Proceedings of ICAST2006:
17th International Conference on Adaptive Structures and Technologies
Oct. 16 ~ Oct. 19, Taipei, Taiwan

Prediction of Unknown Harmonic Force Acting on Beam by PVDF Sensors

Bor-Tsuen Wang[1*], Huiwen Hu[2], Jian-Feng Liu[1]
[1] Department of Mechanical Engineering
[2] Department of Vehicle Engineering
National Pingtung University of Science and Technology, Neipu, Pingtung 91201, Taiwan
* Corresponding author: No. 1, Hseuh-Fu Road, Neipu, Pingtung 91201, Taiwan
Tel: +886-8-7703202#7017, Fax: +886-8-774-0142, E-mail: wangbt@mail.npust.edu.tw

ABSTRACT

This work develops a force predictive model to determine the amplitude and location of the unknown harmonic force acting on a cantilever beam by using PVDF sensors. Both on- and off-resonance excitations are considered. PVDF sensors are distributed at different positions of the beam and used to measure the dynamic response due to the unknown harmonic force. Either theoretical or experimental modal analysis can be performed to obtain modal parameters which are used to predict the dynamic response in comparison to the measured response. An objective function is then defined based on the square of difference between the predictive response and measured response. The amplitude and location of unknown force can therefore be predicted by the minimization of the objective function. Both theoretical simulation and experimental results show that the contents of unknown harmonic forces with various amplitudes, locations and excitation frequencies can be reasonably predicted. The developed methodology can be applied to the dynamic system in harmonic operating conditions such as a rotor system and beneficial to the condition monitoring.

Keywords: harmonic force prediction, PVDF sensor, modal parameter, objective function

1. INTRODUCTION

Force prediction for harmonic excitation can be of interest. Wang [1] developed a general formulation to predict the impact and harmonic forces, respectively, for arbitrary structures. Wang and Lin [2] followed Wang's approach [1] and applied accelerometers to measure the beam response due to harmonic force so as to experimentally determine the amplitude and location of the harmonic force. Ma *et al.* [3] discussed a rotor system with rotating unbalance and loosing bearing problem and estimated their resulting excitation force from the measured displacement. D'Cruz *et al.* [4] considered a plate subject to a harmonic force. They adopted the least-square error method and used Gauss-Newton method to minimize the difference between the predicted and experimentally measured response. The location amplitude and phase angle of the harmonic force can be properly predicted. Karlsson [5] assumed the harmonic force spatially uniform and predicted its complex amplitude through the measured harmonic response. Moller [6] prescribed the harmonic point force location, utilized Betti reciprocal theory for the referred loading and, therefore, obtained the force amplitude and applied location.

The sensor selection is important to force prediction implement. Convectional sensors may have source practical limitations. For examples, accelerometers may have mass inertia effect, and the eddy current displacement sensor generally requires additional auxiliary equipment and limits its measured range. Smart material such as PVDF sensors with light weight can be integrated to structures. This paper adopts PVDF

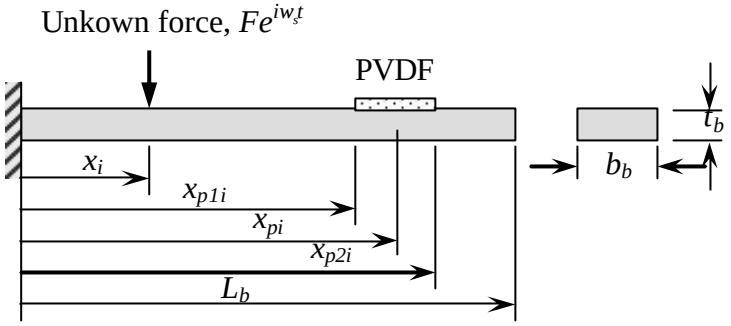


Figure 1. Illustation of harmonic force and PVDF sensor on cantilever beam

sensors for force prediction application. Flatau and Chong [7] introduced NSF-funded projects and research in smart materials and structural systems and their development. Wang [8] presented the active vibration control and compare the control performance for both accelerometers and PVDF sensors.

