Damage Detection of Composite Laminates Using Modal Analysis and Strain Energy Method

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Abstract

A nondestructive damage detection in composite laminates by using modal analysis and strain energy methods is investigated in this paper. Continued fiber-reinforced composite AS4/PEEK was used to fabricate symmetrical laminate plates, $[0]_{16}$ and $[(0/90)_4]_S$. A surface crack was created in one side of the plate. Modal analysis was performed to obtain the mode shapes from both experimental and FEA (finite element analysis) results. The mode shapes were used to calculate strain energy using DQM (differential quadrature method). Consequently, the strain energies of damaged and undamaged plates were used to define a damage index which can successfully locate the surface crack.

Keywords: damage detection, composite laminate, modal analysis, strain energy, DQM

1. Introduction

The application of modal analysis to detect damage in composite materials has been increasingly adopted due to its flexibility of measurement and relatively low cost. The basic idea of these methods is to use the information of modal parameters, such as frequency, mode shape and damping ratio, to access the structural damage.

Cawley and Adams [1] simply used the frequency shifts for different modes to detect the damage in composite structures. Tracy and Pardoen [2] found that the natural frequencies of a composite beam were affected by the size and damage location. Shen and Grady [3] found that local delamination does not have a noticeable effect on global mode shape of vibration of composite beams, but delamination does cause the irregularity of mode shapes. Pandey et al. [4] also showed that the irregular of mode shape is significant for relatively large damage. Zou et al. [5] provided a thorough review in vibration-based techniques and indicated that the above methods were unable to detect very small damage and required large data storage capacity for comparisons.

Cornwell et al. [6] utilized the measured mode shapes to calculate the strain energy of a plate-like structure.

Fractional strain energy was then used to define a damage index which can locate the damage in structure. The method only requires the mode shapes of the structure before and after damage. Nevertheless, the challenge of the method lies in the accuracy of measured modes. A large amount of data points are required for further analysis to locate the damage. To solve this problem, Hu et al. [7] adopted the DQM to rapidly obtain the accurate solution of strain energy and successfully located damage in a composite laminate plate. It was reported that the original DQM was first used in structural mechanics problems by Bert et al. [8]. This method is able to rapidly compute accurate solutions of partial differential equations by using only a few grid points in the respective solution domains [9].

The objective of this paper is to investigate the application of modal analysis and strain energy method to the detection of surface crack damages in composite laminates. Modal analysis was performed to obtain the mode shapes of damaged and undamaged laminates from both experimental and finite element analysis results. The mode shapes were then used to calculate the strain energy using DQM. Consequently, the surface crack was successfully located by using the damage index.

2. Modal Analysis

2.1 Experimental Method

Carbon/epoxy composite AS4/PEEK was used in this study. Symmetrical laminates, $[0]_{16}$ and $[(0/90)_4]_S$, were fabricated by stacking up the prepreg lamina and then cured at a hot-press machine. After curing, the panel was cut to a 209×126×2.4 mm^3 laminate using a diamond saw. A 31 mm long and 1 mm deep surface crack was created at one side near the center by using a laser cutting machine.

Figure 1 shows the experimental setup. Test plate was marked to 13×13 parallel grid points, and vertically hung by two cotton strings to simulate the completely free boundary condition. Modal testing was conducted on the laminate plate. The plate was excited by an impact hammer with a force transducer throughout all grid points. The dynamic responses were measured by an accelerometer fixed at the corner of the plate. Siglab, Model 20-40, was used to record the frequency response functions (FRFs) between the measured acceleration and impact force. ME'Scope, a software for the general purpose curve fitting, was used to extract modal parameters, i.e., natural frequencies, damping ratios and mode shapes, from the FRFs.

