APPLICATION OF MODAL ANALYSIS TO DETECT MATRIX CRACKS IN COMPOSITE LAMINATES

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ABSTRACT

A nondestructive detection of matrix cracks in composite laminates using modal analysis is investigated in this paper. Thermoplastic composite AS4/PEEK was used to fabricate a laminate beam $[0_2/90_n/0_2]$. To create matrix cracks in 90-degree lamina, the laminate beam was subjected to tensile test. Modal test was performed to obtain the mode shapes of laminate beam before and after damage. Mode shape displacements were used to compute the strain energy of the laminate beam. Limited by grid points of measurement, a differential quadrature method was used to calculate the partial differential terms in strain energy formula. A damage index was then defined based on strain energy ratio of the laminate beam before and after damage. Consequently, damage index developed in this study successfully predicts the locations of matrix cracks inside the laminate beam. A pre-study was performed to access this approach by using a 3-D finite element analysis. Good correlation between analytical and experimental results is obtained.

Keywords: modal analysis, matrix crack, composite laminate, strain energy

1. INTRODUCTION

Matrix crack is of concern to composite structure designers especially in structural long term durability. Traditionally, matrix crack initiates the damage in composite laminates and then subsequently triggers other damage modes, such as delamination. Therefore, advance detection of matrix crack is essential in the use of composite structures.

In nondestructive damage detection technology, vibration-based methods have been increasingly adopted due to their flexibility in measurement and relatively low cost. The basic idea of these methods is to use the information of modal parameters, such as frequency, mode shape and damping ratio, to access the structural damage.

Cawley and Adams [1] simply used the frequency shifts for different modes to detect the damage in composite structures. Tracy and Pardoen [2] found that the natural frequencies of a composite beam were affected by the size and damage location. Shen and Grady [3] indicated that local delamination does not have a noticeable effect on global mode shape of composite beams, but delamination does cause the irregularity of mode shapes. Zou et al. [4] provided a thorough review in vibration-based techniques and indicated that the above methods were unable to detect very small damage and required large data storage capacity for comparisons. Cornwell et al. [5] utilized the measured mode shapes to calculate the strain energy of a plate-like structure. Fractional strain energy was then used to define a damage index which can locate the damage in structure. The method only requires the mode shapes of the structure before and after damage. Nevertheless, the challenge of the method lies in the accuracy of measured modes. A large amount of data points are required for further analysis to locate the damage. To solve this problem, Hu et al. [6-9] adopted the DQM to rapidly obtain the accurate solution of strain energy and successfully located surface crack damage in various composite laminate plates, i.e., unidirectional fiber orientation, cross-ply, and quasi-isotropic laminates. It was reported that the original DQM was first used in structural mechanics problems by Bert et al. [10]. This method is able to rapidly compute accurate solutions of partial differential equations by using only a few grid points in the respective solution domains [11].

The objective of this paper is to investigate the detection one of the inside damage, i.e., matrix crack, in composite laminates using modal analysis. Both finite element analysis (FEA) and experimental modal analysis (EMA) were performed to obtain the mode shapes of laminate before and after damaged.

The mode shape displacements were then used to calculate the strain energy which was used to define a damage index for predicting the locations of matrix crack. Finally, x-ray technique was used to validate the prediction.

2. THEORY OF DAMAGE INDEX

A plate-like beam as shown in Figure 1 is subdivided into $N_x \times N_y$ sub-region and denoted the location of each point by (x_i, y_j). For laminate plate theory, the strain energy of beam during elastic deformation is given by

$$U = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4\left(D_{16}\frac{\partial^2 w}{\partial x^2} + D_{26}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x \partial y} + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right] dxdy \quad (1)$$

where w is the transverse displacement; D_{ij} are bending stiffness of the laminate.