The use of PZT actuators and PVDF sensors for structural modal testing is of interest and previously shown its feasibility in beam [9] and plate [10] structures. The structural modal parameters can be successfully obtained. Fukunaga *et al*. [11] also used piezoelectric sensor for the application to damage detection. Ray [12] adopted piezoelectric actuators and sensors for plate vibration control. Hwang and Park [13] presented finite element modeling technique for piezoelectric actuator and sensor implemented to structures.

This paper uses FVDF sensors to measure the harmonic response of a beam subjected to unknown harmonic excitation. The force prediction model is adopted and slightly modified from Wang and Lin [2] to determine the harmonic force location and its amplitude.

2. TRANSIENT RESPONSE ANALYSIS OF PVDF SENSING FOR BEAM

Considering the transverse vibration problem of a cantilever beam, based on Bernoulli-Euler theory, the equation of motion can be expressed as

$$E_b I_b \frac{\partial^4 y(x,t)}{\partial x^4} + C_b \frac{\partial y(x,t)}{\partial t} + r_b A_b \frac{\partial^2 y(x,t)}{\partial t^2} = F(x,t) \tag{1}$$

where E_bI_b is bending stiffness of the beam; C_b is damping coefficient; r_b is material density; A_b is the area of the cross section; $F(x,t)$ is the unknown external force acting on the beam. If the beam is subjected to a harmonic force acting on position x_j as shown in Figure 1, the external force can be expressed as

$$F(x,t) = F_j d(x - x_j) e^{iw_s t} \tag{2}$$

where $d(x- x_j)$ is impulse function; w_s is excitation frequency. The harmonic response is therefore obtained by

$$y(x,t) = \sum_{r=1}^{\infty} f_r(x) q_r(t) = \sum_{r=1}^{\infty} f_r(x) Q_r e^{iw_s t} \tag{3}$$

where $f_r(x)$ is the rth displacement mode shape function and $q_r(t)$ is the time function of the beam. According to the orthogonality of mode shapes of the beam, we have

$$Q_r = \frac{F_j f_r(x_j)}{(w_r^2 - w_s^2) + i(2x_r w_r w_s)} \tag{4}$$

A shape function of PVDF can be expressed as

$$\Gamma(x) = u(x - x_{p1i}) - u(x - x_{p2i}) \tag{5}$$

where x_{p1i} and x_{p2i} are the coordinates of i^{th} PVDF; $u(x)$ is unit step function. Thus, the charge of the PVDF is given by

$$q(t) = \frac{t_b + t_p}{2} b_p e_{31} \int_0^{L_b} \Gamma(x) \frac{\partial^2 y(x,t)}{\partial x^2} dx \tag{6}$$

where t_p and b_p are the thickness and width; e_{31} is the piezoelectric field intensity constant of PVDF. The voltage induced by PVDF is

$$V_p(t) = \frac{q(t)}{eA_p} t_p \tag{7}$$

where e and A_p are the electric capacity and area of the cross section of PVDF. Combining Equations (6) and (7), we have

$$V_p(t) = K_p e^{iw_s t} \sum_{r=1}^{\infty} Q_r \left[f_r'(x_{p2i}) - f_r'(x_{p1i}) \right] \tag{8}$$

where

$$K_p = \frac{t_p (t_b + t_p)}{2eA_p} b_p e_{31} \tag{9}$$

Consequently, the time response of PVDF can be expressed as

$$v(x_i, t) = \sum_{r=1}^{\infty} K_p \left[f_r'(x_{p2i}) - f_r'(x_{p1i}) \right] Q_r e^{iw_s t} = \sum_{r=1}^{\infty} \Phi_r^P(x_i) q_r(t) = \sum_{r=1}^{\infty} \frac{\Phi_r^P(x_i) f_r(x_j) F_j}{\left(w_r^2 - w_s^2\right) + i\left(2x_r w_r w_s\right)} e^{iw_s t} \tag{10}$$

