient Response Analysis
  - 3.5.4 Spectrum Response Analysis

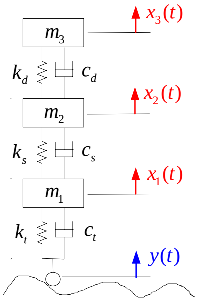
IV. Examples of Vehicle Ride Comfort Analysis: 3-DOF Quarter Car Model

- 4.1 System model definition
- 4.2 Solution of system model
  - 4.2.1 Modal Analysis
  - 4.2.2 Harmonic Response Analysis
  - 4.2.3 Transient Response Analysis
  - 4.2.4 Spectrum Response Analysis
- 4.3 Determination of ride quality parameters
- 4.4 Assessment of ride comfort

4.1 System model definition

- 3-DOF Quarter Car Model:

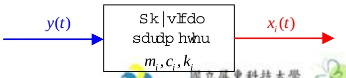
$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\}$$



$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\end{Bmatrix}+\begin{bmatrix} c_t+c_s & -c_s & 0 \\ -c_s & c_s+c_d & -c_d \\ 0 & -c_d & c_d \end{bmatrix}\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{Bmatrix}$$

$$+\begin{bmatrix} k_t+k_s & -k_s & 0 \\ -k_s & k_s+k_d & -k_d \\ 0 & -k_d & k_d \end{bmatrix}\begin{Bmatrix}x_1\\ x_2\\ x_3\end{Bmatrix}$$

$$=\begin{Bmatrix}c_t\\ 0\\ 0\end{Bmatrix}\dot{y}(t)+\begin{Bmatrix}k_t\\ 0\\ 0\end{Bmatrix}y(t)$$



4.1 System model definition: System Equations

- The general form of EOM can be given:

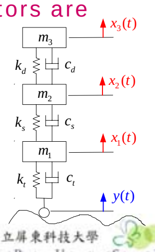
$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\}$$

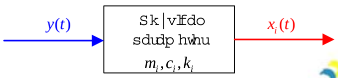
- For proportional damping,

$$[C]=\alpha[M]+\beta[K]$$

- The system input and output vectors are

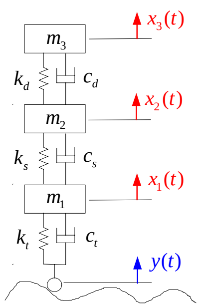
$$\{y(t)\}=\begin{Bmatrix}y_1\\ y_2\\ \vdots\\ y_m\end{Bmatrix} \qquad \{x(t)\}=\begin{Bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{Bmatrix}$$

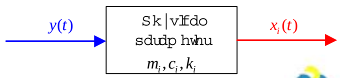




4.1 System model definition: System physical parameters

- 3-DOF Quarter Car Model:
  - m_1=31 kg
  - m_2=229 kg
  - m_3=60 kg;
  - k_t=120 kN/m
  - k_s=20 kN/m
  - k_d=40 kN/m
  - α =0;
  - β =0.01;





4.2 Solution of system model

- 4.2.1 Modal Analysis
- 4.2.2 Harmonic Response Analysis
- 4.2.3 Transient Response Analysis
- 4.2.4 Spectrum Response Analysis

4.2.1 Modal Analysis

- System equation:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\}$$

- For normal mode analysis, let

$$\{x\}=\{X\}e^{i\omega t}$$

- Assume

$$[C]=[0] \qquad \{y\}=\{0\}$$

- The generalized eigenvalue problem can be formulated:

$$[K]\{X\}=\omega^2[M]\{X\}$$

- By solving the above equation, q-pairs of eigenvalues ω_r^2 and eigenvectors $\{X_r\}$ can be obtained. The mass-matrix normalized mode shape can be determined

$$\{\phi_r\}=\frac{1}{\sqrt{m_r}}\{X_r\}$$

where

$$m_r=\{X_r\}^T[M]\{X_r\}$$

4.2.1 Modal Analysis

- Modal parameters can be determined through modal analysis:
  - Natural frequencies ω_r
  - Mode shapes $\{\phi_r\}$
  - Modal damping ratios $\xi_r=\dfrac{\alpha}{2\omega_r}+\dfrac{\beta\omega_r}{2}$
- The orthonormal relationships are summarized:

$$[\Phi]^T[M][\Phi]=\lceil I \rfloor$$

$$[\Phi]^T[C][\Phi]=\lceil 2\xi_r\omega_r \rfloor$$

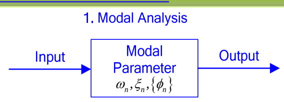
$$[\Phi]^T[K][\Phi]=\lceil \omega_r^2 \rfloor$$

