MODEL VERIFICATION OF FINITE ELEMENT ANALYSIS FOR FREE VIBRATION OF COMPOSITE LAMINATES

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ABSTRACT

A finite element analysis for free vibration of composite laminates is verified using experimental modal analysis in this paper. A three dimensional finite element model was developed to simulate composite laminate plates with unidirectional fiber orientations $[90]_{16}$, cross-ply $[0/90]_{4S}$, and quasi-isotropic $[0/90/\pm45]_{2S}$. Mass effect of accelerometer was considered into the analysis model. Material constants of laminate plates were obtained from tensile test and entered into the solver. A normal mode analysis was performed to obtain the natural frequencies and the associated mode shapes of laminate plates with completely free boundary condition. Modal testing was performed to extract the natural frequencies and associated mode shapes in comparison to FEA results. Thermoplastic composite AS4/PEEK was used to fabricate the laminate plates. Modal Assurance Criterion was used to verify the mode shapes obtained from FEA and EMA results. Consequently, Good correlation between FEA and EMA results is obtained. This verified model can be used for further analysis.

Keywords: finite element analysis, experimental modal analysis, composite laminates, model verification

1. INTRODUCTION

Vibration behavior of composite materials and structures is still one the most complicated problems studied by many researchers due to their anisotropic properties. Traditionally, free vibration of composite laminates is the essential problem to be investigated by significant number of papers which were thoroughly reviewed by Leissa [1], Bert [2], Reddy [3], Kapania and Raciti [4]. Most of the above approaches focused on both analytical (closed-form, Galerkin, Rayleigh-Ritz method) and numerical methods. Especially, the use of FEA commercial codes has increased dramatically over the past decade due to its flexibility of modeling. In these commercial codes, four-node linear shell or plate elements were usually adopted to define the lamina properties and stacking sequence [5-6]. The input mechanical properties of composite material were commonly measured from quasi-static tensile tests of the materials. Nevertheless, the local static properties may not represent the global dynamic behavior of the structures. Thus, a so-call inverse method was proposed to determine the global mechanical properties using modal testing [7-8]. The study of thick cross-ply laminates show that in-plane shear modulus and Poisson's ratio obtained from modal testing are quite different from static test [8]. A question raised here is: can the global properties be applied to other problems of structural analysis? If the global properties are size-dependent or geometry-dependent, which one should we follow? Therefore, the use of local static material properties in the analysis of dynamic global structures is still to be improved and verified.

The objective of this paper is to study the free vibration of various thin composite symmetrical laminates by using FEA and EMA. The material properties obtained from local measurement is used in the global FEA. The dynamic responses obtained from global structure using EMA is used to verify finite element model. This verified FE model can be used in further analysis of the problems of damage detections in composite laminates.

2. TRANSVERSE VIBRATION OF LAMINATE PLATE

The equation of motion for transverse vibration of a composite laminate plate can be derived from classical laminate theory, i.e., [9]

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+\rho\frac{\partial^2 w}{\partial t^2}=0 \quad (1)$$

where w is the transverse displacement; D_{ij} are bending stiffness; ρ is density of the laminate plate. For free harmonic vibration at frequency, ω, the solution of transverse displacement can be assumed as

$$w(x,y,t)=\phi(x,y)e^{i\omega t} \tag{2}$$

where $\phi(x,y)$ is a mode shape function. Substituting equation (2) in equation (1), we have

$$D_{11}\frac{\partial^4\phi}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4\phi}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4\phi}{\partial y^4}-\rho\omega^2\phi=0 \tag{3}$$

For simply supported boundary, transverse displacements and bending moments much vanish at the edges. The solutions is obtained as

$$\phi(x,y)=A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{4}$$

where m and n are mode indices which refer to the number of half wavelengths along the x and y directions, respectively; a and b are the dimensions. Substitution of equation (4) in equation (3) yields the frequency equation

$$\omega_{mn}^2=\frac{\pi^4}{\rho a^4}[D_{11}m^4+2(D_{12}+2D_{66})(mnR)^2+D_{22}(nR)^4] \tag{5}$$

where $R = a/b$. Generally, it is not possible to find exact mode shape function for completely free edges. Approximate solutions must be derived using approaches such as Rayleigh-Ritz method or the Galerkin method [10,11].

