26th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
7-11 July 2019, Montreal

DEVELOPMENT OF PENTAGONAL PLATES WITH HARMONIC SOUND AS PERCUSSION INSTRUMENT

Bor-Tsuen Wang, Po-Chin Chen, Yau-Guang Tsai
Department of Mechanical Engineering, National Pingtung University of Science and Technology, Pingtung, 91201, Taiwan
email: wangbt@mail.npust.edu.tw

Ying-Hui Wu
Machinery Division, National Nei-Pu Senior Agricultural-Industrial Vocational School, Pingtung, 912, Taiwan
e-mail: ch0113@ms42.hinet.net

A bell plate made of a flat sheet of metal may produce a sound like a bell. This work presents the design analysis of pentagonal shape of flat steel plate to produce harmonic sound. The parametric geometry model for the pentagonal plate is first defined to construct finite element model. By performing theoretical modal analysis (TMA), the plate's vibration modes, i.e. natural frequencies and corresponding mode shapes, can be obtained. From vibration modal characteristics of the pentagonal plate, the generated sound modes as well as sound frequencies can be postulated. The generated percussion sound response depends on the structural mode shapes for different striking locations. The optimization problem is then formulated to optimally design the plate dimensions such that the pentagonal plate can produce harmonic sound, i.e. the overtone frequency is twice as the fundamental frequency. The optimum design pentagonal plate is manufactured and performed experimental modal analysis (EMA). The plate's modal parameters obtained from TMA and EMA are compared to verify the design. The percussion sound test on the pentagonal plate is also carried out to show the effective design for harmonic sound. Finally, a set of pentagonal plates consisting of two octaves musical notes is analyzed and made to fabricate the percussion instrument. This work shows the design process for the pentagonal bell plate from design analysis to experimental verification.
Keywords: pentagonal plate, harmonic sound, percussion instrument, vibration mode.

1. Introduction

Percussion instrument is a kind of musical instrument that can be stricken, shaken or rubbed. A typical set of percussion instrument may contain two or three octave musical notes so that the wide range of musical sound can be played. This work aims to develop a set of pentagonal plates with two octave musical notes, including the sharp and flat tones. The special pentagonal plate originally comes from the bell plate [1]. Lavan *et al.* [1] presented the pentagonal plate that can produce the bell-like sound.

With the primary study on the pentagonal plate, we found the percussion sound of plate is dependent on the first few vibration modes. The plate geometry can be optimized to generate the harmonic sound, i.e. the overtone frequency is twice as the fundamental frequency.

McLachlan [2] discussed the relation between sound characteristics of musical bell and its geometry. Both finite element analysis (FEA) and sound measurement were performed to compare structural natural frequencies and realize the sound generation mechanism. The major factors affecting sound characteristics are bell thickness, curvature, angle, height and width. Rabenstein *et al.* [3] showed the mathematical model for the tubular bell and studied physical parameters for sound radiation. Bork [4] modified the undercut of xylophone bar to alter the bar's vibration mode and tune the percussion sound frequency. Lee *et al*. [5] used FEA and experimental modal analysis (EMA) to examine vibration modes of Korea bell and predict bell's vibration and sound radiation. Pan [6] discussed the special sound effect of Chinese bell that can produce different notes for different striking locations. Ansari [7] investigated the irregularity in casting bell such as geometry, material properties and possible local defects. He used FEA to predict the fundamental frequency in comparison with that from EMA to examine the variation effect of geometry on sound response. Bretos *et al.* [8] also applied FEA to modal analysis of wooden bars to discuss the geometry effect on vibration modes as well as material properties, such as Young's modulus for non-uniform material. Jing [9] simulated sound and vibration for a Chinese bell with single and double tones.

For the improvement of musical sound and effective design and manufacture, researchers apply optimization method to musical instrument development. Wang and Jian [10] used FEA and EMA to obtain structural natural frequencies and mode shapes and so forth to design the chord plate that can produce the chord sound. Petrolito and Legge [11] proposed optimization method to target the desired frequencies of percussive beams by changing the undercut. Wang *et al.* [12] developed a new type of Harmonic Glass Plate (HGP) by using FEA and EMA. The HGP can generate several overtone frequencies with harmonics. McLachlan *et al.* [13] studied the bell geometry effect on vibration properties by FEA and optimally design the bell shape to come out with good quality of sound. Wang *et al.* [14] have the design of chord sound plate (CSP) that can produce the triad sound with one strike on the CSP. Wang and Hsieh [15] presented two types of metal bars that can generate the chord sound, and the patent was filed [14]. Wang *et al*. [16] proposed the new method by using multiple sine waves to formulate the geometry shapes of sound plate, in particular for harmonic sound plate (HSP).

