Application of Modal Strain Energy Index to the Damage Detection of Cantilever Beams

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Abstract

A nondestructive detection of surface cracks in aluminum alloy 6061 cantilever beam using modal strain energy index is investigated in this paper. Experimental modal analysis is performed to obtain the modal displacement and modal strain of the beam before and after damage. Modal displacements and modal strains are then used to compute the modal strain energy. A damage index is then defined based on strain energy ratio of the beam before and after damage. This damage index successfully predicts the location of surface crack in aluminum alloy beam. Traditionally, modal displacements measured by accelerometers are used in experimental modal analysis. Two-order derivatives of the mode shapes are necessary to compute the strain energy of the beam. The use of in-plane modal strain saves the partial differential work in the calculation of strain energy and provides more accurate prediction of the damage. A pre-study using a 3-D finite element analysis is also performed to access this approach. Only modal strains of the beam are required in this method, which provides a quick, accurate, inexpensive and flexible approach.

Keywords: surface crack, experimental modal analysis, modal strain, damage index

1. Introduction

Modal analysis has been increasingly adopted to nondestructively detect damages in engineering structures due to its flexibility of measurement and relatively low cost. The basic theory of this approach is to use the information of modal parameters, such as natural frequencies, mode shapes and damping ratios, to access the structural damage.

Cawley and Adams [1] simply used the frequency shifts for different modes to detect the damage in composite structures. Tracy and Pardoen [2] found that the natural frequencies of a composite beam were affected by the size and damage location. Cornwell et al. [3] utilized the measured mode shapes to calculate the strain energy of a steel plate. In his approach, fractional strain energy of the plate before and after damaged was used to define a damage index, which was used to locate the damage in the steel plate. The method only requires the mode shapes of the structure before and after damage. Choi et al. [4] modified this method by using the changes in the distribution of the modal compliance of the plate structure to detect single and multi-cracks in plate. Nevertheless, the challenge of the above approaches lies in the accuracy of measured modes. A large amount of data points are required for further analysis to locate the damage. To solve this problem, Hu et al. [5-8] adapted differential quadrature method (DQM) to obtain a solution of strain energy of a composite plate, and successfully detect the surface and matrix crack locations in various composite laminate plates. It was reported that the original DQM was first used in structural mechanics problems by Bert et al. [9]. This method is able to rapidly compute accurate solutions of partial differential equations by using only a few grid points in the respective solution domains [10]. However, two-order derivatives of the mode shapes are required to compute the strain energy. Thus, numerical errors are inevitable in this approach.

The objective of this paper is to nondestructively detect a surface crack in cantilever beam employing modal strain energy index. The obtained modal strains are used to compute strain energy and damage index of the beam. The prediction of damage index using modal strains is obtained in comparison to the approach using modal displacement.

2. Theory of damage index

A beam structures as shown in Figure 1 is subdivided into N_d sub-region and denoted the associated location of each point by a_j. The strain energy of beam during elastic deformation is given by

$$U = \frac{D}{2}\int_0^l \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{1}$$

where w is transverse displacement; $D = EI$ is bending stiffness of the beam. In modal analysis, transverse displacement can be replaced by modal displacements, which can be usually measured using accelerometers.

2.1 Energy index using modal displacement

Considering a free-free vibration problem, for a particular mode shape displacement, the total strain energy of the beam associated with the k^{th} modal displacement ϕ_k is can be expressed as

$$U_k = \frac{D}{2}\int_0^l \left(\frac{\partial^2 \phi_k}{\partial x^2}\right)^2 dx \tag{2}$$

Cornwell et al. [3] suggested that if the damage is located at a single sub-region then the change of strain energy in sub-region may become significant. Thus, the energy associated with sub-region j for the k^{th} mode is given by

$$U_{k,j} = \frac{D_j}{2}\int_{a_j}^{a_j+1} \left(\frac{\partial^2 \phi_k}{\partial x^2}\right)^2 dx \tag{3}$$

Similarly, U_k^* and $U_{k,j}^*$ represent the total strain energy and sub-regional strain energy of the k^{th} mode shape strain ε_{xk}^* for damaged beam. The fractional energies of the beam are defined as

$$F_{k,j} = \frac{U_{k,j}}{U_k} \quad \text{and} \quad F_{k,j}^* = \frac{U_{k,j}^*}{U_k^*} \tag{4}$$

Considering all measured modes, m, in the calculation, damage index in sub-region j is defined as

$$\beta_j = \frac{\sum_{k=1}^{m} F_{k,j}^*}{\sum_{k=1}^{m} F_{k,j}} \tag{5}$$

Equation (5) is used to predict the damage location in beam structures. In this approach, the partial differential terms can be calculated by means of numerical methods.