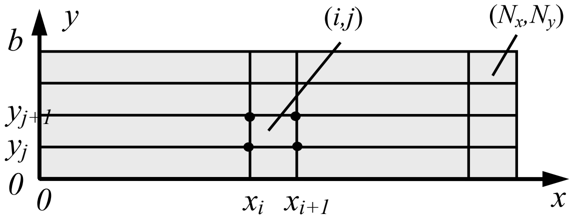


Figure 1 A schematic illustrating of beam

Considering a free-free vibration problem, for a particular normal mode, the total strain energy of the beam associated with the mode shape ϕ_k can be expressed as

$$U_k = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 \phi_k}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2 \phi_k}{\partial x^2}\frac{\partial^2 \phi}{\partial y^2} + 4\left(D_{16}\frac{\partial^2 \phi_k}{\partial x^2} + D_{26}\frac{\partial^2 \phi_k}{\partial y^2}\right)\frac{\partial^2 \phi_k}{\partial x \partial y} + 4D_{66}\left(\frac{\partial^2 \phi_k}{\partial x \partial y}\right)^2 \right] dxdy \quad (2)$$

Cornwell et al. [5] suggested that if the damage is located at a single sub-region then the change of strain energy in sub-region may become significant. Thus, the energy associated with sub-region (i,j) for the k^{th} mode is given by

$$U_{k,ij} = \frac{1}{2}\int_{y_j}^{y_{j+1}}\int_{x_i}^{x_{i+1}} \left[D_{11}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 \phi_k}{\partial y^2}\right)^2 + 2D_{12}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)\left(\frac{\partial^2 \phi_k}{\partial y^2}\right) + 4\left(D_{16}\frac{\partial^2 \phi_k}{\partial x^2} + D_{26}\frac{\partial^2 \phi_k}{\partial y^2}\right)\left(\frac{\partial^2 \phi_k}{\partial x \partial y}\right) + 4D_{66}\left(\frac{\partial^2 \phi_k}{\partial x \partial y}\right)^2 \right] dxdy \quad (3)$$

Similarly, U_k^* and $U_{k,ij}^*$ represent the total strain energy and sub-regional strain energy of the k^{th} mode shape ϕ_k^* for damaged beam. The fractional energies of the beam are defined as

$$F_{k,ij} = \frac{U_{k,ij}}{U_k} \quad \text{and} \quad F_{k,ij}^* = \frac{U_{k,ij}^*}{U_k^*} \quad (4)$$

Considering all modes, m, in the calculation, damage index in sub-region (i,j) is defined as

$$\beta_{ij} = \frac{\sum_{k=1}^{m} F_{k,ij}^*}{\sum_{k=1}^{m} F_{k,ij}} \quad (5)$$

Equation (5) is used to predict the damage location in composite laminate beam. Since the partial differential terms in strain energy formula are difficult to be calculated, an alternative numerical method, differential quadrature method (DQM) was introduced to solve the problem [9].

3. DIFFERENTIAL QUADRATURE METHOD

The basic idea of the DQM is to approximate the partial derivatives of a function $f(x_i, y_j)$ with respect to a spatial variable at any discrete point as the weighted linear sum of the function values at all the discrete points chosen in the solution domain of spatial variable. This can be expressed mathematically as

$$f_x^{(n)}(x_i, y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} f(x_r, y_j) \quad (6)$$

$$f_y^{(m)}(x_i, y_j) = \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_i, y_s) \quad (7)$$

$$f_{xy}^{(n+m)}(x_i, y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_r, y_s) \quad (8)$$

where $i = 1,2,\ldots,N_x$ and $j = 1,2,\ldots,N_y$ are the grid points in the solution domain having $N_x \times N_y$ discrete number of points. $C_{ir}^{(n)}$ and $\overline{C}_{js}^{(m)}$ are the weighting coefficients associated with the n^{th} order and the m^{th} order partial derivatives of $f(x_i, y_j)$ with respect to x and y at the discrete point (x_i, y_j) and n=1, 2,…,N_x-1, m=1, 2,…,N_y-1. The weighting coefficients can be obtained using the following recurrence formulae

$$C_{ir}^{(n)} = n\left(C_{ii}^{(n-1)} C_{ir}^{(1)} - \frac{C_{ir}^{(n-1)}}{x_i - x_r} \right) \quad (9)$$

$$\overline{C}_{js}^{(m)} = n\left(\overline{C}_{jj}^{(m-1)} \overline{C}_{js}^{(1)} - \frac{\overline{C}_{js}^{(n-1)}}{y_j - y_s} \right) \quad (10)$$