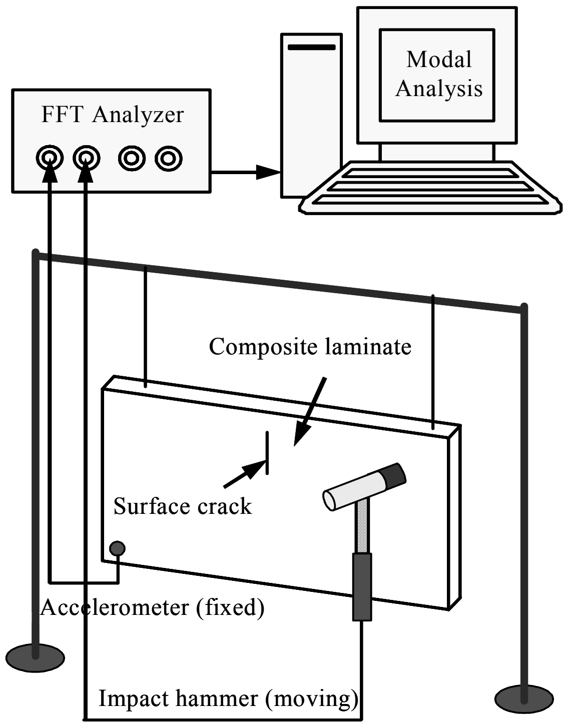

FIGURE 1 Experimental Set-up

2.2 Finite Element Methods

A FE model was established to simulate a composite laminate plate with dimension 209×126×2.4 mm^3. ANSYS, a FEA commercial code, was used in this study. Eight-node linear solid element (SOLID46) was used in the modeling. To simulate a 31 mm long and 1 mm deep surface crack, the nodes at location of surface crack were replaced by separate nodes as shown in Figure 2. Hu et al. [10-11] successfully applied this model to the free vibration analysis of damaged and undamaged composite laminate plates.

Two symmetrical laminate plates, i.e., $[0]_{16}$ and $[(0/90)_4]_S$ were investigated in this study. A convergence study was performed to obtain a 40×30×8 mesh model, which is sufficient to solve the normal mode problem. The mechanical properties (E_1 = 140.3 GPa, $E_2 = E_3$ = 9.4 GPa, $G_{12} = G_{13}$ = 5.4 GPa, $\nu_{12}=\nu_{13}=0.253$) of composite AS4 / PEEK were entered into ANSYS. These data were obtained from the quasi-static tensile tests of composite material. Since the effect of out-of-plane shear modulus G_{23} and Poisson's ratio ν_{23} are not evident in thin plate, the values of G_{23} and ν_{23} were assumed to be the same as G_{12} and ν_{12}. Material density was directly measured from laminate plates, i.e., $\rho=1485 kg/m^3$ for $[0]_{16}$ and $\rho=1537 kg/m^3$ for $[(0/90)_4]_S$.

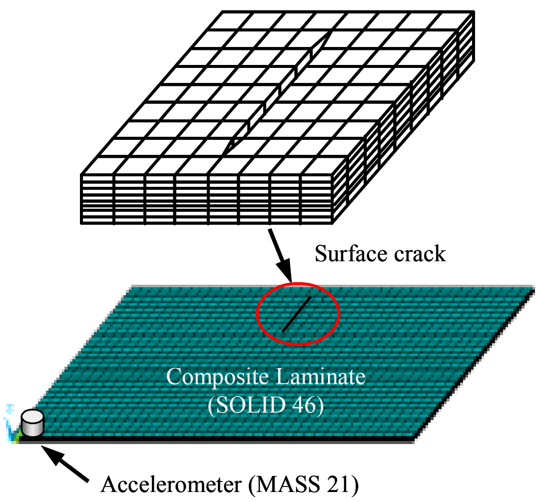

FIGURE 2 Finite element model for damaged composite laminate

A normal mode analysis with completely free boundary condition was performed to obtain the natural frequencies and the associated mode shapes up to 2 kHz. The mass effect of accelerometer was considered into the FE model by assigning a mass element (MASS21) with 0.002 kg to the laminate plate model [10-11].