2.2 Energy index using modal strain

Numerical error seems inevitable in the above calculation of strain energy using equations (1), (2) or (3). In fact, the partial differential terms can be replaced by curvature through the strain-curvature relation, i.e.

$$\kappa = \frac{\partial^2 w}{\partial x^2} = -\frac{\varepsilon_x}{z} \tag{6}$$

where κ is curvature; ε_x is in-plane strains at position z away from the mid-surface of the beam. Considering a free-free vibration problem, for a particular mode k, equation (2) can be rewritten as

$$U_k = \frac{D}{2}\int_0^l \left(\frac{\varepsilon_x}{z}\right)_k^2 dx \tag{7}$$

Thus, the energy associated with sub-region j for the k^{th} mode is given by

$$U_{k,j} = \frac{D_j}{2}\int_{a_j}^{a_j+1} \left(\frac{\varepsilon_x}{z}\right)_k^2 dx \tag{8}$$

Equations (7) and (8) offer much more effective computation of strain energy and no partial differential terms required.

3. Finite element analysis

A pre-study is performed by establishing finite element model. The beam has dimension 300×26×2 mm^3. ANSYS, a FEA commercial code, was used in this study. Eight-node linear solid element (SOLID45) was used to simulate the beam as shown in Figure 2. A convergence study was performed to obtain a 60×2×2 mesh model, which is sufficient to solve the normal mode problem. A surface crack with 26 mm long, 0.5 mm wide and 1 mm deep was created in the beam by separating the nodes at the elements along the crack. Three cases are studies to predict the surface crack close to the fixed end, mid-span and free end. Material properties (E = 70 GPa, ν = 0.33, ρ=2735 kg/m^3) of aluminum alloy 6061 were entered into ANSYS. A normal mode analysis with fixed end (fixed 54 node in the two surface) boundary condition was performed to obtain the natural frequencies, the associated mode shape displacements and the associated mode shape strains up to 2 kHz.

4. Experiment modal analysis

A cantilever Aluminum beam with dimension 260×26×2mm^3 is used in the test. Twelve strain gauges (KYOWA : KGF-5-120-C1-23) are installed through the axial direction of the beam. Modal testing is performed by exciting the beam at a location close to the fixed end using an impact hammer with a force transducer. Dynamic amplifier (KYOWA : DPM-711B) is used to magnify the dynamic strain. Figure 3 shows the experimental setup. Siglab, Model 20-40, is used to record the frequency response functions (FRFs) as shown in Figure 4. ME'Scope, a software for the general purpose curve fitting, is used to extract the natural frequencies and mode shapes from the FRFs. A surface crack with 16 mm long, 0.5 mm wide and 1 mm deep is created by utilizing a knife and located at between 5^{th} and 6^{th} strain gauges as shown in Figure 3.

5. Results and discussion

Table 1 lists the first eight natural frequencies and mode shape of the cantilever beam. Table 2 lists the first eight natural frequencies of the beam before and after damage. Good correlations between FEA and EMA results are obtained. However, mode 3, mode 5 and mode 8 are absent in EMA results. It is because that axial strain gauges can only measure bending mode shapes of the beam, but not lateral and torsional mode shapes. Apparently, mode 3 is lateral mode, and mode 5 and mode 8 are torsional modes as shown in Table 1.

In Table 2, the changes in natural frequencies of the beam before and after damage are almost invisible. In fact, it is very difficult to detect the small damage in beam structures based on the changes of natural frequencies, the associated modal displacements and modal strains. Nevertheless, damage index using the changes of modal strain energy provide more reliable prediction of damage in the beam.

The prediction of surface crack by using FEA results are shown in Figures 5, 6 and 7. The first eight modes are used to compute the strain energy and damage index. In Figure 5, damage indices obtained from both modal displacement and modal strain successfully predict the surface crack location near the fixed end. Figures 6 and 7 show the predictions of surface crack near the mid-span and the free end of the beam, respectively. Apparently, damage indices obtained from both modal displacement and modal strain successfully predict the surface crack location.