where i,r=1,2,…Nx but $r \neq i$; n= 2,3,…,Nx-1; also j,s = 1,2,…,N_y but $s \neq j$; m= 2,3, …,N_y-1. The weighting coefficients when $r=i$ and $s=j$ are given as

$$C_{ii}^{(n)} = -\sum_{r=1, r\neq i}^{N_x} C_{ir}^{(n)} \; ; i = 1,2,...N_x, \text{ and } n = 1,2,...N_x - 1 \quad (11)$$

$$\overline{C}_{jj}^{(m)} = -\sum_{s=1, s\neq j}^{N_y} \overline{C}_{js}^{(m)} \; ; j = 1,2,...N_y, \text{ and } m = 1,2,...N_y - 1 \quad (12)$$

$$C_{ir}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_r) M^{(1)}(x_r)} \; ; i,r = 1,2,...N_x, \text{but} \, r \neq i \quad (13)$$

$$\overline{C}_{js}^{(1)} = \frac{P^{(1)}(y_j)}{(y_j - y_s) P^{(1)}(y_s)} \; ; j,s = 1,2,...N_y, \text{but} \, j \neq s \quad (14)$$

For equations (13) and (14), $M^{(1)}$ and $P^{(1)}$ are denoted by the following expressions

$$M^{(1)}(x_i) = \prod_{r=1, r\neq i}^{N_x} (x_i - x_r) \quad (15)$$

$$P^{(1)}(y_j) = \prod_{s=1, s\neq j}^{N_y} (y_j - y_s) \quad (16)$$

The above equations are applied to calculate the strain energy once the k^{th} mode shape $\phi_{k,ij} = f_k(x_i, y_j)$ is obtained from the experimental results.

4. FINITE ELEMENT ANALYSIS

A pre-study was performed by establishing finite element model for composite laminate, $[0_2/90_{12}/0_2]$, with dimension 222×24.7×2.3mm^3. ANSYS, a FEA commercial code, was used in this study. Eight-node linear solid element (SOLID46) was used to simulate the laminate beam. The element provides a layered version allow up to 250 different material layers. A convergence study was performed to obtain a 23×2×16 mesh model, which is sufficient to solve the normal mode problem. A breadth-wide matrix crack with 0.1 mm wide was created throughout the 90-degree laminate by separating the nodes at the elements along the crack. Figure 2 shows the finite element model.

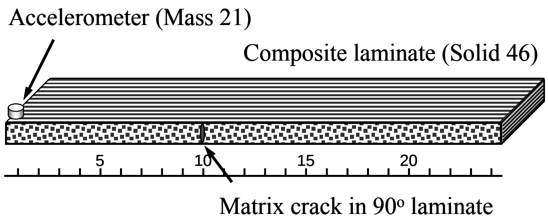


Figure 2 Finite element model

Mechanical properties (E_1 = 117.2 GPa, $E_2 = E_3$ = 9.0 GPa, $G_{12} = G_{13}$ = 4.9 GPa, $\nu_{12} = \nu_{13}$ = 0.315) for composite beam were entered into ANSYS. These data were obtained from the quasi-static tensile tests of the composite material, AS4/PEEK. Hu et al. [12] found that the effects of out-of-plane shear modulus G_{23} and Poisson's ratio ν_{23} on the natural frequencies are not critical in thin plate. Thus, the values of G_{23} and ν_{23} were assumed to be the same as G_{12} and ν_{12} in this study. Material density was directly measured from the specimen, i.e., ρ=1548 kg/m^3. A normal mode analysis with completely free boundary condition was performed to obtain the natural frequencies and the associated mode shapes up to 5 kHz. Hu et al. [12] found that mass effect of accelerometer to the natural frequencies of specimen is significant. Thus, a mass element (MASS21) with 0.0015 kg was assigned to fix at the FE model as shown in Figure 2.

5. EXPERIMENTAL MODAL ANALYSIS

Laminate $[0_2/90_{12}/0_2]$ was fabricated using thermoplastic composite prepreg AS4/PEEK, and then cured at a hot-press machine. After curing, the panel was cut to a specimen with dimension 222×24.7×2.3 mm^3 and marked with 24×3 parallel grid points. The test beam was vertically hung by two cotton strings to simulate a completely free boundary condition as shown in Figure 3. Specimen was excited by an impact hammer with a force transducer throughout all grid points. Dynamic responses were measured by an accelerometer fixed at the corner. Siglab, Model 20-40, was used to record the frequency response functions (FRFs) between measured acceleration and impact force. ME'Scope, a software for general purpose curve fitting, was used to extract modal parameters, i.e., natural frequencies, damping ratios and mode shapes, from the FRFs.

