Theoretical Simulation of Damping Effect Base on Experimental Measurement

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Abstract

In general, damping effect can not be theoretically determined. However, modal damping ratios can be obtained via experimental modal testing. This paper presents two methods to include damping effect into the theoretical model base on the experimentally determined damping data. First, the proportionally, viscously damped model is assumed. The optimization problem is formulated to determine two optimum constants related to mass and stiffness matrices with the known experimental modal damping ratios, and so forth the proportional, viscous damping matrix can be determined from these two constants. Second, the accumulated averaged damping ratio can be obtained from the experiments and adopted to define the constant modal damping ratio for the analytical model. Several case studies are presented to show the implement of damping effect in theoretical analysis such that the analytical model can be more practically simulated in accordance with the experimental test results. The developed methodology is easy to implement and applicable to finite element analysis for arbitrary structures.

Keywords: experimental modal testing, viscously damped model, optimization problem, accumulated averaged damping ratio, proportional damping.

Nomenclature

c_r	r^{th} modal damping
$[C]$	viscous damping matrix
k_r	r^{th} modal stiffness
$[K]$	stiffness matrix
m_r	r^{th} modal mass
$[M]$	mass matrix
$\{X_r\}$	r^{th} mode shape vector
$[X]$	modal matrix
α , β	proportional damping constants
$\{\phi_r\}$	r^{th} mass matrix normalized mode shape vector
$[\phi_r]$	r^{th} mass matrix normalized modal matrix
ω_r	r^{th} undamped natural frequency
ξ_r	r^{th} damping ratio
$\hat{\xi}_r$	r^{th} experimental damping ratio

1. Introduction

Structural damping ratios can be determined via experimental modal analysis (EMA) by extracting the modal parameters from frequency response functions (FRFs). By using either the simple approach such as half-power-point method or complicated curve-fitting algorithms, the modal damping ratios can be obtained and applied to theoretical simulation. If the theoretical model is solved by modal domain approach, the experimentally determined modal damping data can be easily applied to individual mode. However, in practical engineering structures are complicated and required commercial software to perform theoretical response simulation. The modal domain solution technique is not generally supported by software. How to implement the damping effect in theoretical model base on the experimental damping data is a practical engineering issue.

Adhikari [1] proposed the damping identification method that adopting the extracted natural frequencies and modal damping ratios to formulate the damping matrix. The damping matrix is functions of mass and stiffness matrices as well as the modal frequencies and damping ratios. The damping matrix is frequency-dependence and so complex that the damping matrix can be difficult to apply in commercial software application. This work proposes the two approaches that can be implemented in commercial codes. One is the constant damping ratio method, and the other is constants α and β that are related to mass and stiffness matrices, respectively. Gounaris and Anifntis [2] suggested the use of complex modulus of elasticity including loss factor for damping simulation. They used the iteration approach in obtaining structural stresses from both finite element (FE) model and experiments, respectively, by modifying damping values. When the stresses agree to each others, the applied damping ratios can be representative for the structure. This approach may not be practical for most engineering applications.

In modeling material damping, Vantomme [3] adopted the energy balance approach to characterize the damping contributions due to matrix, fiber material, and the interface effects for fiber-reinforced plastics samples. Wang *et al*. [4] applied EMA technique to obtain and validated the mechanical properties of golf club head materials including their material damping ratios. Horr and Schmidt [5] presented a non-linear damping modeling technique to characterize a viscoelastic structural damper and can accurately predict the frequency-dependence damping properties for all structural members and dampers.

This work presents the practical approach by using the experimental damping ratios to be implemented in theoretical analysis, in particular for commercial FE codes. Two methods are proposed, and three types of practical structures are studied by the proposed methods to conduct the FRF simulations and validations. Therefore, further response simulation can be followed base on the FE model with damping effect implemented.

2. Theoretical Analysis

The system equation for a multiple degree-of-freedom vibration system can be as follows:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f(t)\} \tag{1}$$

In particular, for proportional damping, one can assume that:

$$[C]=\alpha[M]+\beta[K] \tag{2}$$

where α and β are some constants.

