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Damage Detection of Surface Crack in Composite Quasi-isotropic Laminate using Modal Analysis and Strain Energy Method

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Abstract. This paper presents a damage detection of surface crack in composite laminate. Carbon/epoxy composite AS4/PEEK was used to fabricate a quasi-isotropic laminate $[0/90/\pm45]_{2s}$. Surface crack was created by using laser cutting machine. Modal analysis was performed to obtain the mode shapes of the laminate before and after damage. The mode shapes were then adopted to compute the strain energy, which was used to define a damage index. Consequently, the damage index successfully predicted the location of surface crack in the laminate. Differential quadrature method (DQM) was introduced to calculate the partial differential terms in strain energy formula.

Introduction

Modal analysis methods have been increasingly adopted to detect the damage in composite materials due to their flexibility in measurement and relatively low cost. The basic idea of these methods is to use the information of modal parameters, such as frequency, mode shape and damping ratio, to access the structural damage.

Cawley and Adams [1] simply used the frequency shifts for different modes to detect the damage in composite structures. Tracy and Pardoen [2] found that the natural frequencies of a composite beam are affected by the size and damage location. Shen and Grady [3] indicated that local delamination does not have a noticeable effect on global mode shape of composite beams, but delamination does cause the irregularity of mode shapes. Zou et al. [4] provided a thorough review in vibration-based techniques and indicated that the above methods were unable to detect very small damage and required large data storage capacity for comparisons. Cornwell et al. [5] utilized the measured mode shapes to calculate the strain energy of a plate-like structure. Fractional strain energy was then used to define a damage index which can locate the damage in structure. The method only requires the mode shapes of the structure before and after damage. Nevertheless, the challenge of the method lies in the accuracy of measured modes. A large amount of data points are required for further analysis to locate the damage. To solve this problem, Hu et al. [6] adopted the DQM to rapidly obtain the accurate solution of strain energy and successfully located damage in a composite laminate plate. It was reported that the original DQM was first used in structural mechanics problems by Bert et al. [7]. This method is able to rapidly compute accurate solutions of partial differential equations by using only a few grid points in the respective solution domains [8].

The objective of this paper is to investigate the detection of surface crack in composite quasi-isotropic laminate using a damage index, which only requires the mode shapes obtained from modal analysis and the strain energy of the laminate before and after damage. Finite element analysis (FEA) was also performed to access this approach.

Experimental modal analysis

The prepreg of carbon/epoxy composite AS4/PEEK was used to stack up a quasi-isotropic laminate, $[0/90/\pm45]_{2s}$ and then cured at a hot-press machine. After curing, the panel was cut to a test plate with dimension $210\times126\times2.4$ mm^3. Marked by 13×13 parallel grid points, the test plate was vertically hung by two cotton strings to simulate a completely free boundary condition as shown in Figure 1. Modal testing was performed by exciting the test plate throughout all grid points using an impact hammer with a force transducer. The dynamic responses were measured by an accelerometer fixing at the corner. Siglab, Model 20-40, was used to record the frequency response functions (FRFs) between measured acceleration and impact force. ME'Scope, a software for the general purpose curve fitting, was used to extract the natural frequencies and mode shapes from the FRFs. A surface crack with 24.5 mm long and 1 mm deep was created using a laser cutting machine.

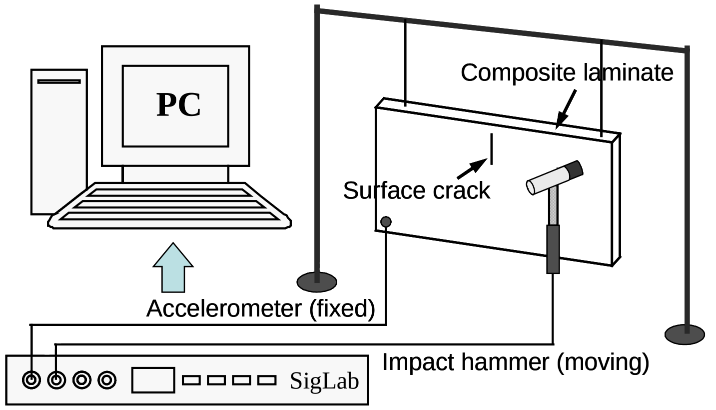


Figure 1: Experimental set-up

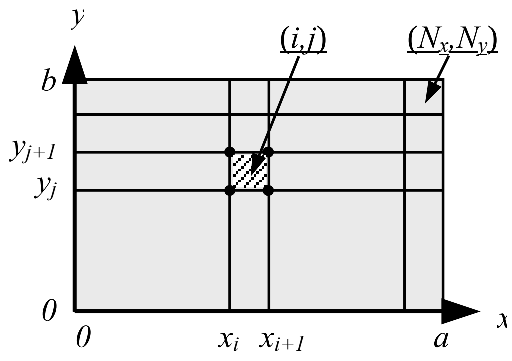


Figure 2: Grid points arrangement

Strain energy method and damage index

The test plate is subdivided into $N_x \times N_y$ sub-region and denoted the location of each point by (x_i, y_j) as shown in Figure 2. For laminate plate theory, the strain energy of the plate during elastic deformation is given by