This work aims to design the pentagonal plate to have two harmonics and reveal better sound response. The original shape is adopted from Lavan *et al.* [1]. Section 2 conveys the design concept and procedures. The parametric geometry model is first defined and performed theoretical modal analysis (TMA) by finite element method (FEM). The optimization problem is then formulated to design the dimensions of pentagonal plate such that the desired percussion sound can reveal two harmonics. The first harmonic is the fundamental frequency and referred to the musical note's frequency, and the second harmonic frequency is twice of the first. The pentagonal plate for C6, which standard frequency is 1046.5 Hz, is shown for its frequency response functions and vibration modes obtained from FEA and EMA, respectively. Section 3 shows the design verification for C6 pentagonal plate and presents the percussion sound characteristics. A complete set of pentagonal plates is manufactured with 25 pieces for two octave musical notes from F5 (698 Hz) to F7 (2794 Hz). This work shows the design and manufacture of pentagonal plate percussion instrument and details the design analysis and experimental verification.

2. Design concept for the pentagonal plate with two harmonics sound

Figure 1(a) shows the basic pattern of pentagonal plate originally from Lavan *et al.* [1]. The key geometry parameters of the pentagonal plate as shown are L, W, H1, H2 and H3. The dimensions shown in Figure 1(a) are the final design of C6 (1046.5 Hz) pentagonal plate. Figure 1(b) is the finite element model, and Figure 1(c) is the real structure.

The objective of design analysis is to obtain the precise geometry of pentagonal plate such that the percussion sound can produce two harmonics sound, i.e. the first and second harmonics. The first harmonic sound should have the same frequency as the standard frequency of musical note, which frequency for C6 is 1046.5Hz and also known as the fundamental frequency. The second harmonic sound frequency is also referred as the overtone frequency and should be twice of the fundamental frequency.

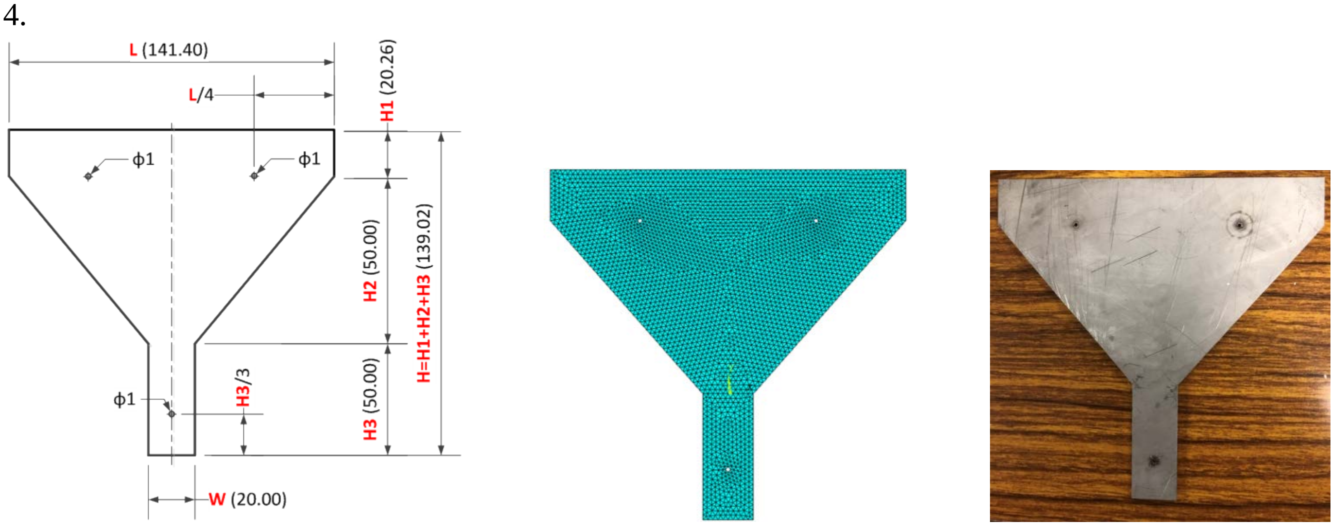
As known, the percussion sound response of such a plate depends on the lateral vibration modes. The primary study is to perform TMA on the pentagonal plate and characterize the vibration modes. In particular, the characteristics of plate mode shapes are important for selection of striking locations. Section 3 will present the pentagonal plate's natural frequencies and corresponding mode shapes obtained from FEA and EMA, respectively.