3. Damage Index

Considering a laminate plate as shown in Figure 3, the plate was subdivided into $N_x \times N_y$ sub-region and denoted the location of each point by (x_i, y_j). The strain energy of laminate plate during elastic deformation is given by

$$U=\frac{1}{2}\int_0^b\int_0^a\left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2+2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right.$$
$$\left.+4\left(D_{16}\frac{\partial^2 w}{\partial x^2}+D_{26}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x\partial y}+4D_{66}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]dxdy \tag{1}$$

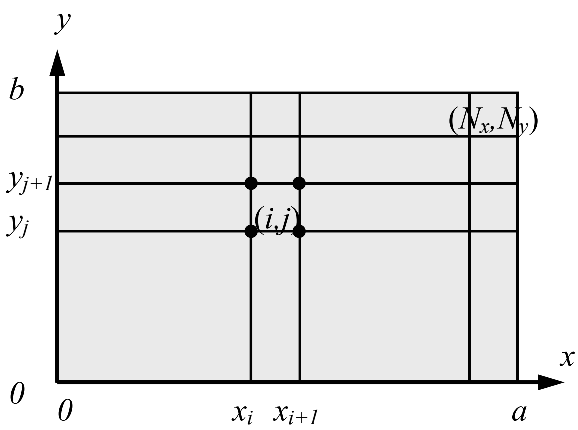

FIGURE 3 A schematic illustrating of plate

where w is the transverse displacement of the laminate, D_{ij} are the bending stiffnesses of the laminate. For a particular mode shape ϕ_k, the energy associated with the mode shape is expressed as

$$U_k = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2\phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2\phi_k}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2\phi_k}{\partial x^2}\frac{\partial^2\phi}{\partial y^2} + 4\left(D_{16}\frac{\partial^2\phi_k}{\partial x^2} + D_{26}\frac{\partial^2\phi_k}{\partial y^2}\right)\frac{\partial^2\phi_k}{\partial x\partial y} + 4D_{66}\left(\frac{\partial^2\phi_k}{\partial x\partial y}\right)^2 \right] dxdy \quad (2)$$

Cornwell *et al*. [6] suggested that if the damage is located at a single sub-region then change in the bending stiffness of the sub-region can be used to locate the damage. Thus, the energy associated with sub-region (i,j) for the k^{th} mode is given by

$$U_{k,ij} = \frac{1}{2}\int_{y_j}^{y_{j+1}}\int_{x_i}^{x_{i+1}} \left[D_{11}\left(\frac{\partial^2\phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2\phi_k}{\partial y^2}\right)^2 + 2D_{12}\left(\frac{\partial^2\phi_k}{\partial x^2}\right)\left(\frac{\partial^2\phi_k}{\partial y^2}\right) + 4\left(D_{16}\frac{\partial^2\phi_k}{\partial x^2} + D_{26}\frac{\partial^2\phi_k}{\partial y^2}\right)\left(\frac{\partial^2\phi_k}{\partial x\partial y}\right) + 4D_{66}\left(\frac{\partial^2\phi_k}{\partial x\partial y}\right)^2 \right] dxdy \quad (3)$$

A fractional energy is defined as

$$F_{k,ij} = \frac{U_{k,ij}}{U_k} \quad (4)$$

The summation of all fractional energies should be equal to unity, i.e.,

$$\sum_{j=1}^{N_y}\sum_{i=1}^{N_x} F_{k,ij} = 1 \quad (5)$$

Similarly, U_k^* and $U_{k,ij}^*$ represent the total strain energy and the sub-regional strain energy of the k^{th} mode shape ϕ_k^* for damaged laminate. Thus, a fractional energy of damage laminate is given by

$$F_{k,ij}^* = \frac{U_{k,ij}^*}{U_k^*} \quad (6)$$

Certainly, we have

$$\sum_{j=1}^{N_y}\sum_{i=1}^{N_x} F_{k,ij}^* = 1 \quad (7)$$

Considering all measured modes, m, in the calculation, damage index in sub-region (i,j) is defined as

$$\beta_{ij} = \frac{\sum_{k=1}^{m} F_{k,ij}^*}{\sum_{k=1}^{m} F_{k,ij}} \quad (8)$$

A normalized damage index is given by

$$Z_{ij} = \frac{\beta_{ij} - \overline{\beta}_{ij}}{\sigma_{ij}} \quad (9)$$

where $\overline{\beta}_{ij}$ and σ_{ij} represent the mean and standard deviation of the damage indices, respectively. It is noted that the partial differential terms in strain energy formula are difficult to be calculated. An alternative numerical method, differential quadrature method (DQM) was therefore introduced to solve problems in structural mechanics field.