3. DEVELOPEMENT OF FORCE PREDICTION MODEL

For a beam structure subjected to an unknown harmonic point force presented in previous section, the beam dynamic response can be measured by the PVDF sensor and used to predict the content of the harmonic force. The optimization is formulated as follows to predict the harmonic force location and its amplitude as well as the phase angle. An objection function is defined as the sum of square difference between the predicted and experimented PVDF response and experessed as follows:

$$Q_t = \sum_{k=1}^{N_t} \left[v_i(t_k) - \hat{v}_i(t_k) \right]^2 \tag{11}$$

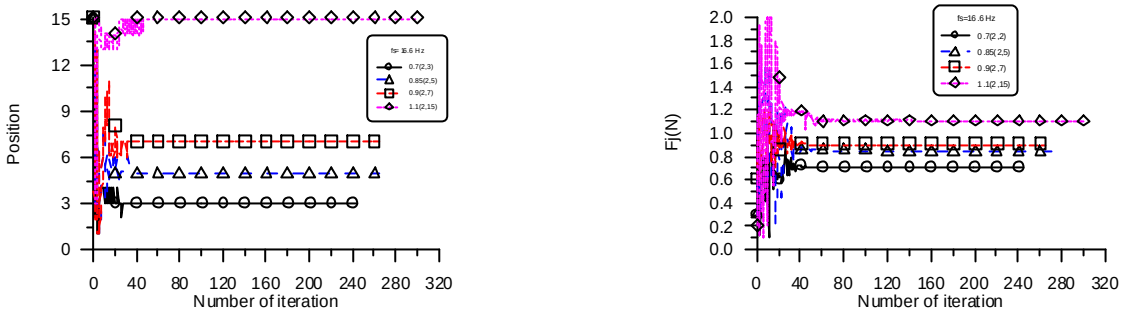
where $v_i(t_k)$ denotes the theoretical prediction of the i^{th} PVDF sensor time response. So shown in equation (10), and $\hat{v}_i(t_k)$ represents the measured PVDF response. N_t is the number of time data included for force prediction. Reinvoke Equation (10) to show the theoretical PVDF sensor response as follows:

$$v_i(t_k) = \sum_{r=1}^{\infty} \frac{\Phi_r^P(x_i) f_r(x_j) F_j}{\left(w_r^2 - w_s^2\right) + i\left(2x_r w_r w_s\right)} e^{iw_s t_k} = V_i e^{iw_s t_k} = |V_i| e^{i(w_s t_k - q)} \tag{12}$$

where $|V_i|$ and q are the amplitude and phase angle of the PVDF harmonic response. As one can observe, the unknown parameters are the force amplitude (F_j), force location (x_j) and phase angle (q) and can be the design varables for the optimization problem. It is noted that the force location is associated with the mode shape vector $f_r(x_j)$, so the location number j is prescribed as the design variable instead of x_j. This modification is different from Wang's approach [1] that used mode shape data $f_r(x_j)$ as design variables. The current approach largely reduces the number of design variables and makes the optimum solution efficient.

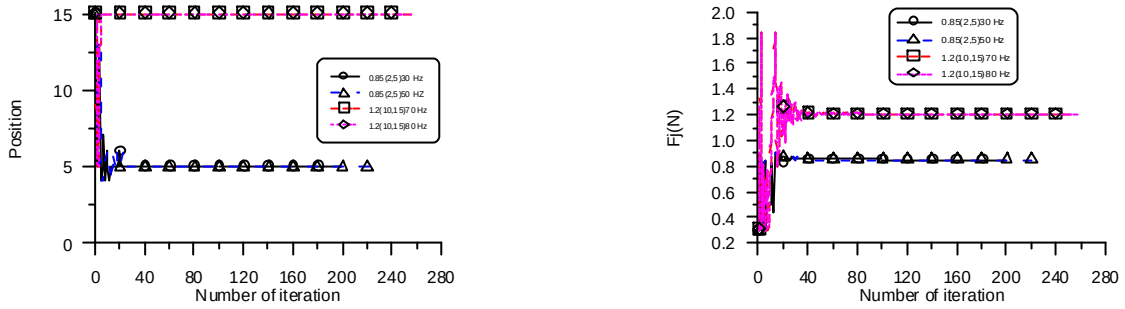
In summary, only the measured PVDF dynamic response $\hat{v}_i(t_k)$ is the input to the developed force prediction algorithm. The objection function as shown in equation (11) can be constructed with the initial guess of design variable i.e. F_j, j, and q. The prediction algorithm is implemented by using compact visual FORTRAN and solved for the optimization problem by IMSC optimization subroutine DBCPOL. The first four modes are employed to compute the theoretical dynamic response.