The design goal for the pentagonal plate is clear for producing two harmonics sound, and structural modal analysis on the parametrized pentagonal plate is ready. The next step is to formulate the optimization problem and carry out the optimum design analysis procedure for the pentagonal plate. There are three items to define, including design variables, objective function and constraints.

First, the design variables are those geometry dimension variables as shown in Figure 1(a). Second, the objective function is to be minimized and defined as the root mean square (RMS) value of frequency errors between the target frequencies and analysis frequencies. The frequency errors include those two desired harmonic modal frequencies. In TMA, the challenge is to identify the vibration modes that will be excited to produce the sound. By prescribing the striking location, the mode shape response at the prescribed location for each vibration mode is monitored and so forth the correct natural frequencies can be extracted to calculate the objective function automatically. If the design variables can be optimally determined, the RMS value of frequency errors is minimized and the design is fit.

Third, the constraints should be properly defined. For the design of the percussion instrument, the first mode of percussion sound should have the same frequency as the standard frequency of musical note. The thumb of rule for the frequency error is $\pm$0.34%. Therefore, the target mode's natural frequency should be less than the criterion and the individual frequency error is as the constraint.

In optimum analysis, the iteration process is essential. The comparison of analysis natural frequencies and target musical note's frequencies is conducted to determine if the design is appropriate. Section 3 will show the designed C6 pentagonal plate's sound response that fulfils the design goal. The complete set of percussion instrument consisting of 25 pieces of pentagonal plates is shown in Section 4.



(a) Parametric geometry model (b) Finite element model (c) real structure photo

Figure 1: Pentagonal plate for C6, where the dimensions are for C6.

3. Design verification of pentagonal plate for C6

This section shows the design verification for the C6 pentagonal plate. FEA is adopted to perform TMA on the pentagonal plate, which finite element model is shown in Figure 1(b). The pentagonal plate is made of stainless steel with 3mm thickness and manufactured by CNC laser cut. Figure 1(c) shows the real structure of C6 pentagonal plate.

3.1 Model verification

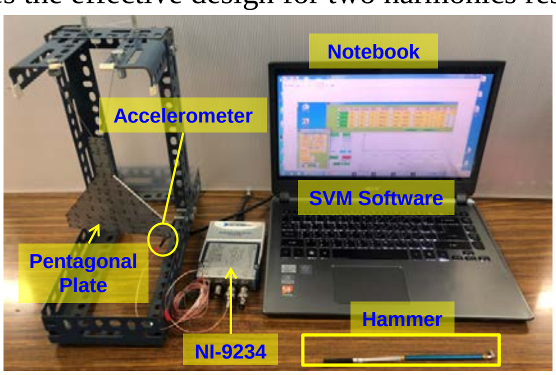
Figure 2(a) shows the experimental setup for EMA on the pentagonal plate. The traditional modal testing by roving impact hammer and fixed the acceleration on the suspended plate in free boundary is conducted. Figure 2(b) shows the grid points on the pentagonal plate.