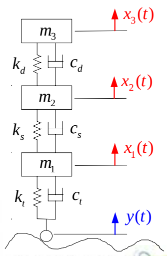
where

$$[\Phi]=[\{\phi_1\},\{\phi_2\},\cdots,\{\phi_n\}]$$

4.2.1 Modal Analysis 3-DOF Quarter Car Model

- Solution of system model: modal analysis
  - Natural frequencies
    - f_1 =1.2125 Hz
    - f_2 =4.6601 Hz
    - f_3 =10.7113 Hz
  - Modal damping ratios
    - $\xi_1=0.0381$
    - $\xi_2=0.1464$
    - $\xi_3=0.3365$
  - Mode shape vectors





1st, 2nd, 3rd

$$[\Phi]=\left[\{\phi_1\},\{\phi_2\},\{\phi_3\}\right]=\begin{bmatrix} 0.0083 & -0.0057 & 0.1793 \\ 0.0576 & -0.0322 & -0.0037 \\ 0.0631 & 0.1126 & 0.0006 \end{bmatrix}\begin{matrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{matrix}$$

4.2.2 Harmonic Response Analysis

- For system subject to harmonic excitation

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\} \qquad y_j(t)=Y_je^{i\omega t}$$

- The system output will also be harmonic:

$$\{y(t)\}=\{Y\}e^{i\omega t}=\begin{Bmatrix}Y_1\\ Y_2\\ \vdots\\ Y_m\end{Bmatrix}_{m\times 1}e^{i\omega t} \qquad \{x(t)\}=\{X\}e^{i\omega t}=\begin{Bmatrix}X_1\\ X_2\\ \vdots\\ X_n\end{Bmatrix}_{n\times 1}e^{i\omega t}$$

where

- ω is the excitation frequency
- $\{Y\}$ is the magnitude of input vector
- $\{X\}$ is the magnitude of output vector

4.2.2 Harmonic Response Analysis

- The system frequency response function (FRF) matrix can be determined:

$$[H]_{n\times m}=[A]^{-1}_{n\times n}[B]_{n\times m} \qquad H_{ij}(\omega)=\frac{X_i}{Y_j}$$

where

$$[A]_{n\times n}=\left([K]_{n\times n}-\omega^2[M]_{n\times n}+i\omega[C]_{n\times n}\right)$$

$$[B]_{n\times m}=\left([K']_{n\times m}+i\omega[C']_{n\times m}\right)$$

$$[H(\omega)]=\begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1m} \\ H_{21} & H_{22} & \cdots & H_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1} & H_{n2} & \cdots & H_{nm} \end{bmatrix}$$

- The system equation can be obtained:

$$[A]_{n\times n}\{X\}_{n\times 1}=[B]_{n\times m}\{Y\}_{m\times 1}$$

$$\{X\}_{n\times 1}=[A]^{-1}_{n\times n}[B]_{n\times m}\{Y\}_{m\times 1}$$

$$\{X\}_{n\times 1}=[H]_{n\times m}\{Y\}_{m\times 1}$$

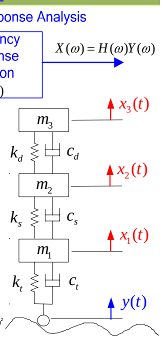
$$\{Y\}=\begin{Bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{Bmatrix}_{m\times 1} \qquad \{X\}=\begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{Bmatrix}_{n\times 1}$$

- The transmissibility is also defined:

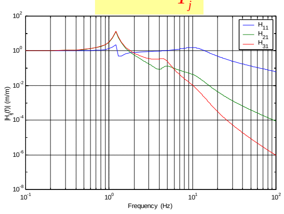
$$Tr_{ij}(\omega)=\frac{X_i}{X_j}$$

4.2.2 Harmonic Response Analysis: 3-DOF Quarter Car Model

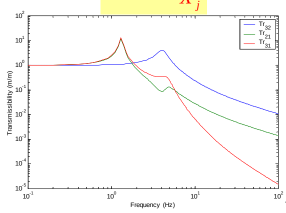
- Solution of system model:



$$H_{ij}(\omega)=\frac{X_i}{Y_j} \qquad Tr_{ij}(\omega)=\frac{X_i}{X_j}$$



Frequency Response Function



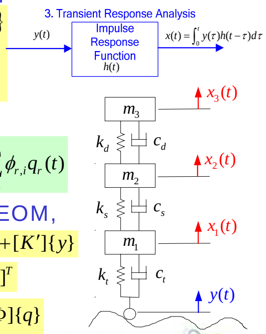
Transmissibility

4.2.3 Transient Response Analysis

- The general form of system equation:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\}$$

$$\text{I.C.:}\begin{cases}\{x(0)\}=\{x_0\}\\ \{\dot{x}(0)\}=\{v_0\}\end{cases}$$



- From expansion theorem,

$$\{x(t)\}=[\Phi]\{q(t)\}=\sum_{r=1}^{n}\{\phi_r\}q_r(t) \qquad x_i(t)=\sum_{r=1}^{n}\phi_{r,i}q_r(t)$$

- By the substitution of $\{x(t)\}$ to EOM,

$$[M][\Phi]\{\ddot{q}\}+[C][\Phi]\{\dot{q}\}+[K][\Phi]\{q\}=[C']\{\dot{y}\}+[K']\{y\}$$

- By the pre-multiplication of $[\Phi]^T$

$$[\Phi]^T[M][\Phi]\{\ddot{q}\}+[\Phi]^T[C][\Phi]\{\dot{q}\}+[\Phi]^T[K][\Phi]\{q\}$$
$$=[\Phi]^T[C']\{\dot{y}\}+[\Phi]^T[K']\{y\}$$

4.2.3 Transient Response Analysis

- By the employment of orthonormal relationships of modal matrix, the EOM is simplified

$$[\,^{\backslash}I_{\backslash}]\{\ddot{q}\}+[\,^{\backslash}2\xi_r\omega_{r\backslash}]\{\dot{q}\}+[\,^{\backslash}\omega_r^2{}_{\backslash}]\{q\}=\{\dot{z}(t)\}+\{z(t)\}$$

where

$$\{\dot{z}(t)\}=[\Phi]^T[C']\{\dot{y}\} \qquad \dot{z}_r(t)=\sum_{j=1}^{n}\sum_{k=1}^{n}\phi_{r,k}C'_{kj}\dot{y}_j(t)$$

$$\{z(t)\}=[\Phi]^T[K']\{y\} \qquad z_r(t)=\sum_{j=1}^{n}\sum_{k=1}^{n}\phi_{r,k}K'_{kj}y_j(t)$$

- The r-th decoupled equation becomes:

$$\ddot{q}_r+2\xi_r\omega_r\dot{q}_r+\omega_r^2q_r=\dot{z}_r(t)+z_r(t)$$

$$q_{r0}=q_r(0),r=1,2,\ldots,n \qquad \{q(0)\}=[\Phi]^T[M]\{x_0\}=\{q_0\}$$

$$\dot{q}_{r0}=\dot{q}_r(0),r=1,2,\ldots,n \qquad \{\dot{q}(0)\}=[\Phi]^T[M]\{v_0\}=\{\dot{q}_0\}$$

$$z_{r0}=z_r(0),r=1,2,\ldots,n \qquad z_r(0)=\sum_{j=1}^{n}\sum_{k=1}^{n}\phi_{r,k}K'_{kj}y_j(0)$$