It is noted that two phases of the implement in force prediction are carried out. First, the theoretical PVDF sensor response is used as the measured one and input to the prediction program to validate the



(a) force locations **(b)** force amplitudes

Figure 2. Phase I: different force locations for on-resonance excitation



(a) force locations **(b)** force amplitudes

Figure 3. Phase I: different excitation frequencies for off-resonance excitations

feasibility of the developed algorithm. Second, the experimental modal testing on the cantilever beam is performed to obtain experimental modal parameters. The PVDF sensor response due to a controlled, prescribed harmonic force is recorded and used as the input to predict the harmonic force content.

4. RESULTS AND DISCUSSIONS

4-1 Phase I: theoretical simulation

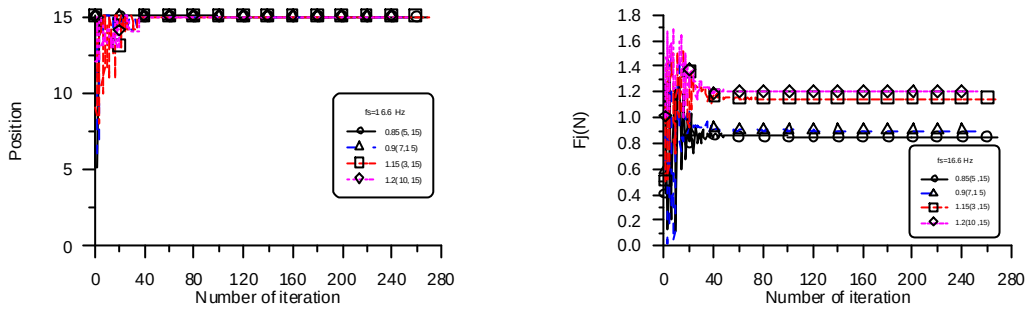
This section presents the theoretical simulation results for harmonic force prediction by using the theoretical PVDF sensor response as the measured ones. Different forcing conditions and sensor locations are simulated to study the feasibility of the developed algorithm.

Consider a cantilever beam subjected to harmonic point forces at different locations, i.e. positions j=3, 5, 7 and 15, respectively. Figure 2 shows the prediction results for on-resonance excitation, f_s=16.6 Hz near the first resonance frequency. The symbols in Figure 2 "0.7(2,3)" denote the force amplitude 0.7N and (i,j)=(2,3), where i is the sensor location, and j is the force location. As one can observe, both force location and force amplitude converge to the prescribed values alter about 40-50 times of iteration during optimization solution. The force predictions are successful.

Consider the same sensor location and prescribed force location and amplitude for (i,j)=(2,5) and (10,15), respectively. Figure 3 shows those for off-resonance excitation f_s=30, 50, 70, and 80 Hz, respectively. One can see both the predicted force locations and amplitudes almost exactly the same to the prescribed values.

The sensor location can be critical to achieve satisfactory prediction in practice. Here, through the theoretical simulation to test the sensitivity for different sensor locations. Figure 4 shows the prediction results for on-resonance cases, f_s=16.6 Hz. The PVDF sensors are chosen at positions i=3, 5, 7, and 10, respectively. Results show the prediction of force locations and amplitudes converges to the prescribed values very well.