4. FINITE ELEMENT ANALYSIS

A three dimensional finite element model was established to analyze the free vibration of composite laminate plates, $[90]_{16}$, $[0/90]_{4S}$, $[0/90/\pm45]_{2S}$, with dimension 222×247×2.3 mm^3. ANSYS, a FEA commercial code, was used in this study. Eight-node linear solid element (SOLID46) was used to simulate the laminate plates. The element provides a layered version allow up to 250 different material layers. Traditionally, shell or plate elements are adopted in solving the problem. However, they are not convenient to model a local damage not penetrating the plate such as surface crack or inside matrix crack. This is why solid element is used in this study. Figure 1 shows the finite element model. A convergence study was performed for laminate plate $[90]_{16}$. A 36×24×8 mesh model is sufficient to solve the first eight normal modes problem.

Material constants listed in Table 1 are obtained from quasi-static tensile tests and are entered into ANSYS. Hu et al. [12] found that the effects of out-of-plane shear modulus G_{23} and Poisson's ratio ν_{23} on the natural frequencies are not critical in thin plate. Thus, the values of G_{23} and ν_{23} were assumed to be the same as G_{12} and ν_{12} in this study. Material density was directly measured from the test plates. A normal mode analysis with completely free boundary condition was performed to obtain the natural frequencies and the associated mode shapes up to 5 kHz. Hu et al. [12] found that mass effect of accelerometer to the natural frequencies of specimen is significant. Thus, a mass element (MASS21) with 0.0015 kg was assigned to fix at the FE model as shown in Figure 1.

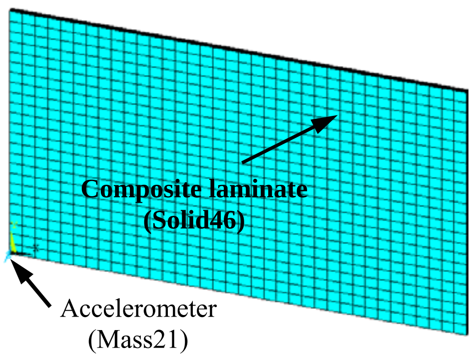


Figure 1 Finite element model

Table 1 Material properties for laminate plates

Laminates / properties	$[90]_{16}$	$[0/90]_{4S}$	$[0/90/\pm45]_{2S}$
Density (kg/m^3)	1526	1537	1576
Material constants	E_{11}=117.2 GPa E_{22}=9.0 GPa $G_{12}=G_{23}=G_{13}$=4.9 GPa $\nu_{12}=\nu_{23}=\nu_{13}$=0.315		

5. EXPERIMENTAL MODAL ANALYSIS

Laminate plates $[90]_{16}$, $[0/90]_{4S}$, $[0/90/\pm45]_{2S}$ were fabricated using thermoplastic composite prepreg AS4/PEEK, and then cured at a hot-press machine. After curing, the panel was cut to a plate with dimension 222×24.7×2.3 mm^3 and marked with 13×13 parallel grid points. The test beam was vertically hung by two cotton strings to simulate a completely free boundary condition as shown in Figure 2. Laminate plate was excited by an impact hammer with a force transducer throughout all grid points. Dynamic responses were measured by an accelerometer fixed at the corner. Siglab, Model 20-40, was used to record the frequency response functions (FRFs) between measured acceleration and impact force. ME'Scope, a software for general purpose curve fitting, was used to extract modal parameters, i.e., natural frequencies, damping ratios and mode shapes, from the FRFs.