4.2.3 Transient Response Analysis

- The modal coordinate $q_r(t)$ can be solved and so is the system transient response :

$$q_r(t)=q_{r,IRF}(t)+q_{r,IC1}(t)+q_{r,IC2}(t) \qquad x_i(t)=\sum_{r=1}^{n}\phi_{r,i}q_r(t)$$

where

$$\{x(t)\}=[\Phi]\{q(t)\}=\sum_{r=1}^{n}\{\phi_r\}q_r(t)$$

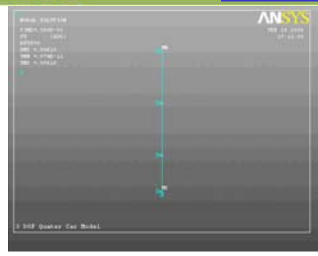
$$q_{r,IRF}(t)=\int_0^t z_r(\tau)h_r(t-\tau)d\tau$$

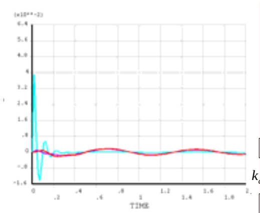
$$q_{r,IC1}(t)=q_{r0}\left[e^{-\xi_r\omega_r t}\left(\cos\omega_{dr}t+\frac{\xi_r\omega_r}{\omega_{dr}}\sin\omega_{dr}t\right)\right]+\dot{q}_{r0}\left(\frac{1}{\omega_{dr}}e^{-\xi_r\omega_r t}\sin\omega_{dr}t\right)$$

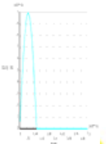
$$q_{r,IC2}(t)=-z_{r0}\left(\frac{1}{\omega_{dr}}e^{-\xi_r\omega_r t}\sin\omega_{dr}t\right)$$

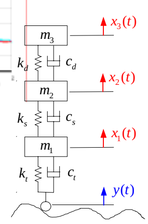
$$h_r(t)=e^{-\xi_r\omega_r t}\left[\cos\omega_{dr}t+\frac{1-\xi_r\omega_r}{\omega_{dr}}\sin\omega_{dr}t\right] \qquad \omega_{dr}=\omega_r\sqrt{1-\xi_r^2}$$

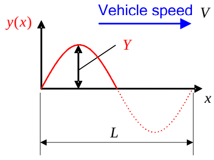
4.2.3 Transient Response Analysis Half-Sine Bump











$L=0.1$ (m)

$Y=0.1$ (m)

$V=10$ (km/hr)

4.2.4 Spectrum Response Analysis

- From harmonic analysis,

$$\{X(\omega)\}_{n\times 1}=[H(\omega)]_{n\times m}\{Y(\omega)\}_{m\times 1}$$

$$\begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{Bmatrix}_{n\times 1}=\begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1m} \\ H_{21} & H_{22} & \cdots & H_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1} & H_{n2} & \cdots & H_{nm} \end{bmatrix}_{n\times m}\begin{Bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{Bmatrix}_{m\times 1} \qquad H_{ij}(\omega)=\frac{X_i}{Y_j} \qquad Tr_{ij}(\omega)=\frac{X_i}{X_j}$$

- For a random input, the input PSD matrix can be given by $[G_{yy}(\omega)]$
- The system output PSD matrix can be obtained:

$$[G_{xx}(\omega)]_{n\times n}=[H(\omega)]^*_{n\times m}[G_{yy}(\omega)]_{m\times m}[H(\omega)]^T_{m\times n}$$

and

$$[G_{yx}(\omega)]_{m\times n}=[G_{yy}(\omega)]_{m\times m}[H(\omega)]^T_{m\times n}$$

4.2.4 Spectrum Response Analysis

- The r.m.s. value of the i-th DOF output $x_i(t)$ can be determined by:

$$x_{i,r.m.s.}=\sqrt{\overline{x_i^2}}=\sqrt{\frac{1}{2\pi}\int_0^\infty G_{x_ix_i}(\omega)d\omega}=\sqrt{\int_0^\infty G_{x_ix_i}(f)df}$$

- For zero mean of random input, $x_{i,rms}=\sigma_{x_i}$
- The PSD and r.m.s. value of velocity can be obtained as follows:

$$G_{\dot{x}_i\dot{x}_i}(\omega)=(\omega)^2G_{x_ix_i}(\omega) \qquad G_{\dot{x}_i\dot{x}_i}(f)=(2\pi f)^2G_{x_ix_i}(f)$$

$$\dot{x}_{i,r.m.s.}=\sqrt{\frac{1}{2\pi}\int_0^\infty G_{\dot{x}_i\dot{x}_i}(\omega)d\omega}=\sqrt{\int_0^\infty G_{\dot{x}_i\dot{x}_i}(f)df}$$