In summary, the developed force prediction algorithm can successfully determine the force location



(a) force locations **(b)** force amplitudes

Figure 4. Phase I: different sensor locations for on-resonance excitation

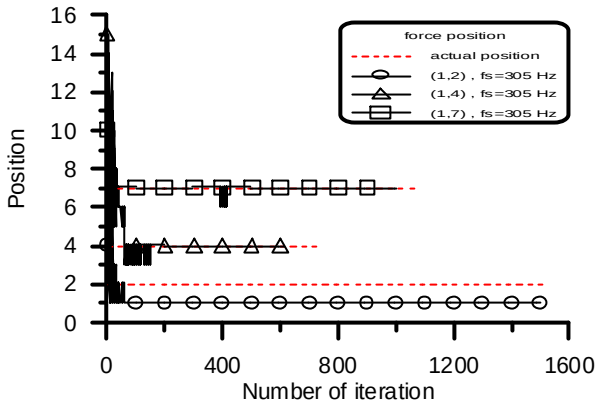


Figure 5. Phase II: prediction of force locations for different force locations

and amplitude exactly for theoretical simulation. Different force locations and amplitudes, excitation frequencies, and PVDF sensor locations are tested and revealed perfect predictions. Next section will show the experimental prediction results.

4-2 Phase II: experimental validation

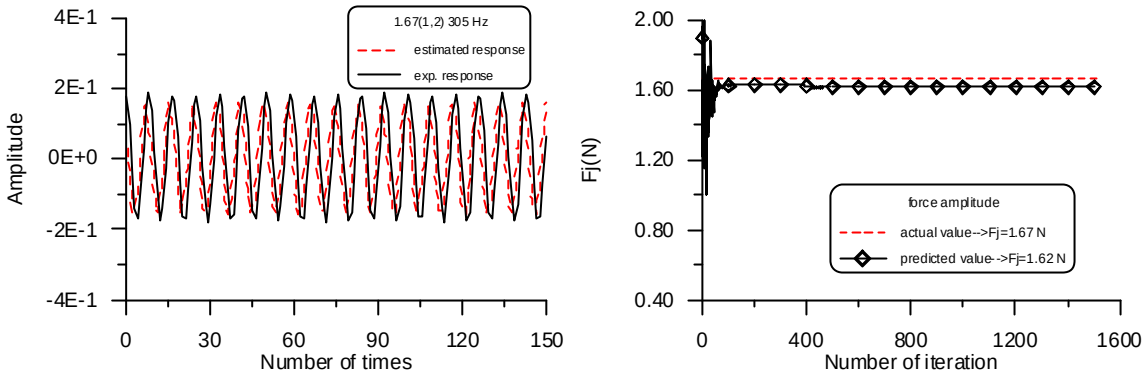
In this section, the experimental modal parameters obtained from modal testing of the beam are used to predict the PVDF sensor response $v_i(t_k)$ as shown in Equation (12). The first four natural frequencies are 16.6, 107, 300 and 587 Hz, respectively. Experiments for the cantilever beam subjected to prescribed harmonic excitation forces are carried out to record PVDF sensor dynamic response that is used to predict the harmonic amplitude and its location, simultaneously, through the predictive program.

Consider the unknown harmonic force excited at 305 Hz near the third resonance. The PVDF sensor is at position i=1, and the harmonic forces are applied at positions i=2, 4 and 7, respectively. Figure 5 shows the optimization solution process for the convergence of force location, and Figure 6 reveals the PVDF sensor time response and the convergence of force amplitude, respectively, for case (i,j)=(1,2). Table 1 summarizes the prediction results. The predicted force amplitudes are with in $\pm$4% except the case (i,j)=(1,4) with 25% error because of the small force amplitude. The predicted locations for case (i,j)=(1,2) deviates to j=1 close to prescribed j=2. The other two cases reveal very good location prediction. The phase angles are also predicted to adjust the sensor response accordingly as shown in Figure 9(a) for case (i,j)=(1,2).