Figure 2 Experimental Set-up

6. MODEL VERIFICATION

The first eight natural frequencies of the laminate plates, $[90]_{16}$, $[0/90]_{4S}$, and $[0/90/\pm45]_{2S}$, are listed in Tables 2, 3 and 4, respectively. Superscript symbol "*" denotes the analytical results of FE models without accelerometer. Symbol "△" denotes the difference between FEA and EMA results. The data show that accelerometer mass significantly affects the natural frequency. Adding the mass element to FE model, the differences between FEA and EMA results reduce to less than 4 % for laminate plates $[90]_{16}$ and $[0/90]_{4S}$, and less than 6 % for laminate plate $[0/90/\pm45]_{2S}$. Ignoring accelerometer mass effect, the difference between FEA and EMA could increase up to more than 10%. In fact, the mass ratio of accelerometer to laminate plates is about 0.016; however, the changes of natural frequencies in laminate plates are in the range from 2% to 12%.

The first eight associated mode shapes of the laminate plates are compared using Modal Assurance Criterion (MAC), which is an approach to compare the mode shapes in terms of vector form, i.e.,

$$\mathrm{MAC}(\{\phi_a\},\{\phi_e\}) = \frac{\left(\{\phi_a\}^T,\{\phi_e\}\right)^2}{\left(\{\phi_a\}^T,\{\phi_a\}\right)\left(\{\phi_e\}^T,\{\phi_e\}\right)} \tag{6}$$

where $\{\phi_a\}$ and $\{\phi_e\}$ are mode shapes obtained from FEA and EMA, respectively. Basically, good correlation between FEA and EMA results is obtained, if the value of MAC is greater than 0.9. Less than 0.05, correlation is poor. Table 5, 6 and 7 list the MAC values of three different laminate plates. The data at the diagonal show that good correlations between FEA and EMA results are obtained. The mode shapes obtained from FEA and EMA results can also be verified by checking mode shape contours as shown in table 8, 9 and 10. The contours show that FEA (with accelerometer) and EMA results are very much alike. Consequently, a finite element model has been verified. This reliable model can be used for further analysis of damage detection problems.

Table 2 Natural frequencies of laminate $[90]_{16}$

Mode	FEA(Hz)		EMA(Hz)	△(%)	
(3,1)	144*	138	134	7.3*	3.0
(2,2)	181*	173	175	4.2*	-1.1
(4,1)	397*	364	371	7.1*	-1.9
(3,2)	399*	398	387	3.1*	2.8
(4,2)	689*	638	658	4.6*	-3.0
(5,1)	783*	768	740	5.7*	3.8
(5,2)	1082*	997	1030	5.1*	-3.2
(6,1)	1301*	1255	1220	6.6*	2.9

Table 3 Natural frequencies of laminate $[0/90]_{4S}$

Mode	FEA(Hz)		EMA(Hz)	△(%)	
(2,2)	173*	160	154	12.2*	3.9
(3,1)	382*	366	361	5.8*	1.4
(3,2)	519*	476	486	6.7*	-2.1
(1,3)	920*	817	846	8.8*	-3.4
(2,3)	981*	941	924	6.1*	1.8
(4,1)	1056*	1023	1000	5.6*	2.3
(4,2)	1175*	1119	1120	4.9*	-0.1
(3,3)	1223*	1201	1170	4.5*	2.6

Table 4 Natural frequencies of laminate $[0/90/\pm45]_{2S}$

Mode	FEA(Hz)		EMA(Hz)	△(%)	
(2,2)	254*	242	212	19.8*	14.2
(3,1)	304*	295	280	8.6*	5.4
(3,2)	600*	567	536	11.9*	5.8
(4,1)	833*	808	778	7.1*	3.9
(1,3)	883*	857	804	9.8*	6.6
(2,3)	1030*	960	921	11.8*	4.2
(4,2)	1139*	1112	1060	7.5*	4.9
(3,3)	1398*	1339	1280	9.2*	4.6