Modal testing was conducted on test specimen before damage. After test, specimen was subjected to tensile test to create matrix crack in 90-degree but not in 0-degree laminate. To achieve this, a tiny pre-crack was created at both sides of the grip point 10. Loading was stopped once the loading curve suddenly dropped accompanied a harsh noise. The location of matrix crack can be verified by using an x-ray machine.

Figure 3 Experimental Set-up

6. RESULTS AND DISCUSSION

A pre-study of finite element analysis was performed to evaluate this approach. The first six mode shapes obtained from normal mode analysis were used to compute the strain energy and damage index of the laminate beam. Figure 4 show the damage index of laminate for putting a breadth-wide matrix crack at 90-degree laminate near the grid point 10. The peak values of damage index occur around the location of matrix crack. Subsequently, Figures 5 and 6 show the predictions of one half crack and one quarter lengths of matrix crack at grid point 10, respectively. Analytical results show that damage indices successfully predict the locations of different sizes of matrix cracks. These encouraging outcomes lead to the following experimental results.

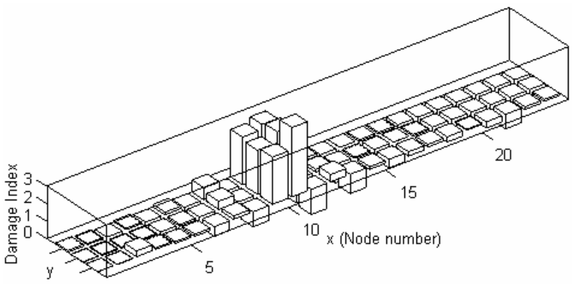


Figure 4 Damage index for a full crack at grid point 10 (FEA result)

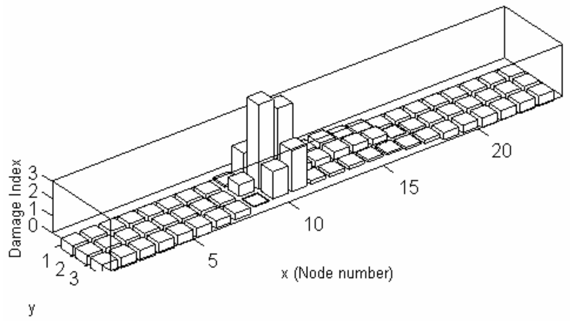


Figure 5 Damage index for one half crack at grid point 10 (FEA result)

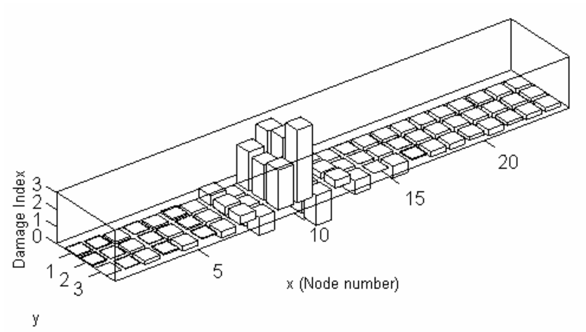


Figure 6 Damage index for a one quarter crack at grid point 10 (FEA result)

Table 1 and 2 list the first eight natural frequencies of the laminate beam before and after damage, respectively. Checking the difference between FEA and EMA results, very good correlations are obtained. In FEA results, there are not much different in natural frequencies of laminate beam before and after damage. However, in EMA results, matrix crack damage significantly decreases the natural frequencies of laminate beam. In fact, the change of natural frequencies is not able to detect the damage location in laminate beam. But the irregularity of mode shapes caused by damage may contribute to this detection. It is noted that mode $(3,1)_y$ describes the first bending mode in y-direction. This particular mode only appears in FEA result but not in EMA result, since finite element model provides a normal mode analysis for three dimensional problems.