By performing theoretical modal analysis, the normal modes of the system can be solved, therefore, the r^{th} modal parameters, including natural frequencies (ω_r), mode shape vectors ($\{X_r\}$), and damping ratios (ξ_r) can be obtained. The modal matrix can then be defined as follows:

$$[X]=[\{X_1\},\{X_2\},\ldots,\{X_n\}] \tag{3}$$

The orthogonality properties of mode shape vectors in matrix form can be obtained as follows:

$$[X]^T[M][X]=\left[{}^{\diagdown}m_{r\,\diagdown}\right] \tag{4}$$

$$[X]^T[C][X]=\left[{}^{\diagdown}c_{r\,\diagdown}\right] \tag{5}$$

$$[X]^T[K][X]=\left[{}^{\diagdown}k_{r\,\diagdown}\right] \tag{6}$$

where m_r, c_r, and k_r can vary and be dependent on the choice of scalable mode shape vector $\{X_r\}$, and, therefore, and redefines the mode shape vectors as follows:

$$\{\phi_r\}=\frac{1}{\sqrt{m_r}}\{X_r\} \tag{7}$$

$\{\phi_r\}$ is termed the mass-matrix normalized mode shape vector, and the corresponding modal matrix can be defined as follows:

$$[\phi]=[\{\phi_1\},\{\phi_2\},\ldots,\{\phi_n\}] \tag{8}$$

The orthonormality properties of mode shape vector can then be expressed:

$$[\phi]^T[M][\phi]=\left[{}^{\diagdown}I_{\diagdown}\right] \tag{9}$$

$$[\phi]^T[C][\phi]=\left[{}^{\diagdown}2\xi_r\omega_{r\,\diagdown}\right] \tag{10}$$

$$[\phi]^T[K][\phi]=\left[{}^{\diagdown}\omega_{r\,\diagdown}^2\right] \tag{11}$$

where

$$\xi_r=\frac{\alpha}{2\omega_r}+\frac{\beta\omega_r}{2} \tag{12}$$

and ξ_r is the r^{th} modal damping ratio that is functions of modal natural frequency (ω_r) and two constants (α, β).

In theoretical analysis, the modal damping ratio can be well defined; however, it must be determined from the experiments for the real structure. This paper will address the definition of damping effect in theoretical analysis according to the experimental measurement of damping ratios.

3. Simulation of Damping Effect in Analysis

The structural damping ratios can be experimentally determined via experimental modal testing. For examples, when n modes are obtained, there will be n modal damping ratios. Ideally, one can employ these modal damping ratios for structural response analysis by modal domain analytical approach. However, in practice complicated structures are generally performed analysis by commercial finite element code, such as ANSYS software.

There are two ways to implement damping effect in ANSYS software as follows:

(1) Define constants α and β: Both constants are as defined in Equation (2). The proportional damping effect is assumed. The damping effect can be implemented in solution accordingly. The commands for setting α and β in ANSYS are "ALPHAD" and "BETAD."

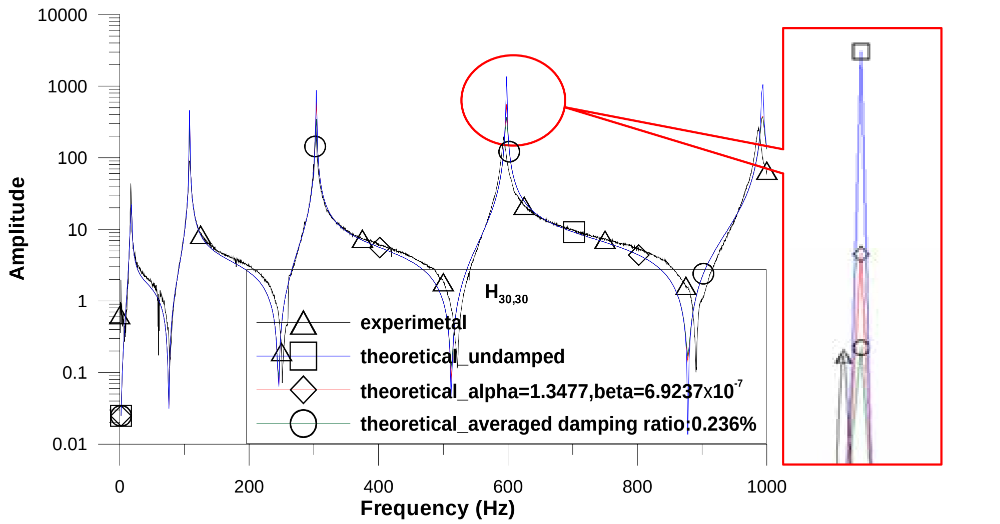
(2) Define constant modal damping ratios: The command is "DMPART." For the case, all of the modal damping ratios are assumed to be the same. This results in different α and β for each mode.