Table 5 MAC of laminate $[90]_{16}$

Mode		FEA							
		(3,1)	(2,2)	(4,1)	(3,2)	(4,2)	(5,1)	(5,2)	(6,1)
EMA	(3,1)	**0.967**	0.030	0.015	0.001	0.001	0.047	0.019	0.002
	(2,2)	0.001	**0.976**	0.023	0.001	0.008	0.018	0.019	0.003
	(4,1)	0.006	0.001	**0.808**	0.117	0.043	0.002	0.001	0.009
	(3,2)	0.001	0.000	0.101	**0.867**	0.010	0.002	0.011	0.057
	(4,2)	0.004	0.004	0.006	0.005	**0.905**	0.036	0.040	0.009
	(5,1)	0.002	0.001	0.000	0.000	0.002	**0.876**	0.023	0.002
	(5,2)	0.004	0.001	0.008	0.000	0.002	0.017	**0.847**	0.074
	(6,1)	0.000	0.001	0.006	0.000	0.000	0.000	0.001	**0.745**

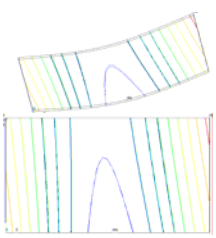
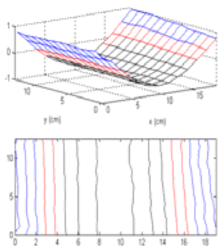
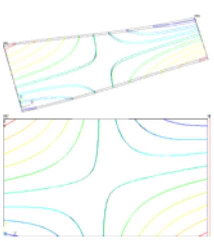
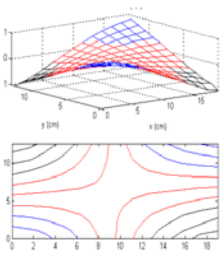
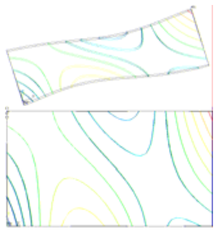
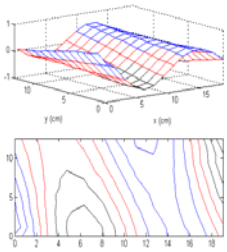
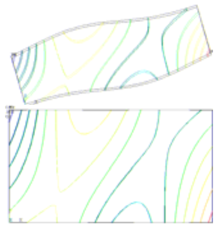
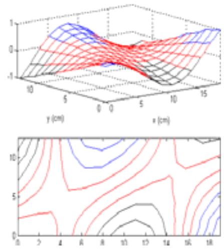
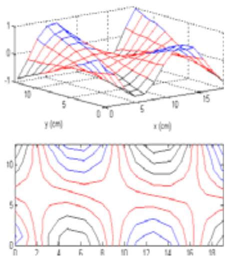
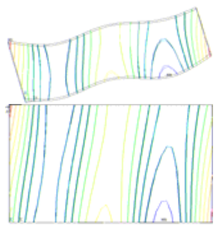
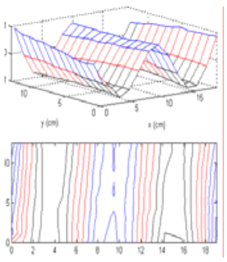
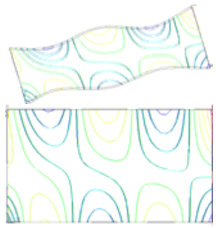
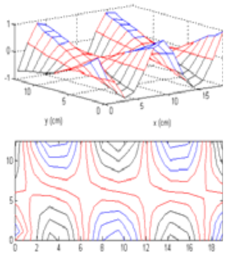
Table 6 MAC of laminate $[0/90]_{4S}$