In EMA result, the first five mode shape strains, i.e., 1, 2, 4and 6-7, are used to compute the strain energy and damage index as shown in Figure 8-(a)~(d). Peak values occur around surface crack location, but some noises appear at undamaged areas. Hu et al [6] suggest that the resolution of damage index can be magnified by summing up all damage indices measured at different times in practical application of real-time structural healthy monitor. Figure 8-(e) shows that the damage index obtained from the summation of those of in (a)~(d).

6. Conclusions

A surface crack in aluminum alloy 6061 cantilever beam is successfully predicted by using modal strain energy method. Significant contributions of this approach are as follows,

- Modal strain energy and damage index of the beam can be computed by using either modal displacement or modal strain.
- Both damage indices can successfully locate the surface cracks in the beam.
- The approach only requires a few mode of the beam before and after damage.
- Damage index obtained from the modal strains provides more accurate prediction of the surface crack location than that of using modal displacement.
- Potential damages in overall structure can be evaluated.
- Real-time monitor of structure health in practical service is available.
- Measurement is flexible and cost is relative low.

7. References

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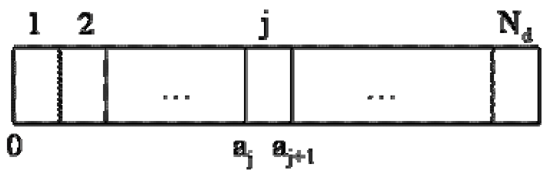


Fig.1 A schematic illustrating of beam

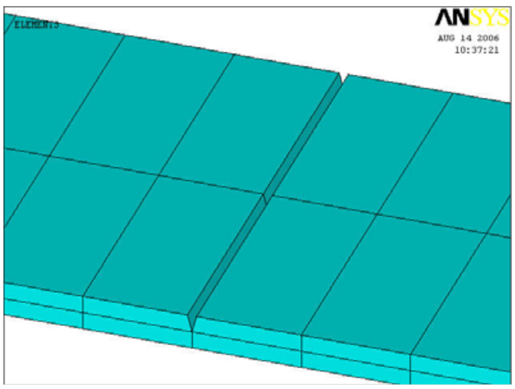


Fig.2 Finite element model of cracked beam

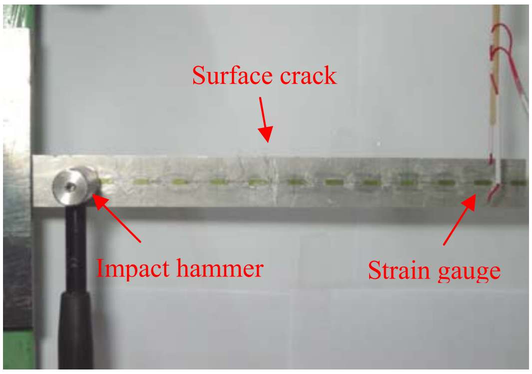
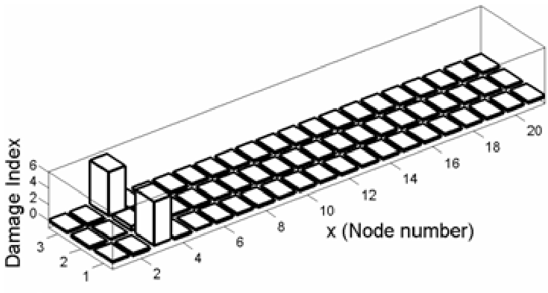
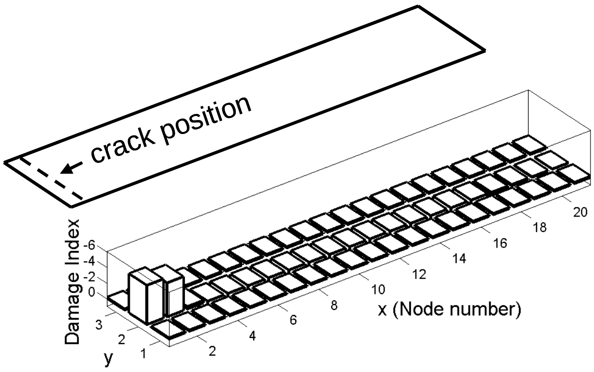