4. Differential quadrature method

The basic idea of the DQM is to approximate the partial derivatives of a function $f(x_i,y_j)$ with respect to a spatial variable at any discrete point as the weighted linear sum of the function values at all the discrete points chosen in the solution domain of spatial variable. This can be expressed mathematically as

$$f_x^{(n)}(x_i, y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} f(x_r, y_j) \quad (10)$$

$$f_y^{(m)}(x_i, y_j) = \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_i, y_s) \quad (11)$$

$$f_{xy}^{(n+m)}(x_i, y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_r, y_s) \qquad (12)$$

where i = 1,2,…,N_x and j =1,2,…,N_y are the grid points in the solution domain having $N_x \times N_y$ discrete number of points. $C_{ir}^{(n)}$ and $\overline{C}_{js}^{(m)}$ are the weighting coefficients associated with the n^{th} order and the m^{th} order partial derivatives of $f(x_i, y_j)$ with respect to x and y at the discrete point (x_i, y_j) and n=1, 2,…,N_x-1, m=1, 2,…,N_y-1. The weighting coefficients can be obtained using the following recurrence formulae

$$C_{ir}^{(n)} = n\left(C_{ii}^{(n-1)} C_{ir}^{(1)} - \frac{C_{ir}^{(n-1)}}{x_i - x_r} \right) \qquad (13)$$

$$\overline{C}_{js}^{(m)} = n\left(\overline{C}_{jj}^{(m-1)} \overline{C}_{js}^{(1)} - \frac{\overline{C}_{js}^{(n-1)}}{y_j - y_s} \right) \qquad (14)$$

where i,r=1,2,…Nx but $r \neq i$; n= 2,3,…,Nx-1; also j,s = 1,2,…,N_y but $s \neq j$; m= 2,3, …,N_y-1. The weighting coefficients when $r=i$ and $s=j$ are given as

$$C_{ii}^{(n)} = -\sum_{r=1, r\neq i}^{N_x} C_{ir}^{(n)} \text{ ; } i = 1,2,...N_x, \text{ and } n = 1,2,...N_x - 1 \qquad (15)$$

$$\overline{C}_{jj}^{(m)} = -\sum_{s=1, s\neq j}^{N_y} \overline{C}_{js}^{(m)} \text{ ; } j = 1,2,...N_y, \text{ and } m = 1,2,...N_y - 1 \qquad (16)$$

$$C_{ir}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_r) M^{(1)}(x_r)} \text{ ; } i, r = 1,2,...N_x, \text{but} r \neq i \qquad (17)$$

$$\overline{C}_{js}^{(1)} = \frac{P^{(1)}(y_j)}{(y_j - y_s) P^{(1)}(y_s)} \text{ ; } j, s = 1,2,...N_y, \text{but} j \neq s \qquad (18)$$

For equations (17) and (18), $M^{(1)}$ and $P^{(1)}$ are denoted by the following expressions

$$M^{(1)}(x_i) = \prod_{r=1, r\neq i}^{N_x} (x_i - x_r) \qquad (19)$$

$$P^{(1)}(y_j) = \prod_{s=1, s\neq j}^{N_y} (y_j - y_s) \qquad (20)$$

The above equations are applied to calculate the strain energy once the k^{th} mode shape $\phi_{k,ij} = f_k(x_i, y_j)$ is obtained from the experimental results.

5. Results and Discussions

The first five mode shapes obtained from Ref. [10] and Ref. [11] were adopted to calculate the strain energy and damage index of the laminate plates. Figures 4 and 5 show the damage indices using FEA results. The peak values clearly indicated the location of surface crack in laminate plates $[0]_{16}$ and $[(0/90)_4]_S$ respectively. High resolution of damage index could be achieved through a fine mesh of FE model. Nevertheless, it is restricted in experiments due to limited points of measurement.