The question here is how to apply experimentally determined modal damping ratios ($\hat{\xi}_r$) to the response simulation in software without losing the generic nature of damping from experimental results. For the above two approaches in defining damping effect, the proposed strategy is as follows:

3.1 Define constants α and β

In order to find a set of the two constants that can best simulate the damping effect according to the experiments. The optimization problem is formulated as follows:

Table1 Simulation results of damping effects for Cantilever Beam

Mode	Natural frequency (Hz)	Experimental Damping ratio	Predicted Damping ratio $\alpha = 1.3477$ $\beta = 6.9237\times10^{-7}$	Error (%)
1	17.2	0.582%	0.627%	7.84
2	108	0.124%	0.123%	-0.91
3	303	0.097%	0.101%	4.00
4	594	0.160%	0.147%	-8.22
5	988	0.217%	0.226%	4.12
Averaged	—	0.236%	0.245%	3.74%
FRF				

(1) Design variable: α , β

(2) Objective functions:

$$\Phi(\alpha,\beta)=\sum_{r=1}^{n}\left(\frac{\xi_r-\hat{\xi}_r}{\hat{\xi}_r}\right)^2 \tag{13}$$

where $\hat{\xi}_r$ is the experimental modal damping ratio, and ξ_r is the optimized predicted modal damping ratio according to Equation (12) that is functions of α and β as well as the modal frequency (ω_r). The objective is to optimally determine the set of α and β such that the objective function, which is defined as the sum of square errors between the predicted and experimental damping ratios, is minimum. Therefore, the optimum set of α and β can be implemented in software simulation.

3.2 Define constant modal damping ratio

From the experiments, there can be n modes of damping ratios. The constant damping ratio to be applied can be defined as the accumulated average of all modes as follows:

$$\xi=\frac{\sum_{r=1}^{n}\hat{\xi}_r}{n} \tag{14}$$

This section provides the practical implement of damping effect in response simulation according to experimentally determined damping ratios.

4. Case Study Results and discussions

This section will present the theoretical simulation of frequency response function (FRF) by adopting the above mentioned damping implement approaches for different materials and structures, including a steel cantilever beam, a carbon fiber-reinforced composite plate in free boundary, and a print circuit board (PCB) in fixed condition.

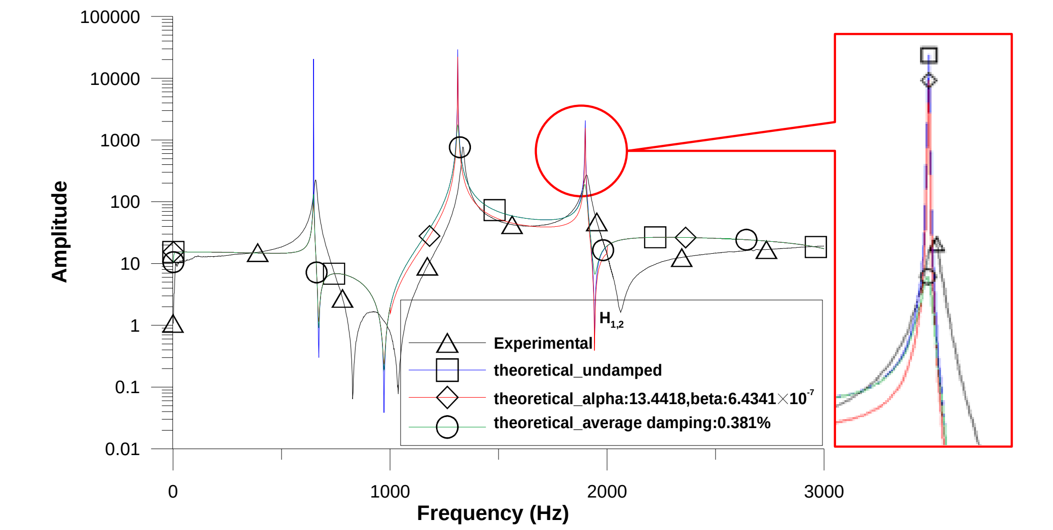
4.1 Steel Cantilever Beam

Figure 1 shows the experimental setup for the steel cantilever beam performed by conventional modal testing. There are 5 modes up to 1000 Hz. Table 1 shows experimental natural frequencies and damping ratios. The averaged damping ratio is 0.236% that can be used to define the constant damping ratio. Also, according to the formulated optimization problem in Equation (13), the optimal constants are $\alpha = 1.3477$ and the $\beta = 6.9237\times10^{-7}$. The predicted damping ratios and their corresponding errors with respect to the experimental ones are also revealed in Table 1.