Mode		FEA							
		(2,2)	(3,1)	(3,2)	(1,3)	(2,3)	(4,1)	(4,2)	(3,3)
EMA	(2,2)	**0.992**	0.025	0.022	0.026	0.001	0.000	0.010	0.027
	(3,1)	0.001	**0.950**	0.064	0.022	0.000	0.000	0.000	0.009
	(3,2)	0.007	0.010	**0.934**	0.111	0.005	0.008	0.006	0.001
	(1,3)	0.003	0.001	0.000	**0.601**	0.278	0.061	0.021	0.001
	(2,3)	0.001	0.002	0.000	0.134	**0.687**	0.154	0.036	0.000
	(4,1)	0.001	0.004	0.000	0.031	0.012	**0.756**	0.131	0.001
	(4,2)	0.002	0.004	0.002	0.033	0.003	0.046	**0.775**	0.038
	(3,3)	0.001	0.005	0.000	0.003	0.004	0.001	0.017	**0.922**

Table 7 MAC of laminate $[0/90/\pm45]_{2S}$

Mode		FEA							
		(2,2)	(3,1)	(3,2)	(1,3)	(2,3)	(4,1)	(4,2)	(3,3)
EMA	(2,2)	**0.990**	0.000	0.003	0.005	0.000	0.001	0.006	0.004
	(3,1)	0.010	**0.992**	0.000	0.001	0.001	0.004	0.002	0.001
	(3,2)	0.017	0.007	**0.983**	0.000	0.000	0.006	0.002	0.005
	(4,1)	0.006	0.008	0.031	**0.905**	0.013	0.035	0.000	0.000
	(1,3)	0.002	0.003	0.009	0.003	**0.958**	0.003	0.001	0.003
	(2,3)	0.001	0.002	0.019	0.116	0.016	**0.866**	0.003	0.002
	(4,2)	0.022	0.002	0.006	0.002	0.005	0.024	**0.967**	0.001
	(3,3)	0.011	0.006	0.006	0.000	0.001	0.040	0.011	**0.934**

Table 8 Mode shapes of laminate $[90]_{16}$

Mode	FEA	EMA
(3,1)	138 Hz	134 Hz
		
(2,2)	173 Hz	175 Hz
		
(4,1)	364 Hz	371 Hz
		
(3,2)	398 Hz	387 Hz
		
(4,2)	638 Hz	658 Hz
		
(5,1)	768 Hz	740 Hz
		
(5,2)	997 Hz	1030 Hz
		

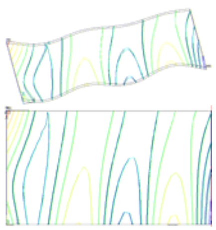
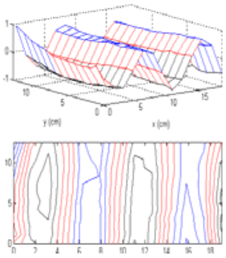
	1255 Hz	1220 Hz
(6,1)		