Fig.3 Experimental set-up

Fig.4 Siglab and dynamic amplifier

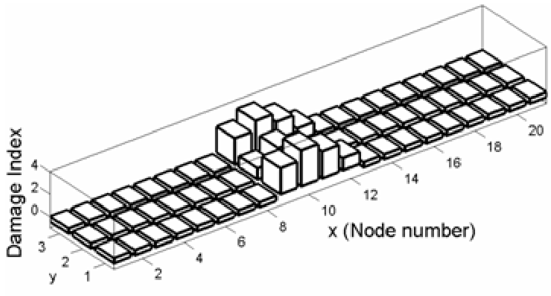


(a) by Modal displacement

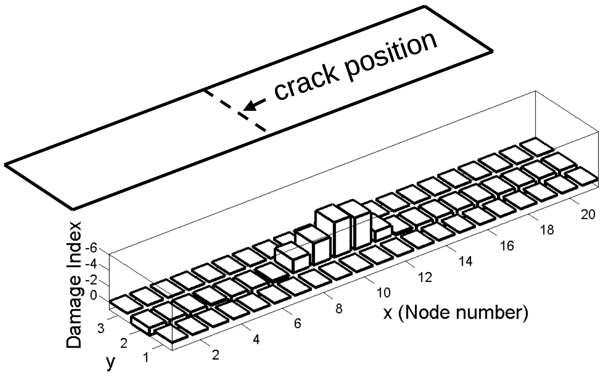


(b) by Modal strain

Fig.5 Damage index of surface crack close to the fixed end (FEA)

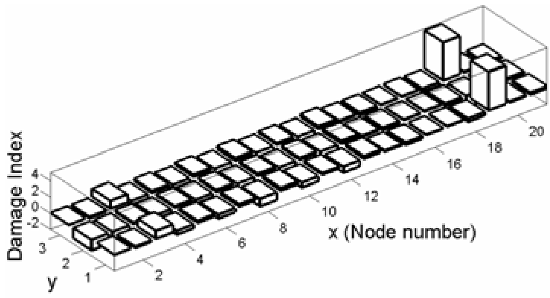


(a) by Modal displacement

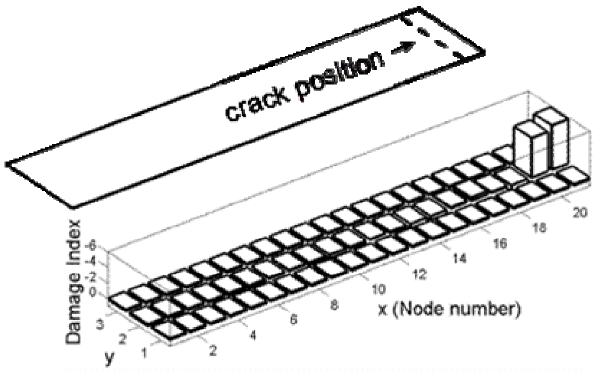


(b) by Modal strain

Fig.6 Damage index of surface crack close to the mid-span (FEA)

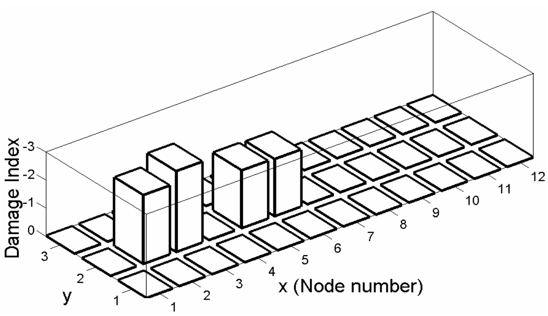


(a) by Modal displacement

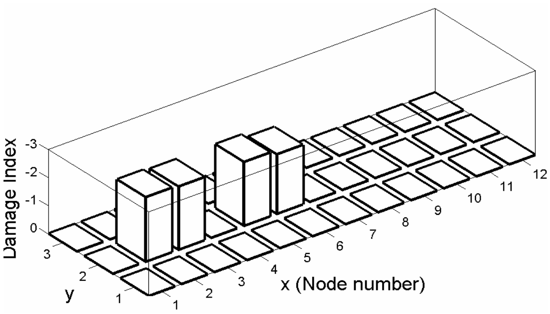


(b) by Modal strain

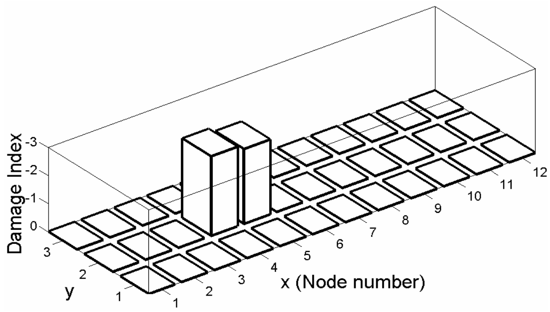
Fig.7 Damage index of surface crack close to the free end (FEA)