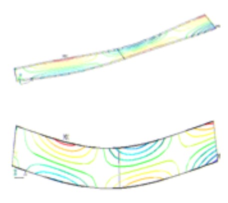
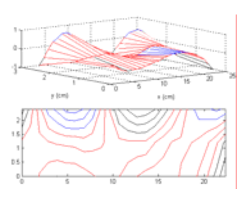
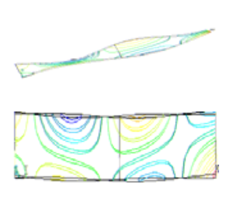
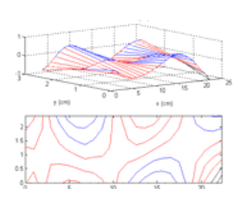
Table 1 Natural frequencies of the laminate specimen (before damage)

Mode	FEA (Hz)	EMA (Hz)	△(%)
(3,1)	281	281	0.0
(2,2)	607	638	-4.9
(4,1)	823	804	2.4
(3,2)	1294	1350	-4.1
(5,1)	1633	1570	4.0
$(3,1)_y$	2065	2090	-1.2
(4,2)	2125	2210	-3.8
(6,1)	2684	2570	4.4

Table 2 Natural frequencies of the laminate specimen (after damage)

Mode	FEA (Hz)	EMA (Hz)	△(%)
(3,1)	281	276	1.8
(2,2)	607	633	-4.1
(4,1)	823	802	2.6
(3,2)	1294	1340	-3.4
(5,1)	1633	1570	4.0
$(3,1)_y$	2063	2040	1.1
(4,2)	2125	2170	-2.1
(6,1)	2685	2570	4.5

Table 3 Mode shapes of $(3,1)_y$ and (4,2)

Mode	FEA	EMA
$(3,1)_y$	2065 Hz	2090 Hz
		
(4,2)	2125 Hz	2210 Hz
		

Comparing the mode shapes of $(3,1)_y$ and (4,2), Table 3 shows that contours of these two modes are very much alike in both FEA and EMA. It is easy to distinguish mode $(3,1)_y$ from mode (4,2). However, in EMA results, frequency response functions can only produce the mode shapes in z-direction. This is why two associated mode shapes of EMA in Table 3 are almost the same, even though the natural frequencies are different.

In EMA result, the prediction of matrix crack of laminate beam subjected to tensile test is shown in Figure 7. The first six mode shapes were used to compute the strain energy and damage index. The peak values clearly locate the matrix crack; however, many peak values also emerged at some other undamaged areas. The deviation in measurement may attribute to these pseudomorphs. Cornwell et al. [5] suggested that damage indices with values greater than two are associated with potential damage locations. Figure 8 shows the damage index after truncation. The improved outcome clearly located the matrix crack damages. Apparently, damage indices in Figures 7 and 8 indicate two potential damage locations at grid point 10 and the area from grid points 14 to 16. It looks like that more than one matrix cracks occur inside the laminate beam. The prediction has been verified using an x-ray machine, Eresco 200MF. Before irradiation, damaged beam was treated with developer solution, 1,4-Diiodobutane, 99+%, which can permeate the beam through the cracks. Output voltage and exposure time for the irradiation of damaged beam are 25 kV and 36 sec. The picture shown in Figure 9 reveals that three matrix cracks occurred inside the laminate beam after tensile test. One breadth-wide crack is located between grid point 9 and 10; the other two short cracks are located between grid points 11 and 12.

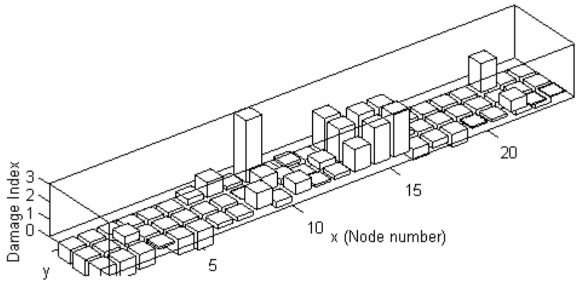


Figure 7 Damage index of EMA result (before truncation)