$$U = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4\left(D_{16}\frac{\partial^2 w}{\partial x^2} + D_{26}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x \partial y} + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right] dxdy \quad (1)$$

where w is the transverse displacement and D_{ij} are the bending stiffnesses of the laminate.

Considering a free vibration problem, for a particular normal mode, the total strain energy of the plate associated with the mode shape ϕ_k can be expressed as

$$U_k = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 \phi_k}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2 \phi_k}{\partial x^2}\frac{\partial^2 \phi}{\partial y^2} + 4\left(D_{16}\frac{\partial^2 \phi_k}{\partial x^2} + D_{26}\frac{\partial^2 \phi_k}{\partial y^2}\right)\frac{\partial^2 \phi_k}{\partial x \partial y} + 4D_{66}\left(\frac{\partial^2 \phi_k}{\partial x \partial y}\right)^2 \right] dxdy \quad (2)$$

Cornwell *et al*. [5] suggested that if the damage is located at a single sub-region then the change of strain energy in sub-region may become significant. Thus, the energy associated with sub-region (i,j) for the k^{th} mode is given by

$$U_{k,ij} = \frac{1}{2}\int_{y_j}^{y_{j+1}}\int_{x_i}^{x_{i+1}} \left[D_{11}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 \phi_k}{\partial y^2}\right)^2 + 2D_{12}\left(\frac{\partial^2 \phi_k}{\partial x^2}\right)\left(\frac{\partial^2 \phi_k}{\partial y^2}\right) + 4\left(D_{16}\frac{\partial^2 \phi_k}{\partial x^2} + D_{26}\frac{\partial^2 \phi_k}{\partial y^2}\right)\left(\frac{\partial^2 \phi_k}{\partial x \partial y}\right) + 4D_{66}\left(\frac{\partial^2 \phi_k}{\partial x \partial y}\right)^2 \right] dxdy \quad (3)$$

Similarly, U_k^* and $U_{k,ij}^*$ represent the total strain energy and sub-regional strain energy of the k^{th} mode shape ϕ_k^* for damaged plate. A fractional energy of undamaged or damage plate is given by

$$F_{k,ij} = \frac{U_{k,ij}}{U_k} \quad \text{and} \quad F_{k,ij}^* = \frac{U_{k,ij}^*}{U_k^*} \quad (4)$$

Considering all modes, m, in the calculation, damage index in sub-region (i,j) is defined as

$$\beta_{ij} = \frac{\sum_{k=1}^{m} F_{k,ij}^{*}}{\sum_{k=1}^{m} F_{k,ij}} \tag{5}$$

Equation (5) is used to predict the damage location in composite laminate plate in this study. To obtain the partial differential terms in strain energy formula, an alternative numerical method DQM is introduced to solve the problem.

Differential quadrature method

The basic idea of DQM is to approximate the partial derivatives of a function $f(x_i,y_j)$ with respect to a spatial variable at any discrete point as the weighted linear sum of the function values at all the discrete points chosen in the solution domain of spatial variable. This can be expressed mathematically as

$$f_x^{(n)}(x_i,y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} f(x_r,y_j)\,; \quad f_y^{(m)}(x_i,y_j) = \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_i,y_s)\,; \quad f_{xy}^{(n+m)}(x_i,y_j) = \sum_{r=1}^{N_x} C_{ir}^{(n)} \sum_{s=1}^{N_y} \overline{C}_{js}^{(m)} f(x_r,y_s) \tag{6}$$

where i = 1,2,…,N_x and j =1,2,…,N_y are the grid points in the solution domain having $N_x \times N_y$ discrete number of points. $C_{ir}^{(n)}$ and $\overline{C}_{js}^{(m)}$ are the weighting coefficients associated with the n^{th} order and the m^{th} order partial derivatives of $f(x_i,y_j)$ with respect to x and y at the discrete point (x_i, y_j) and n=1, 2,…,N_x-1, m=1, 2,…,N_y-1. The weighting coefficients can be obtained using the following recurrence formulae