Table 1 shows natural frequencies of C6 pentagonal plate obtained from FEA and EMA, and Figures 3(a) and 3(b), respectively, show corresponding mode shapes. The discussions are as follows:

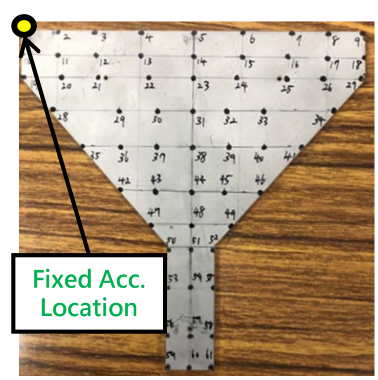
- Both FEA and EMA results reveal good agreement in term of corresponding mode shape physical meanings.
- Natural frequencies obtained from EMA are generally smaller than those from FEA is the cause of accelerometer mass effect.
- Although the pentagonal plate is not really a rectangle, the physical meaning of mode shape can be interpreted as (x,y)=(m,n). It is reasonable that there is no (1,1), (2,1) and (1,2) modes for free boundary plate.
- The design striking location as indicated in Figure 3 is at the middle of horizontal direction. The even mode in x-direction will not be excited. The excited modes are remarked in Figure 3.
- From Table 1, mode number in FEA is started from F-07 because there are six rigid body modes that are not presented. F-08 and F-10 are the target modes that will produce two harmonics sound.

Figure 4(a) and 4(b) show frequency response functions (FRFs) for (i,j)=(1,1) and (i,j)=(1,23), respectively, where i is the accelerometer location, and j is the impact hammer applied location. Discussions are as follows:

- FRFs obtained from FEA and EMA show very good agreement. The simulation model can well predict the plate modal response and frequency response.
- For FRF at (i,j)=(1,1), all of vibration modes can be excited because of the accelerometer being fixed at i=1 where is not a nodal point. Mode shapes are depicted at the top of peaks in FRFs.
- For FRF at (i,j)=(1,23), where j=23 is the prescribed striking location, modes F-08 and F-10, which are desired modal response, have relatively higher peaks as well as mode F-14. This indicates the effective design for two harmonics response.



(a) Instrument setup

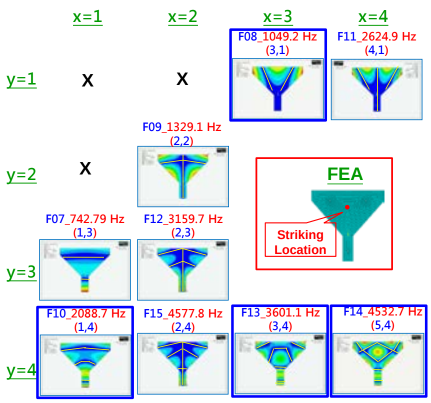


(b) Measurement grid points

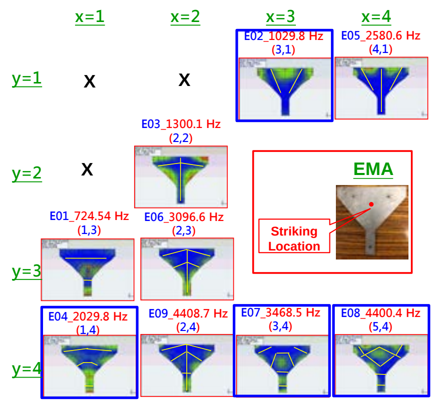
Figure 2: Experimental setup for EMA on the pentagonal plate.

Table 1: Comparison of natural frequencies from FEA and EMA for C6 pentagonal plate

FEA		EMA		Natural Frequency Error (%)	Modal Damping Ratio (%)	Physical Meaning of Mode Shapes
Mode	Natural Frequency (Hz)	Mode	Natural Frequency (Hz)			
F-07	742.79	E-01	724.54	2.52	0.0597	(x,y)=(1,3)
F-08	1049.2	E-02	1029.8	1.88	0.1278	(x,y)=(3,1)
F-09	1329.1	E-03	1300.1	2.23	0.3823	(x,y)=(2,2)
F-10	2088.7	E-04	2029.8	2.90	0.7746	(x,y)=(1,4)
F-11	2624.9	E-05	2566.0	2.30	0.5977	(x,y)=(4,1)
F-12	3159.7	E-06	3096.6	2.04	0.7997	(x,y)=(2,3)
F-13	3601.1	E-07	3468.5	3.82	1.2570	(x,y)=(3,4)
F-14	4532.7	E-09	4408.7	2.81	0.1081	(x,y)=(5,3)
F-15	4577.8	E-08	4400.4	4.03	1.2157	(x,y)=(2,4)