Table 2 shows the prediction results for different excitation frequencies. Figures 7 and 8 show the corresponding results for Table 2. For the case (i,j)=(5,2) with f_s=107 Hz, the predictive force amplitude is 4% error, and the predictive force location is exact. For case (i,j) =(1,7) with f_s=202 Hz, the predictive location j=8 is slightly different from the prescribed j=7, and the predictive force amplitude is reasonably predicted in 7% error. As revealed in Figure 8(a), there may have some measured noise in experiments for the slight distortion of harmonic wave. This can be the cause of the prediction errors. Similar phenomenon can be

Table 1. Phase II: prediction of force locations and amplitudes for different force locations

(i, j)	Applied force (N)	Predicted force (N)	Predicted location	Phase angle (degree)
(1,2)	1.67	1.62(-3%)	1	17.20
(1,4)	0.377	0.473(25%)	4	42.42
(1,7)	1.68	1.61(4%)	7	19.75



(a) PVDF time response for (i,j)=(1,2) **(b)** convergence of predicted amplitude for (i,j)=(1,2)

Figure 6. Phase II: prediction of force amplitudes for different force locations

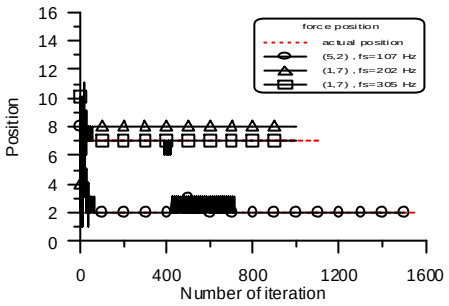
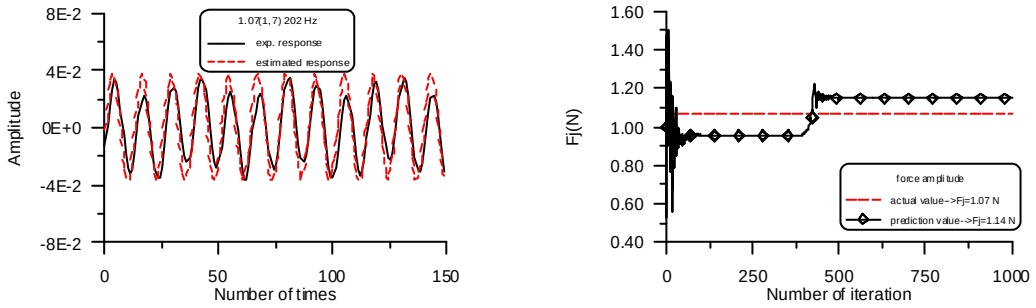


Figure 7. Phase II: prediction of force locations for different excitation frequencies



(a) PVDF time response for (i,j)=(5,2) **(b)** convergence of predicted amplitude for (i,j)=(5,2)

Figure 8. Phase II: prediction of force amplitudes for different excitation frequencies

Table 2. Phase II: prediction of force position and amplitude for different excitation frequencies

(i, j)	Excitation freqency (Hz)	Applied force (N)	Predicted force (N)	Predicted location	Phase angle (degree)
(5,2)	107	2.32	2.42(4%)	2	16.41
(1,7)	202	1.07	1.14(7%)	8	31.23
(1,7)	305	1.68	1.61(-4%)	7	19.75

observed in other cases. However, the predictive model does accommodate the signal noise to reasonably predict the harmonic force amplitude and location. Although the phase angle of the harmonic force may not be as important as the amplitude or location, the phase angle can properly adjust the time shift of the estimated PVDF sensor response to fit the experimental one and is crucial to the optimum solution to ensure the appropriate construction of the objective function. The third case of (i,j) =(1,7) with f_s=305 Hz also reveals reasonable prediction results.