$$C_{ir}^{(n)} = n\left(C_{ii}^{(n-1)} C_{ir}^{(1)} - \frac{C_{ir}^{(n-1)}}{x_i - x_r} \right) \quad \text{and} \quad \overline{C}_{js}^{(m)} = n\left(\overline{C}_{jj}^{(m-1)} \overline{C}_{js}^{(1)} - \frac{\overline{C}_{js}^{(n-1)}}{y_j - y_s} \right) \tag{7}$$

where i,r=1,2,…N_x but $r \neq i$; n= 2,3,…,N_x-1; also j,s = 1,2,…,N_y but $s \neq j$; m= 2,3, …,N_y-1. The weighting coefficients when $r=i$ and $s=j$ are given as

$$C_{ii}^{(n)} = -\sum_{r=1, r\neq i}^{N_x} C_{ir}^{(n)}\ ;\ i=1,2,\ldots,N_x, \text{ and } n=1,2,\ldots,N_x-1 \quad \text{and} \quad \overline{C}_{jj}^{(m)} = -\sum_{s=1, s\neq j}^{N_y} \overline{C}_{js}^{(m)}\ ;\ j=1,2,\ldots,N_y, \text{ and } m=1,2,\ldots,N_y-1 \tag{8}$$

$$C_{ir}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_r) M^{(1)}(x_r)}\ ;\ i,r=1,2,\ldots,N_x, \text{but } r \neq i \quad \text{and} \quad \overline{C}_{js}^{(1)} = \frac{P^{(1)}(y_j)}{(y_j - y_s) P^{(1)}(y_s)}\ ;\ j,s=1,2,\ldots,N_y, \text{but } j \neq s \tag{9}$$

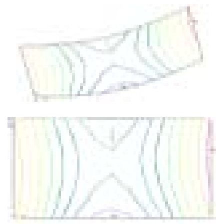
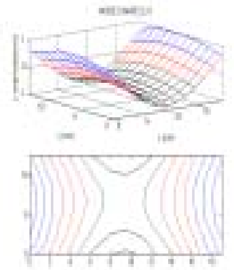
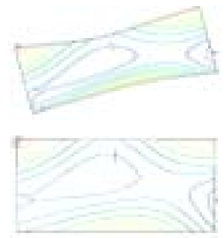
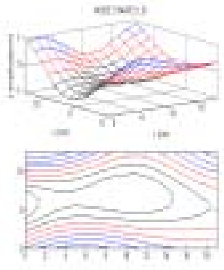
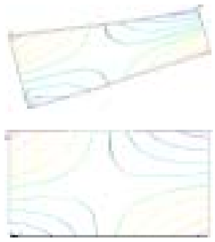
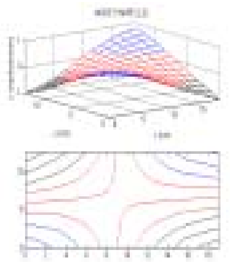
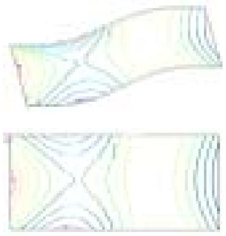
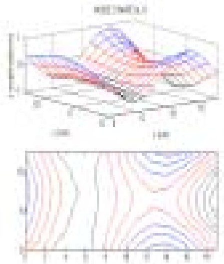
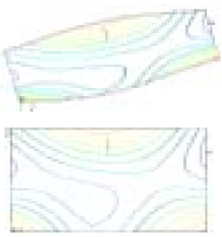
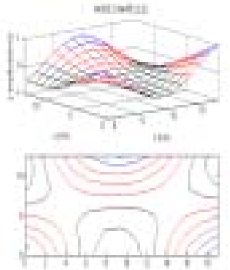
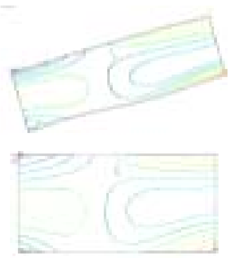
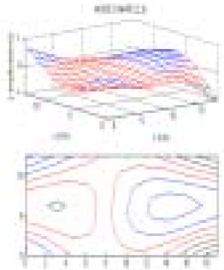
where $M^{(1)}(x_i) = \prod_{r=1, r\neq i}^{N_x} (x_i - x_r)$ and $P^{(1)}(y_j) = \prod_{s=1, s\neq j}^{N_y} (y_j - y_s)$. The above equations are applied to compute the strain energy once the k^{th} mode shape $\phi_{k,ij} = f_k(x_i,y_j)$ is obtained.