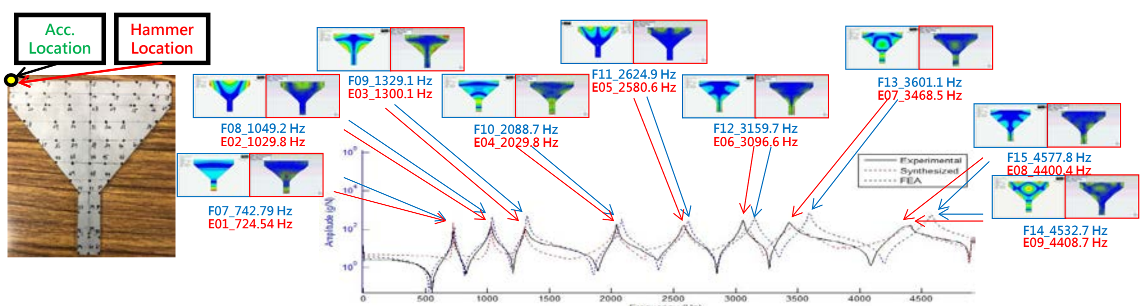


(a) FEA

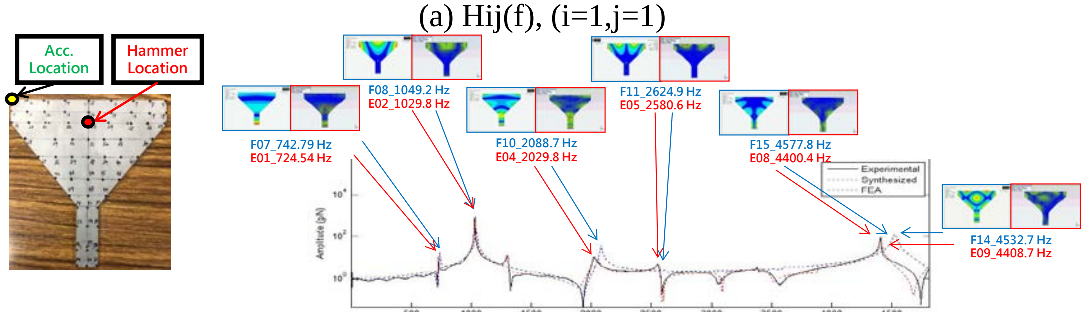


(b) EMA

Figure 3: Natural frequencies and corresponding mode shapes of pentagonal plate for C6.



(a) Hij(f), (i=1,j=1)



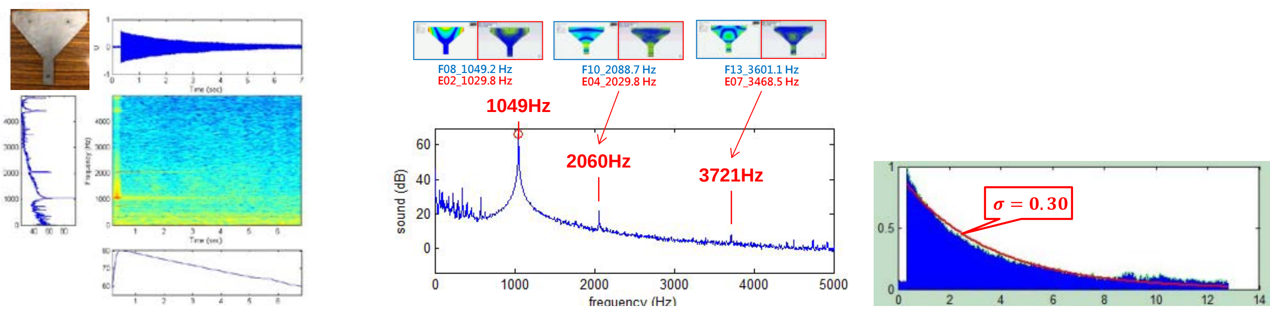
(b) Hij(f), (i=1,j=23)

Figure 4: Frequency response functions, with natural frequencies and mode shapes for C6 pentagonal plate.