Table 9 Mode shapes of laminate $[0/90]_{4S}$

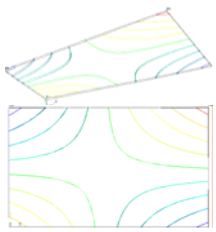
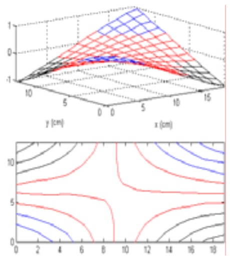
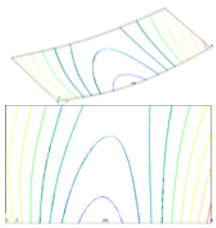
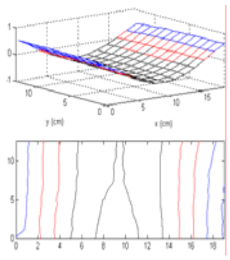
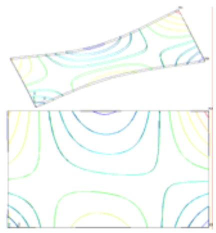
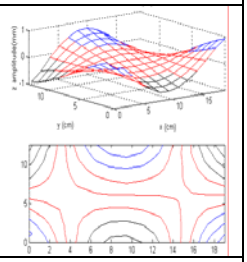
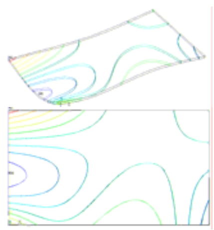
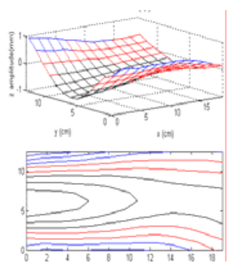
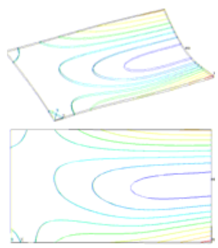
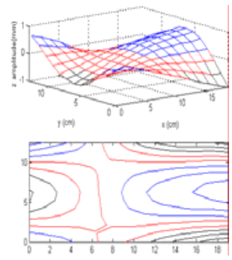
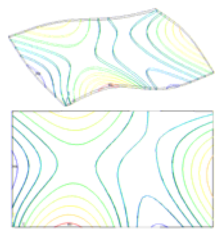
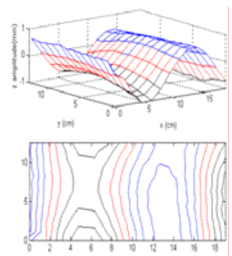
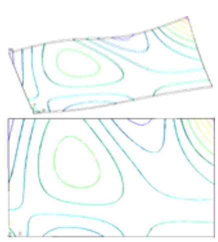
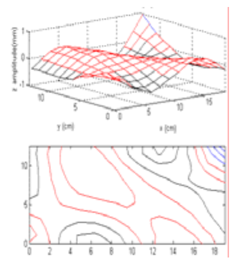
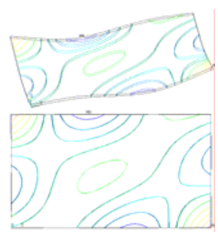
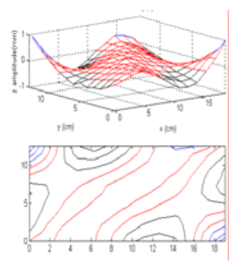
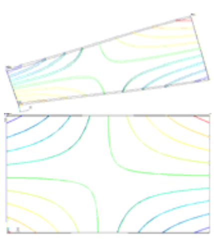
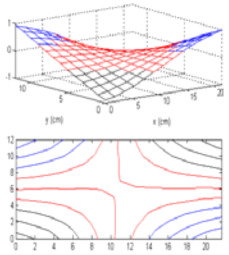
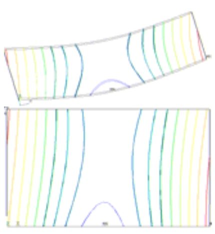
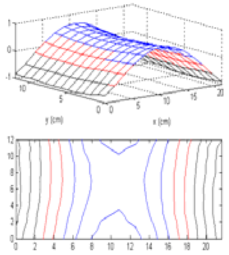
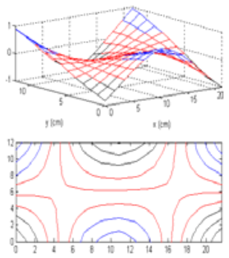
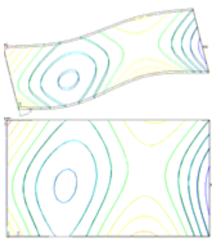
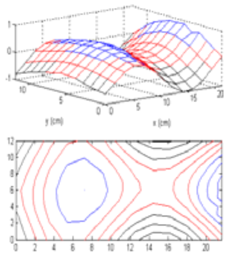
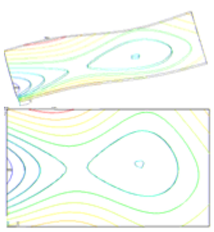
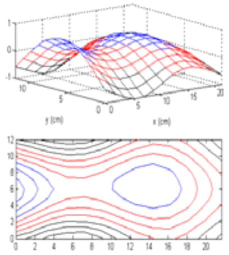
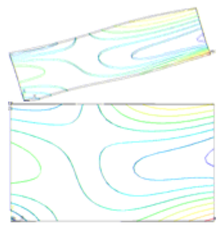
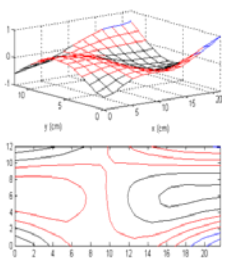
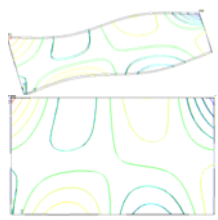
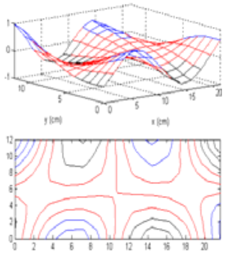
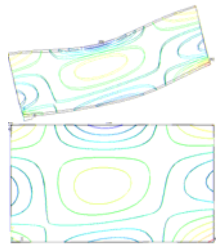
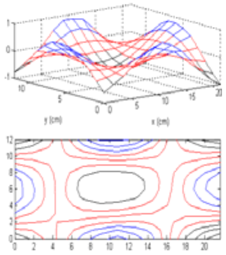
Mode	FEA	EMA
	160 Hz	154 Hz
(2,2)		
	366 Hz	361 Hz
(3,1)		
	476 Hz	486 Hz
(3,2)		
	817 Hz	846 Hz
(1,3)		
	941 Hz	924 Hz
(2,3)		
	1023 Hz	1000 Hz
(4,1)		
	1119 Hz	1120 Hz
(4,2)		
	1201 Hz	1170 Hz
(3,3)		