The damping effects by using constant damping ratio $\xi = 0.236\%$ and setting both α and β are implemented into finite element model, respectively, as shown in Figure 2. The beam model is constructed by linear hexahedron elements (solid 45) in ANSYS software. The FRF plots are shown in Table 1 as well and discussed as follows:

(1) Different damping effects do affect the FRF, especially near the resonance peaks.

Table 2 Simulation results of damping effects for Fiber-Reinforced Composite Plate

Mode	Natural frequency (Hz)	Experimental Damping ratio	Predicted Damping ratio $\alpha = 13.4418$ $\beta = 6.4341 \times 10^{-7}$	Error (%)
1	658	0.309%	0.296%	-4.33
2	1340	0.304%	0.351%	15.53
3	1910	0.529%	0.442%	-16.44
Averaged	—	0.381%	0.356%	-6.38%
FRF				

(2) The enlarged plot around the 4th mode is also displayed and revealed that the constant damping ratio provides with the best fit, while the undamped case reveals exceptional high value at peak resonances. The α and β definition approach is right between the both cases.

(3) Base on the FRF simulation, the constant damping ratio method has simulated the damping effect better than others in this case study.

4.2 Carbon Fiber-Reinforced Composite Plate

Figure 3 shows the carbon plate as well as the experimental rig for the impact modal testing. Figure 4 is the corresponding FE model for the carbon plate. The plate model is constructed by linear hexahedron elements. The plate is considered as in free boundary condition.

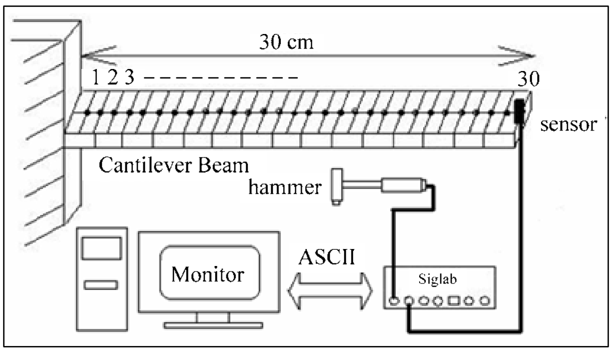


Figure1 Experimental setup for cantilever beam EMA

Table 2 shows the experimental natural frequencies and damping ratios for the first three modes as well as the FRF plots. The discussions are as follows:

(1) For constant damping ratio method, the damping ratio is set as the averaged damping ratio $\xi = 0.381\%$.