3.2 Percussion sound of pentagonal plate for C6

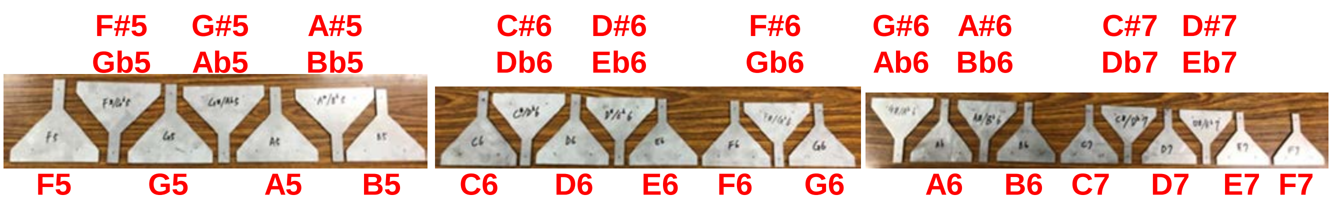
Since the plate is designed to suspend with three thin strings to tie the plate through the small holes. The percussion sound test is on the string-tie boundary condition. The impact sound is measured for the mallet striking at the prescribed location. Figure 5 shows the percussion sound response for C6 pentagonal plate discussed as follows:

- Figure 5(a) reveals the time waveform of percussion sound and spectrogram. The typical decay signal is analysed for its decay rate σ=0.30 as shown in Figure 5(c).
- From Figure 5(b), there are three peaks in the sound spectrum, i.e. 1049Hz, 2060Hz and 3721Hz. Their corresponding mode shapes are depicted on the top of each peak. The first two peak frequencies are the designed target frequencies as expected.
- From the spectrogram, the first harmonic frequency last longer through the time history.

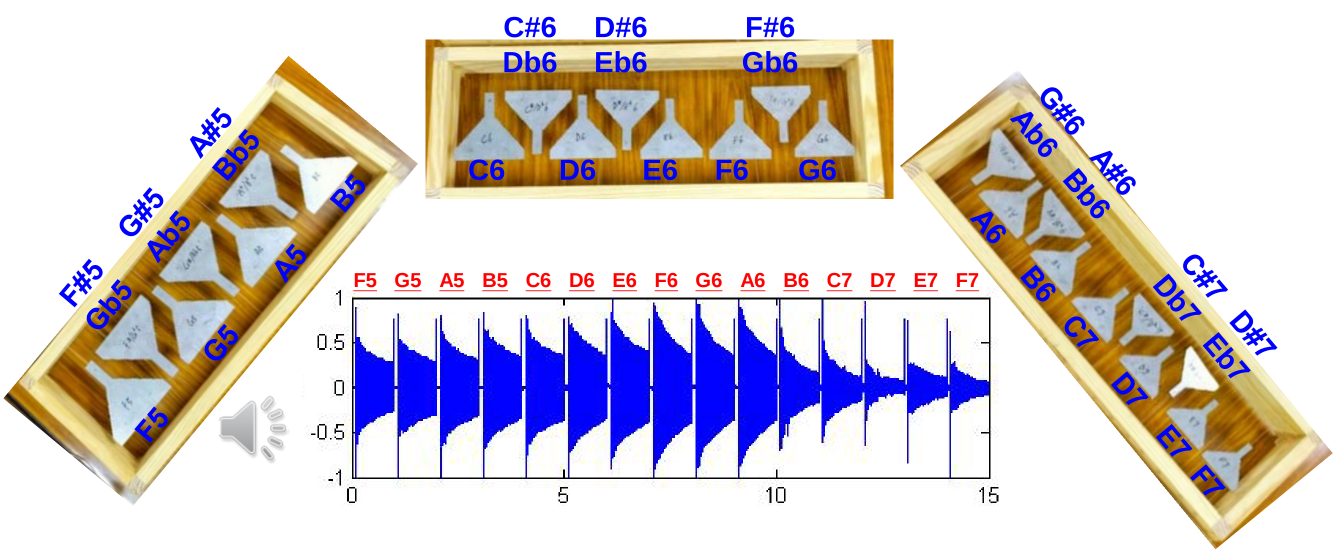


(a) Time wave form and spectrogram (b) Sound spectrum (c) Decay rate analysis

Figure 5: Percussion sound response for C6 pentagonal plate.