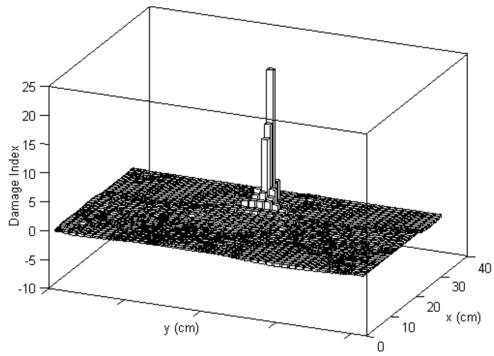

FIGURE 4 Damage index of laminate $[0]_{16}$ (FEA)

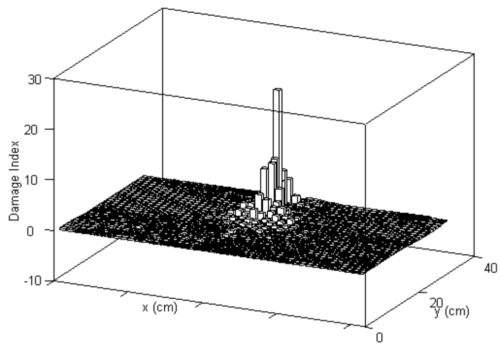

FIGURE 5 Damage index of laminate $[(0/90)_4]_S$ (FEA)

Figures 6 and 7 show the damage indices of laminate plates $[0]_{16}$ and $[(0/90)_4]_S$ using the first five modes of experimental results. The peak values occur at the location of surface crack; however, many peak values also emerged at some other undamaged areas. Poor resolution of mode shapes obtained from the experiment may attribute to these pseudomorphs. Cornwell *et al*. [6] suggested that damage indices with values greater than two are associated with potential damage locations. Therefore, the damage indices at surface cracks are significant.

Damage indices using particular mode is able to clearly identify the surface crack. For instance, the mode shape (2,3) of laminate $[0]_{16}$ depicts a bending mode in the direction perpendicular to surface crack as shown in Figure 8. Certainly, surface crack will induce significant losses of stiffness in this mode. The damage index using mode shape (2,3) clearly identifies the location of surface crack as shown in Figure 9. Similarly, in laminate plate $[(0/90)_4]_S$, mode shape (3,1) depicts a bending mode in the direction perpendicular to surface crack as shown in Figure 10. The damage index using mode shape (3,1) clearly identifies the location of surface crack as shown in Figure 11.

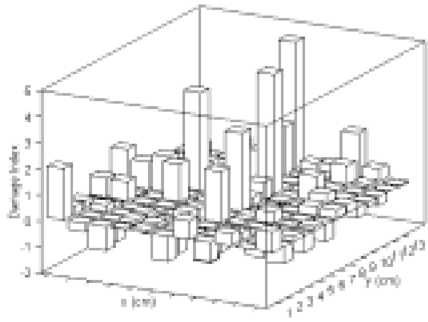

FIGURE 7 Damage index of laminate $[(0/90)_4]_S$ (Experiment)

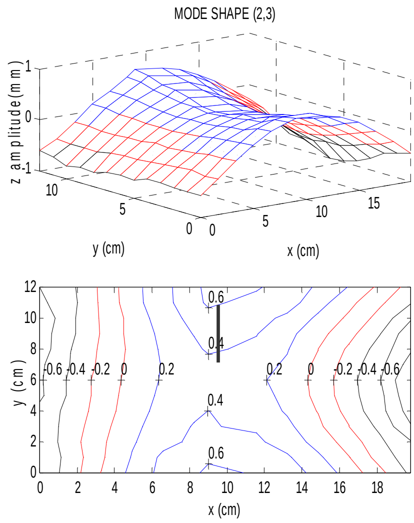

FIGURE 8 Mode shape (2,3) of laminate $[0]_{16}$ (Experiment)