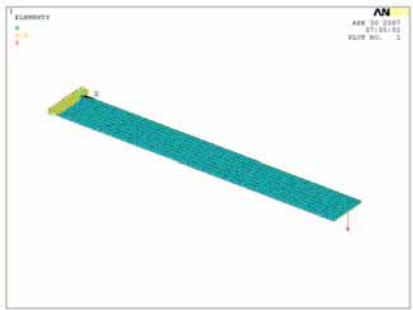


Figure 2 FE model for cantilever beam

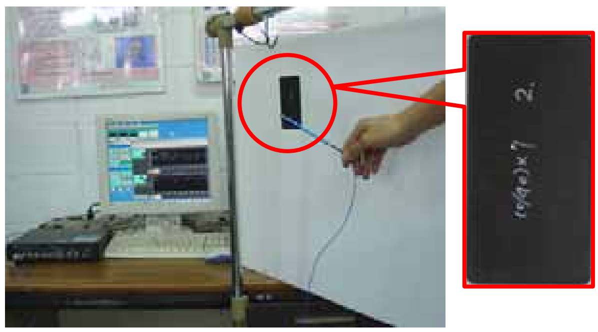
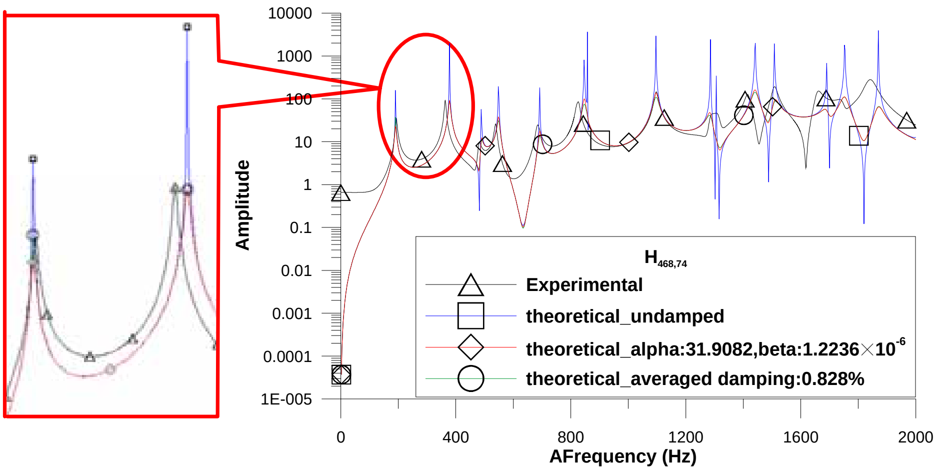


Figure3 Experimental setup for carbon plate EMA

Table 3 Simulation results of damping effects for PCB

Mode	Natural frequency (Hz)	Experimental Damping ratio	Predicted Damping ratio $\alpha = 31.9082$ $\beta = 1.2236\times10^{-6}$	Error (%)
1	189	1.418%	1.416%	-0.11
2	363	0.688%	0.839%	21.99
3	481	1.065%	0.713%	-33.06
4	539	0.673%	0.678%	0.78
5	685	0.797%	0.634%	-20.46
6	822	0.698%	0.625%	-10.51
7	1100	0.573%	0.654%	14.01
8	1290	0.695%	0.693%	-0.40
9	1310	0.781%	0.697%	-10.71
10	1420	0.683%	0.725%	6.14
11	1510	0.627%	0.749%	19.46
12	1660	0.766%	0.791%	3.22
13	1750	0.788%	0.818%	3.81
14	1820	1.335%	0.839%	-37.14
Averaged	—	0.828%	0.776%	-6.19%
FRF				

(2) Base on the first three modes, the optimal values of $\alpha = 13.4418$ and $\beta = 6.4341\times10^{-7}$ can result in minimum errors in terms of predicted damping ratios as revealed in Table 2. The averaged error is -6.38%.

(3) For the comparison of FRFs obtained from different damping effects, one can still see that the constant damping ratio method has better fit than others near the peak resonances.

4.3 Print Circuit Board in Fixed Boundary

Figure 5 shows the PCB setup for experimental modal testing. The PCB is fixed at four corners as shown. The corresponding FE model as shown in Figure 6 uses liner hexahedrons elements (solid 45) for the board and spring elements (combin 14) to simulate the screw-fixed boundary. Both constant damping ξ and optimal constants α and β determined from the experiments are shown in Table 3 as well as FRF simulation plots. Discussions are as follows:

(1) The averaged damping ratio from experiments is 0.828% that can be adopted as the constant damping ratio for FRF prediction.

(2) The optimal values of $\alpha = 31.9082$ and $\beta = 1.2236\times10^{-6}$ determined from Equation (13), results in the averaged errors of damping ratios about -6.19%.