(a) Individual plate for each musical note



(b) The percussion instrument set

Figure 6: The set of pentagonal plates as percussion instrument.

4. Design of percussion instrument with pentagonal plates

The pentagonal plate for C6 has been shown design verification in previous section. From sound spectrum frequency domain response as well as modal parameters comparison from FEA and EMA, the design of pentagonal plate can be validated. In order to design a set of percussion instrument, we choose two octaves of musical notes from F5 to F7 because the note range is suitable for playing variety of songs. Table 2 lists the standard frequencies of musical notes. In particular, the first harmonic is the fundamental frequency, and the second harmonic is the overtone frequency.

Different fundamental frequencies of pentagonal plates can be obtained by scaling the size of the plate by using FEA to predict the desired target frequencies for different musical notes. The rule of thumb in tuning different sizes of plates is the square root of length ratio being the inverse of frequency ratio between musical notes. Therefore, different pentagonal plates for all of the musical notes can be design and manufacture.

Figure 6(a) shows the 25 pieces of pentagonal plates from F5 to F7. The lower frequency for the musical note, the larger the plate size is. Figure 6(b) shows the outfit of the complete set of percussion instrument. From Table 2, those notes from F6 to F7 reveals frequency errors smaller than -0.34%, i.e. the sound frequencies are not fulfilled for musical tone. This may be due to the manufacturing error and need to tune for correct frequency response. Nevertheless, the design of pentagonal plate for percussion instrument is basically successful.

Table 2: Comparison of target and measured frequencies for percussion instrument of pentagonal plates

Target Frequency (Hz)			Measured Frequency (Hz)		Frequency Error (%)	
Musical Notes	**1st Harmonic**	**2nd Harmonic**	**1st Harmonic**	**2nd Harmonic**	**1st Harmonic**	**2nd Harmonic**
F5	**698**	**1397**	**696**	**1375**	**-0.29**	**-1.57**
F#5/Gb5	**740**	**1480**	**742**	**1455**	**0.27**	**-1.69**
G5	**784**	**1568**	**783**	**1542**	**-0.13**	**-1.66**
G#5/Ab5	**831**	**1661**	**834**	**1635**	**0.41**	**-1.58**
A5	**880**	**1760**	**882**	**1733**	**0.23**	**-1.53**
A#5/Bb5	**932**	**1865**	**935**	**1835**	**0.29**	**-1.59**
B5	**988**	**1976**	**991**	**1943**	**0.33**	**-1.65**
C6	**1047**	**2093**	**1049**	**2058**	**0.24**	**-1.67**
C#6/Db6	**1109**	**2217**	**1108**	**2179**	**-0.07**	**-1.73**
D6	**1175**	**2349**	**1176**	**2306**	**0.11**	**-1.84**
D#6/Eb6	**1245**	**2489**	**1245**	**2442**	**0.04**	**-1.89**
E6	**1319**	**2637**	**1318**	**2586**	**-0.04**	**-1.93**
F6	**1397**	**2794**	**1389**	**2729**	**-0.57**	**-2.32**
F#6/Gb6	**1480**	**2960**	**1472**	**2892**	**-0.54**	**-2.30**
G6	**1568**	**3136**	**1556**	**3058**	**-0.76**	**-2.49**
G#6/Ab6	**1661**	**3322**	**1646**	**3237**	**-0.92**	**-2.57**
A6	**1760**	**3520**	**1747**	**3432**	**-0.74**	**-2.50**
A#6/Bb6	**1865**	**3729**	**1850**	**3636**	**-0.79**	**-2.50**
B6	**1976**	**3951**	**1962**	**3855**	**-0.68**	**-2.43**
C7	**2093**	**4186**	**2077**	**4081**	**-0.76**	**-2.51**
C#7/Db7	**2217**	**4435**	**2200**	**4320**	**-0.79**	**-2.59**
D7	**2349**	**4699**	**2325**	**4566**	**-1.03**	**-2.82**
D#7/Eb7	**2489**	**4978**	**2461**	**4831**	**-1.13**	**-2.95**
E7	**2637**	**5274**	**2604**	**5112**	**-1.25**	**-3.07**
F7	**2794**	**5588**	**2774**	**5431**	**-0.71**	**-2.80**