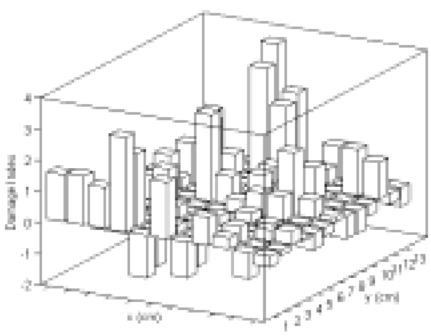

FIGURE 6 Damage index of laminate $[0]_{16}$ (Experiment)

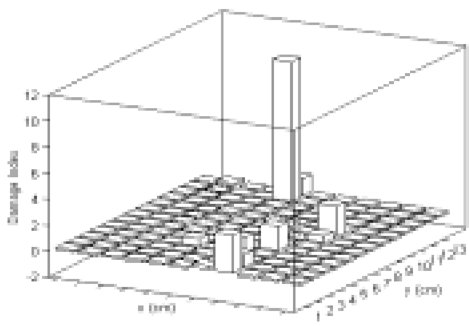

FIGURE 9 Damage index of laminate $[0]_{16}$ using mode (2,3) (Experiment)

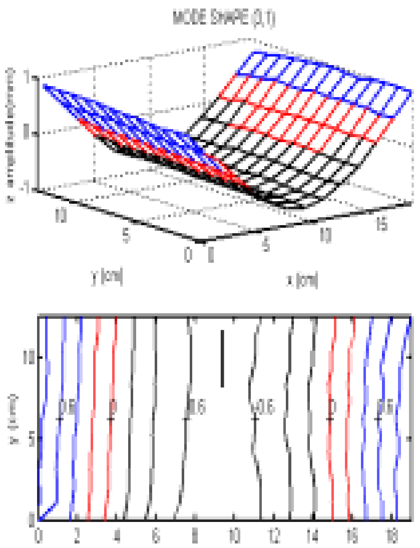

FIGURE 10 Mode shape (3,1) of laminate $[(0/90)_4]_S$ (Experiment)

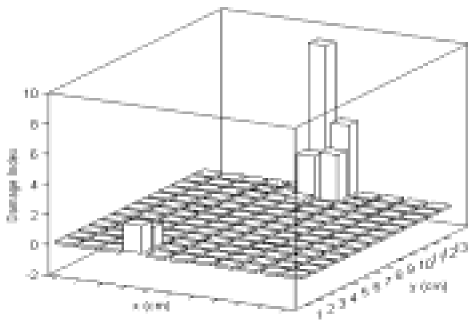

FIGURE 11 Damage index of laminate $[(0/90)_4]_S$ using mode (3,1) (Experiment)

5. Conclusions

Damage index using modal analysis and strain energy methods was developed to detect a surface crack in composite laminate plate in this paper. This method only requires a few mode shapes of the plate before and after damage. Both FEA and experimental results show that surface cracks were successfully located by using damage indices. Since the number of measured points was limited, DQM provides us an accurate approach to calculate strain energy by using only a few grid points in the test plate. Further research interests lie in the implementation of sensitivity of results using various sensors. Future work will focus on a similar study for various types of damage as well.

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應用模態分析與應變能法在複合材料疊層板之破壞檢驗

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摘要

本論文應用複合材料疊層板在破壞前後之模態振型來計算其應變能，並利用破壞前後之應變能比值所定義的破壞指標來找出複合材料疊層板的破壞位置。本文採用碳纖維/聚二醚酮(AS4/PEEK)，疊層型式為 $[0]_{16}$ 以及$[(0/90)_4]_S$。破壞模型為表面裂縫，破壞前後之模態振型可由有限元素分析與模態實驗獲得，兩種方法均在本文中討論。應變能之計算則採用微分值積法(DQM)來分析。結果顯示，模態分析與應變能法所定義之破壞指標成功地預測出複合材料疊層板之表面裂縫位置。

關鍵字：破壞檢測、複合材料疊層板、模態分析、應變能法、微分值積法