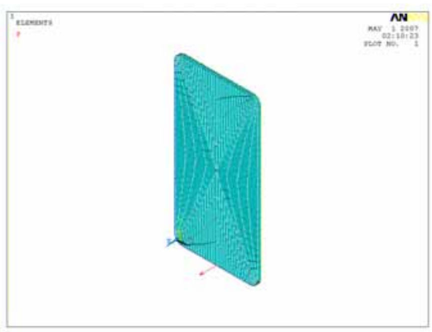


Figure 4 FE model for carbon plate

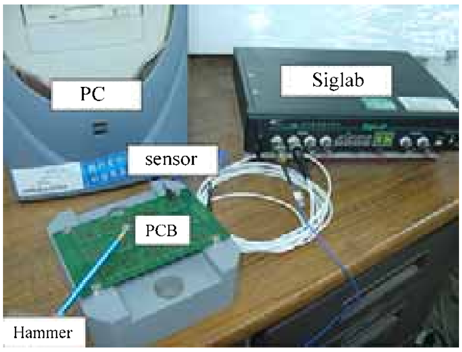


Figure5 Experimental setup for PCB EMA

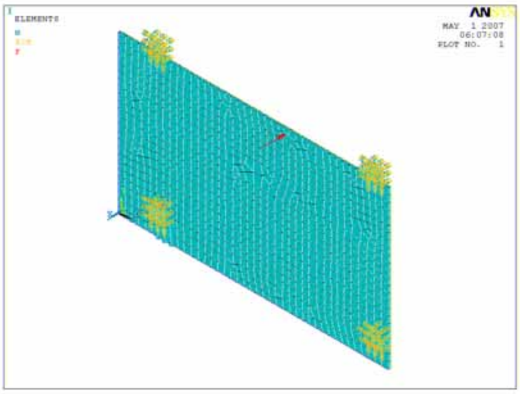


Figure 6 FE model for PCB

(3) The FRF plots generally agree each other. The undamped case reveals unreasonable high peak at resonance. Therefore, the damping effect should be properly included for FRF simulation.
(4) For the first two resonances, both constant ξ and constants α and β approaches reveal about the same FRF response at peaks in this case study.

5. Conclusion

This paper introduces two types of methods to include damping effect in response simulation for finite element analysis base on experimentally determined damping ratios. Three kinds of different material and structures are presented to study the FRF simulation for different damping effects. Some conclusions are summarized as follows:

(1) The FRF simulation for undamped case is generally in adequate, especially near the peak resonance frequency range.
(2) Both constant damping ratio method and constants α and β method for damping effect implement can provide with more realistic FRF simulation results in comparison to experimental FRFs.
(3) Using the averaged damping ratio from experiments as the constant damping ratio can be the easiest way and can better reasonably simulate the FRFs.
(4) This work discusses the approaches to implement damping effect in finite element analysis and can be useful for practical engineering applications.

6. 參考文獻

[1] Adhikari, S., “Damping Modeling and Identification Using Generalized Proportional Damping,” Proceedings of the 23rd International Modal Analysis Conference, Orlando, Florida, USA, 2005.
[2] Gounaris, G. D., and Anifantis, N. K., “Structural Damping Determination by Finite Element Approach,” Computers and Structures, Vol. 73, pp. 445-452, 1999.
[3] Aantomme, J., “A Parametric Study of Material Damping in Fiber-Reinforced Plastics,” Composite, Vol. 26, No. 2, pp. 147-153, 1995.
[4] Wang, B. T., Zhao, Z. Y., and Wu, J. D., “Determination of Golf Club Material Properties by Experimental Modal Analysis,” Bulletin of National Pingtung University of Science and Technology, Vol. 8, No. 4, pp. 311-317, 1999. (in Chinese)
[5] Horr, A. M., and Schmidt, L. C., “Modeling of Nonlinear Damping Characteristics of a Viscoelastic Structural damper,” Engineering Structures, Vol. 18, No, 2, pp. 154-161, 1996.

基於實驗模態量測對阻尼比效應之模擬

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摘要

阻尼效應一般而言無法由理論分析求得，而經由實驗模態測試可獲得模態阻尼比。本文介紹兩種由實驗量測所得阻尼比套入理論分析模型之模擬方法。第一種方法係假設結構為比例黏滯阻尼模型，可基於實驗模態分析所量得之阻尼比對質量與勁度矩陣相關之兩常數進行最佳化分析，可由所得最佳常數值進一步求得比例黏滯阻尼系統之黏滯阻尼矩陣。第二種方法則將實驗模態分析所得之各模態阻尼比加以累加並平均，可假設阻尼比為常數且應用於理論模型分析。最後並以實際案例說明實務上如何於理論模型分析中模擬阻尼比效應，並與實驗模態分析所得之頻率響應函數結果相比較。未來可於任意結構之阻尼比效應模擬中，以此兩種方法，使阻尼比參數有一參考值作為輸入。

關鍵詞：實驗模態測試、黏滯阻尼模型、最佳化分析、累加平均阻尼比、比例阻尼