Table 3. Phase II: prediction of force position and amplitude by different sensor locations

(i, j)	Applied force (N)	Predicted force (N)	Predicted location	Phase angle (degree)
(2,2)	2.34	2.17(-7%)	2	26.72
(3,2)	2.34	2.51(7%)	2	5.57
(6,2)	2.32	2.25(-3%)	3	10.16
(8,2)	2.34	2.30(-2%)	2	14.69

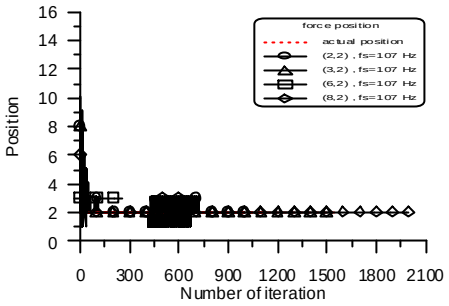
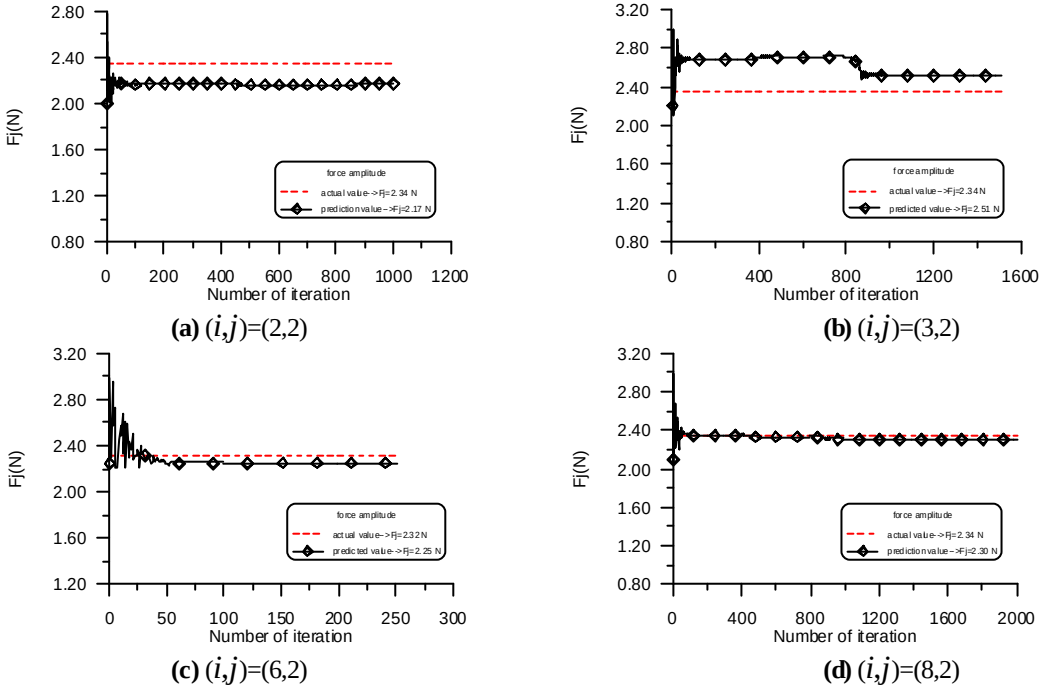


Figure 9. Phase II: prediction of force locations for different sensor locations



(a) (i,j)=(2,2) **(b)** (i,j)=(3,2)

(c) (i,j)=(6,2) **(d)** (i,j)=(8,2)

Figure 10. Phase II: convergence of predicted force amplitudes for different sensor locations

To experimentally study the effect of PVDF sensor location on the force prediction, the sensor is arranged to be at position i=2, 3, 6, or 8, respectively, which the harmonic force is prescribed at j=2 with f_s=107 Hz. Table 3 summarizes the prediction results, and Figures 9 and 10 show the corresponding convergence results, respectively. The predictive force amplitudes are within $\pm$7% errors, the predicted force location is the same as the prescribed except for case (i,j)=(6,2) where the predicted j=3. The number of iteration in optimum solution is generally about 50-100, except the case (i,j)=(3,2) up to 800 times. In summary, the force prediction can be reasonably successful.