5. Conclusions

This work aims to redesign the pentagonal plate dimension such that the fundamental frequency, i.e. the first sound radiated mode, can fulfil the musical notes exactly for being the percussion instrument. The second sound radiated mode referred to the overtone frequency of percussion sound is optimally controlled by dimension optimum design. The overtone frequency can have twice of the fundamental frequency. This kind of percussion sound can be said as harmonic sound. In design analysis, the finite element model of pentagonal plate is built to perform theoretical modal analysis and obtain the design for the pentagonal with two harmonics sound. Design verification is also carried out by comparing modal and frequency response with experiments. Finally, the complete set of percussion instrument consisting of 25 different sizes of plates are designed and manufactured. The design and manufacture processes are detailed and applicable to percussion instrument development.

REFERENCES

1 Lavan, D., Hogg, S. and Wolfe, J. Why do bell plate ring?, *Acoustics Australia*, **31**, No. 2–1, (2003).

2 McLachlan, N. The Application of new analyses and design methods to musical bells, *Proceedings of the 75th Acoustical Society of America Conference*, New York, 1–8, (2002).

3 Rabenstein, R., Koch, T. and Popp, C. Tubular Bells: A Physical and Algorithmic Model, *IEEE Transactions on audio Speech And Language Processing*, **18**, 881–890, (2010).

4 Bork, I. Practical tuning of xylophone bars and resonators, *Applied Acoustics*, **46**, 103–127, (1995).

5 Lee, J. M., Kim, S. H., Lee S. J., Jeong, J. D. and Choi, H. G. A study on the vibration characteristics of a large size korean bell, *Journal of Sound and Vibration*, **257** (4), 779–790, (2002).

6 Pan, J. Acoustical properties of ancient chinese musical bells, *The Journal of the Acoustical Society of America*, **84**, No. S1, (2009).

7 Ansari J. Finite element vibration analysis and modal testing of bells, *Proceedings of the IJME Intertech 2006 International Conference*, Union, NJ, USA, (2006)..

8 Bretos, J., Santamaria, C. and Moral, J. A. Finite element analysis and experimental measurements of natural eigenmodes and random responses of wooden bars used in musical instruments, *Applied Acoustics*, **56**, 141–156, (1999).

9 Jing M. A Theoretical study of the vibration and acoustics of ancient chinese bells, *Journal of the Acoustical Society of America*, **114**(3), 1622–1628, (2003).

10 Wang, B. T., and Jian, X. M. Model verification and percussion sound characteristics of metallophone with chord sound, *Proceedings of the 17th International Congress on Sound and Vibration*, Cairo, Egypt, 18–22 July, (2010).

11 Petrolito, J. and Legge, K.A. Optimal undercuts for the tuning of percussive beams, *Journal of Acoustical Society of America*, **102**(4), 2432–2437, (1997).

12 Wang, B. T., Yu, G. D., Chen, P. H. and Chang, H.Y. Design and percussion sound evaluation of harmonic glass plate," *Proceedings of the 20th International Congress on Sound & Vibration*, Bangkok, Thailand, 7–11 July, (2013).

13 McLachlan, N., Nigjeh, B. K. and Hasell, A. The design of bells with harmonic overtones, *The Journal of the Acoustical Society of America*, **114**, No.1, (2003).

14 Wang, B. T., Chao, T. C., and Hsieh, M. H. *Manufacturing Method for a Metallophone with a Chord Plate and a Chord Plate of Metallophones*, Taiwan Patent No. I 409801, Date of Patent: 02/01/2013, Effective Date: 08/26/2028, (2016).

15 Wang, B. T., and Hsieh, M. H. Sound characteristics and design analysis of the metallophone bar with chord, *Conference on 2009 Annual Meeting and 22th Symposium of Acoustical Society of the Republic of China* ,Taipei, A006, (2009). (in Chinese)

16 Wang, B. T., Dung, S. P., Wu, M. F., Tsai, Y. L. and Chang, C. H. *Method for Design a Board with Harmonics Sound and Percussion Instruments with Boards*, Taiwan Patent No. I 530939, Date of Patent: 04/21/2016, Effective Date: 08/20/2034, (2016).