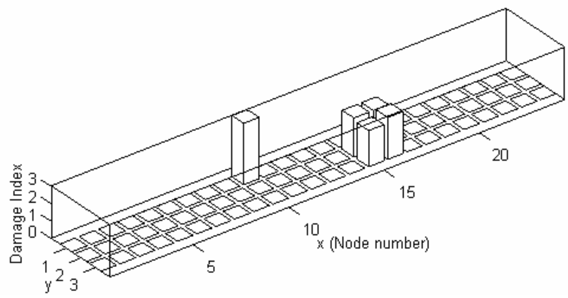


Figure 8 Damage index of EMA result (after truncation)

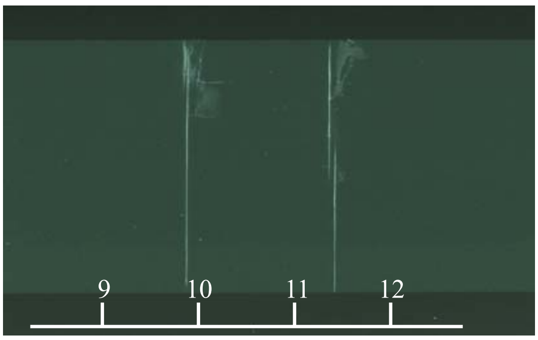


Figure 9 Three matrix cracks occur in laminate beam (x-ray picture)

To simulate the real case shown in Figure 9, a FE model was rebuilt and three matrix cracks were created in the model. Analytical result in Figure 10 indicates that peak values of damage indices around the three cracks clearly indicate the damage area. Comparing the predictions in Figure 8 and Figure 10, damage indices obtained from EMA seem to deviate from the real locations of matrix cracks a little bit. The outcomes indicate that if the locations of two matrix cracks are extremely adjacent to each other, the damage index using experiment may not precisely predict all damage locations. Nevertheless, the peak values of damage indices nearby the matrix cracks are good enough to predict the damage in laminate beam.

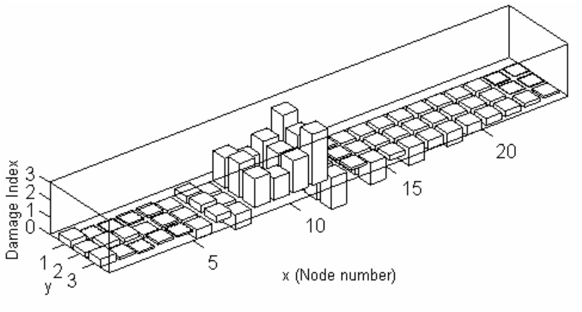


Figure 10 Damage index of FEA result (three matrix cracks model)

7. CONCLUSIONS

Damage index using modal analysis is developed to detect a matrix crack in composite laminate beam in this paper. This method only requires a few mode shapes of the laminate beam before and after damage. In fact, the changes of mode shapes before and after damage are almost invisible. However, the irregularities of mode shapes due to matrix crack become significant from the perspective of strain energy approach. Both FEA and EMA results show that matrix crack locations were successfully predicted by using damage indices. Since the number of measured points was limited, DQM provides us an accurate approach to compute strain energy by using only a few grid points in the test specimen. This nondestructive method provides a reliable, cost-effective approach for damage detection in the utilization of composite structures. Further research interests lie in the application of this method to various composite laminates.

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應用模態分析於複合材料疊層板之基材裂縫檢驗

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摘要

本論文應用複合材料疊層樑在破壞前後之模態振型位移來計算疊層樑之應變能，並利用破壞前後應變能的比值定義的破壞指標，用來辨認複合材料疊層樑的破壞位置。本文採用碳纖維/聚二醚酮(AS4/PEEK)，疊層型式為 $[0_2/90_n/0_2]$。破壞模型為疊層樑內部 90 度疊層之基材裂縫，破壞前後之模態振型可由有限元素分析與模態實驗分析獲得，兩種方法均在本文中討論。由於模態實驗量測的點數有所限制，本文採用微分值積法(DQM)來計算疊層樑之應變能，此方法可以迅速精確地計算應變能公式中的偏微分項。由於基材裂縫是發生在疊層板內部，所以裂縫的位置最終須由x-ray辨認。結果顯示，本研究發展之破壞指標可以成功地預測出複合材料疊層樑之基材裂縫位置。

關鍵字：模態分析、基材裂縫、複合材料疊層樑、應變能法