5. CONCLUSIONS

This work modifies the predictive algorithm from Wang [1]. The force location index j is adopted as the design variables making the optimal solution efficient. In addition, the current approach reveals more flexible in selecting the sensor locations and ensure more stable the convergence of optimal solution than Wang and Lin [2]. Both theoretical simulation and experimental verification for the prediction algorithm in

using the PVDF sensors are carried out, respectively, and shown the feasibility and effectiveness of the unknown harmonic force prediction. Both the harmonic force amplitude and location can be simultaneously determined in solving the formulated optimization problem. The predictive algorithm can reasonably predict the force contents that can be useful for structural diagnosis. The off-line force prediction is adopted in this work, and the on-line force prediction is of great interest in the future for practical applications.

ACKNOWLEDGEMENT

The authors gratefully thank the financial support of this work from National Science Council, TAIWAN under the contract number: NSC93-2212-E-020-006.

REFERENCES

1. Wang, B. T., 2002, "Prediction of Impact and Harmonic Forces Acting on Arbitrary Structure: Theoretical Formulation," *Mechanical Systems and Signal Processing*, Vol. 16, No. 6, pp. 935-953.
2. Wang, B. T., and Lin, K. Y., "Prediction of Harmonic Force Acting on Cantilever Beam," *Proceedings of the 18th International Modal Conference*, Dearborn, Michigan, Paper No. 64, Session 3 (2004).
3. Ma, C. K., Liu, J. J., Kung, I. C., and Lin, D. C., "The Exciting Force Estimation and Validation of a Rotary Machine," *Journal of Taiwan Society of Naval Architects and Marine Engineers*, Vol. 19, No. 3, pp. 43-51 (2000)
4. D'Cruz, J., Crisp, J. D. C., and Ryall, T. G., "On the Identification of a Harmonic Force on a Viscoelastic Plate From Response Date," *ASME Journal of Applied Mechanics*, Vol. 59, pp. 722-729 (1992).
5. Karlsson, S. E. S., "Identification of External Loads from Measured Harmonic Responses," *Journal of Sound and Vibration*, Vol. 196, pp.59-74 (1996).
6. Moller, P. W., "Load Identification Through Structure Modification," *Transactions of the ASME Journal of Applied Mechanics*, Vol. 66, pp. 236-241 (1999).
7. Flatau, A. B., and Chong, K. P., "Dynamic Smart Material and Structural Systems," Engineering Structures, Vol. 24, No. 3, pp. 261-270 (2002)
8. Wang, B. T., "The Performance of Accelerometers and PVDF Sensor in Active Structural Vibration Control," *Bulletin of National Pingtung Polytechnic Institute*, Vol. 3, pp. 81-91 (1999).
9. Wang, B. T., and Wang, C. C., 1997, "Feasibility Analysis of Using Piezoceramic Transducers for Cantilever Beam Modal Testing," *Journal of Smart Materials and Structures*, Vol. 6, pp. 106-116.
10. Wang, B. T., and Chen, R. L., 2000, "The Use of Piezoceramic Transducers for Smart Structural Testing," *Journal of Intelligent Material Systems and Structures*, Vol. 11, No. 9, pp. 713-724.
11. Fukunaga, H., Hu, N., and Chang, F. K., "Structural Damage Identification Using Piezoelectric Sensor," *International Journal of Solid and Structures*, Vol. 39, pp. 393-418 (2002).
12. Ray, M. C., "Optimal Control of Laminated Plate with Piezoelectric Sensor and Actuator Layers," *AIAA Journal*, Vol. 36, No. 12, pp. 2204-2208 (1998).
13. Hwang, W. S., and Park, H. C., "Finite Element Modeling of Piezoelectric Sensors and Actuators," *AIAA Journal*, Vol. 31, No. 5, pp. 930-936 (1993).