- The PSD and r.m.s. value of acceleration can be obtained as follows:

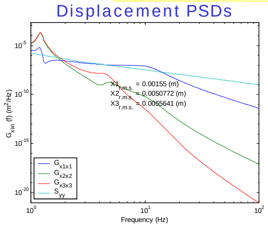
$$G_{\ddot{x}_i\ddot{x}_i}(\omega)=(\omega)^4G_{x_ix_i}(\omega) \qquad G_{\ddot{x}_i\ddot{x}_i}(f)=(2\pi f)^4G_{x_ix_i}(f)$$

$$\ddot{x}_{i,r.m.s.}=\sqrt{\frac{1}{2\pi}\int_0^\infty G_{\ddot{x}_i\ddot{x}_i}(\omega)d\omega}=\sqrt{\int_0^\infty G_{\ddot{x}_i\ddot{x}_i}(f)df}$$

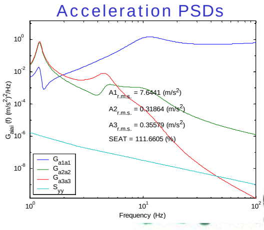
4.2.4 Spectrum Response Analysis: 3-DOF Quarter Car Model

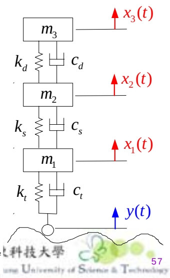
- For the vehicle running at 60 km/hr speed on Road Class A, the example outputs:
  - Displacement PSDs and $x_{i,r.m.s.}$ values
  - Acceleration PSDs and $a_{i,r.m.s.} = \ddot{x}_{i,r.m.s.}$ values
  - $SEAT(\%) = \frac{\text{seat r.m.s.}}{\text{floor r.m.s.}} = \frac{\ddot{x}_{3,r.m.s.}}{\ddot{x}_{2,r.m.s.}} \times 100$

Displacement PSDs



Acceleration PSDs

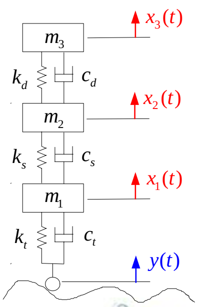




4.3 Determination of Ride Quality Parameters

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[C']\{\dot{y}\}+[K']\{y\}$$

- 1. Driver/passenger acceleration
  $\ddot{x}_3(t) \rightarrow G_{\ddot{x}_3\ddot{x}_3}(f) \rightarrow \ddot{x}_{3,i-1/3-r.m.s.}$, $\ddot{x}_{3,r.m.s.}$
- 2. SEAT (Seat Effective Amplitude Transmissibility)
  $SEAT(\%) = \frac{\text{seat r.m.s.}}{\text{floor r.m.s.}} = \frac{\ddot{x}_{3,r.m.s.}}{\ddot{x}_{2,r.m.s.}} \times 100$
- 3. Road holding
  $h(t) = x_1(t) - y(t) \rightarrow G_{hh}(\omega) \rightarrow h_{i-1/3-r.m.s.}$, $h_{r.m.s.}$
- 4. Suspension travel
  $s(t) = x_2(t) - x_1(t) \rightarrow G_{ss}(\omega) \rightarrow s_{i-1/3-r.m.s.}$, $s_{r.m.s.}$
- 5. Tire force
  $f_t(t) = k_t h(t) + c_t \dot{h}(t) \rightarrow G_{f_t f_t}(f) \rightarrow f_{t,i-1/3-r.m.s.}$, $f_{t,r.m.s.}$
- 6. Suspension force
  $f_s(t) = k_s s(t) + c_s \dot{s}(t) \rightarrow G_{f_s f_s}(f) \rightarrow f_{s,i-1/3-r.m.s.}$, $f_{s,r.m.s.}$