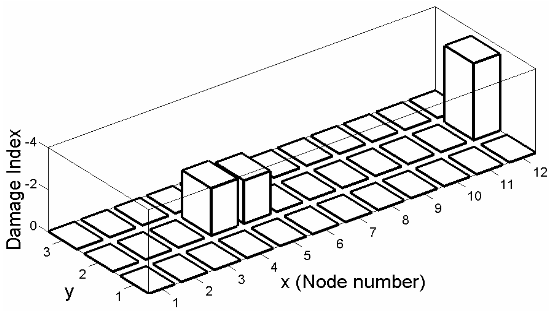
(a) using the first five modes



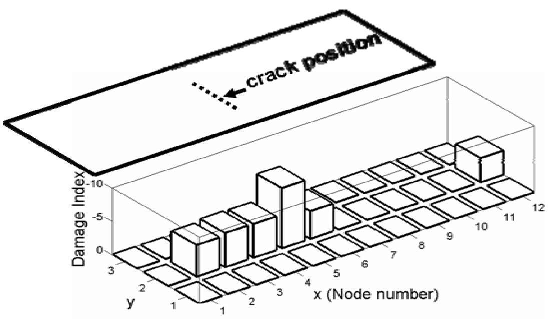
(b) using the first four modes



(c) using the first three modes



(d) using the first two modes



(e) Summation of damage indices (a), (b), (c) and (d)
Fig.8 Damage index of surface crack close to mid-span (EMA)

Table 1 Natural frequencies and mode shapes

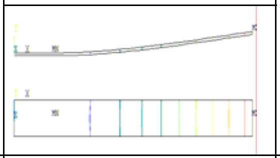
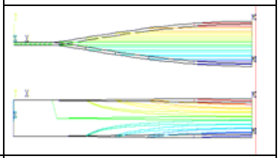
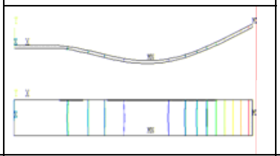
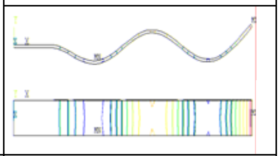
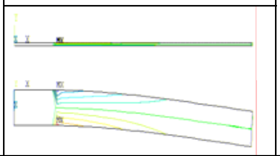
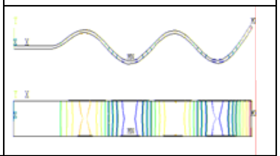
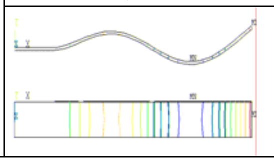
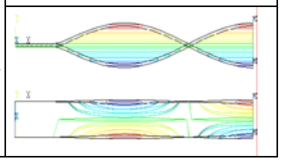
	FEA		FEA
	24.45Hz		470.85 Hz
1		5	
	153.25 Hz		846.67 Hz
2		6	
	312.35 Hz		1408.1 Hz
3		7	
	430.15 Hz		1424.4 Hz
4		8	

Table 2 Natural frequencies before and after damage

	Before damage			After damage		
mode	FEA (Hz)	EMA (Hz)	△ (%)	FEA (Hz)	EMA (Hz)	△ (%)
1	24.45	23.9	-2.25	24.36	23.9	-1.88
2	153.25	149	-2.77	151.2	149	-1.46
3	312.35	-	-	311.96	-	-
4	430.15	417	-3.06	430.02	418	-2.80
5	470.85	-	-	470.46	-	-
6	846.67	817	-3.50	835.73	812	-284
7	1408.1	1350	-4.13	1406.3	1360	-3.29
8	1424.4	-	-	1423.8	-	-

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利用模態應變能指標於懸臂樑之破壞偵測

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摘要

本論文應用鋁合金6061懸臂樑在破壞前後之模態振型位移與模態振型應變分別來計算樑之應變能，並利用破壞前後應變能的比值定義的破壞指標來辨認樑的表面裂縫破壞位置。破壞前後之模態振型位移與模態振型應變可由有限元素分析或實驗模態分析獲得。結果顯示，無論使用模態振型位移與模態振型應變計算知之破壞指標皆可以成功地預測出懸臂樑之表面裂縫位置。

關鍵字：表面裂縫、模態應變能、破壞指標、有限元素分析、實驗模態分析