Table 10 Mode shapes of laminate $[0/90/\pm 45]_{2S}$

Mode	FEA	EMA
	242Hz	212Hz
(2,2)		
	295Hz	280Hz
(3,1)		
	567Hz	536Hz
(3,2)		
	808Hz	778Hz
(4,1)		
	857Hz	804Hz
(1,3)		

	960Hz	921Hz
(2,3)		
	1112Hz	1060Hz
(4,2)		
	1339Hz	1280Hz
(3,3)		

7. CONCLUSIONS

A finite element model for free vibration analysis of composite laminate plates has been verified using experimental modal analysis. This reliable model successfully simulates three types of laminate plates, unidirectional fiber orientations $[90]_{16}$, cross-ply $[0/90]_{4S}$, and quasi-isotropic $[0/90/\pm 45]_{2S}$. The mass of accelerometer significantly affects natural frequencies of the laminate plates. Considering the mass effect of accelerometer, good correlations of natural frequencies and the associate mode shapes between FEA and EMA results are obtained. This model verifies that local measured static material properties can still be used to represent the global structural dynamic behaviors. The reliable model provides us to further analyze the dynamic problem such as damage detection problems of composite laminates in the future work.

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有限元素於複合材料疊層板自由振動分析之模型驗證

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摘要

本論文探討如何應用實驗模態來驗證複合材料疊層板之有限元模型在自由振動問題之分析。疊層板之有限元模型採用實體元素，並將加速度計質量以單點質量元素模擬。有限元模型之材料常數則以靜態拉伸試驗量測獲得。實驗材料是採用碳纖維/聚二醚酮(AS4/PEEK)，疊層型式為單一纖維方向 $[90]_{16}$，十字疊層$[0/90]_{4S}$，以及類似等向性疊層 $[0/90/\pm45]_{2S}$三種。經比對實驗與有限元分析結果之自然頻率與模態振型，兩種結果相當吻合。模態振型的比較則採用模態保證指標。

關鍵字：有限元訴分析、實驗模態分析、複合材料疊層板、模型驗證