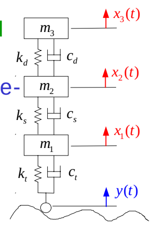
4.3 Ride Quality Parameter: 1. Driver/passenger acceleration

- To comply with the ISO 2631-1, the r.m.s. acceleration at each one-third octave band shall be obtained.
- The r.m.s. value of acceleration in each one-third octave band is determined by:

$$\ddot{x}_{3,i-1/3-r.m.s.} = \left[\int_{f_l}^{f_u} G_{\ddot{x}_3\ddot{x}_3}(f)df\right]^{\frac{1}{2}}$$

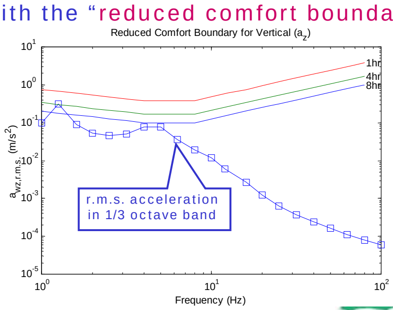
where

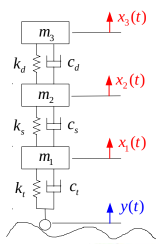
- f_u , f_l are the upper and lower bounds of the one-third octave band with central frequency f_c , respectively.
- $\ddot{x}_{3,i-1/3-r.m.s.}$ denotes the r.m.s. acceleration of $\ddot{x}_3(t)$ corresponding to the i-th central frequency f_c



4.4 Assessment of Ride Comfort: Driver/passenger acceleration

- For adopting ISO 2631-1 (1985), the r.m.s. accelerations $\ddot{x}_{3,i-1/3-r.m.s.}$ at the i-th central frequency are plotted over the frequency range to compare with the "reduced comfort boundary".





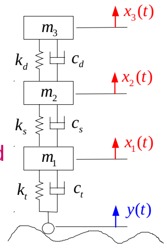
4.4 Assessment of Ride Comfort: Driver/passenger acceleration

- For adopting ISO 2631-1 (1997), the single value of r.m.s. acceleration is calculated by:

$$a_{w,r.m.s.} = \left[\sum_i (W_i a_i)^2\right]^{\frac{1}{2}} \qquad \ddot{x}_{3,r.m.s.} = \left[\sum_i (W_i \ddot{x}_{3,i-1/3-r.m.s.})^2\right]^{\frac{1}{2}}$$

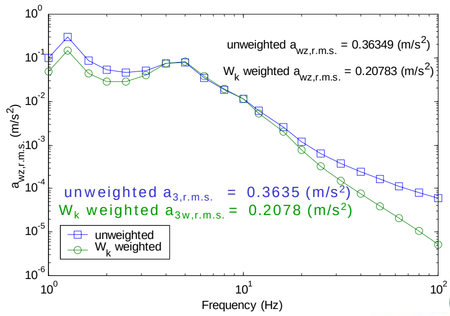
where

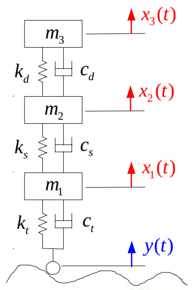
- $a_i = \ddot{x}_{3,i-1/3-r.m.s.}$ is the r.m.s. value at central frequency of 1/3 octave band
- W_i is the frequency weighting factor at the i-th central frequency of one-third octave band
- $a_{w,r.m.s.} = \ddot{x}_{3,r.m.s.}$ is the frequency-weighted r.m.s. acceleration.



4.4 Assessment of Ride Comfort: Driver/passenger acceleration

- For adopting ISO 2631-1 (1997), the single value of r.m.s. acceleration is compared with ISO 2631-1 (1997)





V. Conclusions

- Analytical approach for Vehicle Ride Comfort Analysis is presented.
  - Vehicle Riding Model
  - Road Model
  - Ride Quality Parameters
  - Ride Comfort Criterion
- The solution of vibration system for four types of analyses regarding to ride quality analysis is illustrated.
- An examples of vehicle ride comfort analysis to assess a 3-DOF Quarter Car Model is shown.

Thank you for your attention.