Results and discussions

The first six natural frequencies ω_n and the associated mode shapes contours of the test plate are listed in Table 1. The natural frequencies in brackets denote the results of damaged plate. Both FEA and EMA results show that bending mode (3, 1) loses certain degree of the natural frequency. This implies that surface crack decreases the bending stiffness of the test plate in x-direction. However, it is not enough to preciously predict the surface crack location. Mode shape contour is also unable to locate the surface crack. In fact, the mode shapes of damage plate are almost the same to the undamaged.

A pre-study was performed using FEA software, ANSYS. Figure 3 shows the finite element model. Eight-node solid element (SOLID 46) was used to simulate the quasi-isotropic laminate plate. Mass element (MASS 21) was assigned to accelerometer. To simulate the modal testing, FE model was meshed using the same grid points to EMA. The first six mode shapes of the laminate plate were then adopted to compute the strain energy in obtaining the damage index. Figure 4 shows the damage index obtained from FEA result. The peak value successfully indicates the surface crack location. The encouraging outcome leads to the following experimental results.

Table 1: Natural frequency and mode shape of laminate plate

	FEA	EMA		FEA	EMA
(3,1)	 $\omega_n = 313(312) Hz$	 $\omega_n = 241(238) Hz$	(1,3)	 $\omega_n = 873(872) Hz$	 $\omega_n = 701(699) Hz$
(2,2)	 $\omega_n = 369(369) Hz$	 $\omega_n = 281(280) Hz$	(4,1)	 $\omega_n = 920(920) Hz$	 $\omega_n = 717(717) Hz$
(3,2)	 $\omega_n = 779(779) Hz$	 $\omega_n = 648(647) Hz$	(2,3)	 $\omega_n = 1119(1119) Hz$	 $\omega_n = 956(956) Hz$

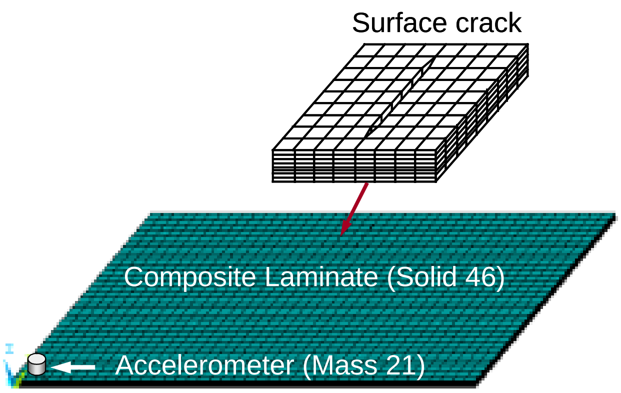


Figure 3: Finite element model

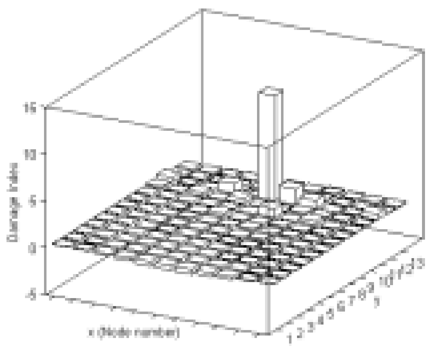


Figure 4: Damage index (FEA)

Figure 5 shows the damage index obtained from EMA. Peak values occur at the location of surface crack; however, many peak values also emerged at some other undamaged areas. The deviation in measurement may attribute to these pseudomorphs. Cornwell *et al*. [5] suggested that damage indices with values greater than two are associated with potential damage locations. Thus, we tried to truncate the peaks of damage index less than two. The improving outcome is obtained in Figure 6. Two more EMA results are shown in Figures 7 and 9. After truncation, improving outcomes are also obtained in Figures 8 and 10. Practicably, structural damage can be long term monitored. Damage indices obtained in different times may be summed to amplify the signal of surface crack. Figure 11 shows the summation of two damage indices in first and second EMA results. The damage index by adding the third EMA result is shown in Figure 12. Both combinations clearly indicate the surface crack location in the test plate.

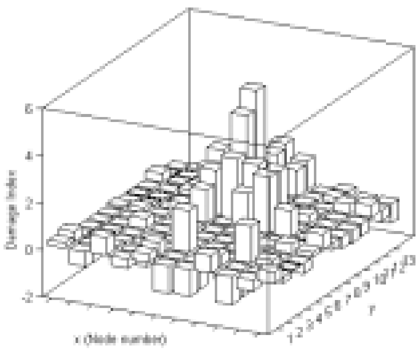


Figure 5: Damage index (1st EMA)

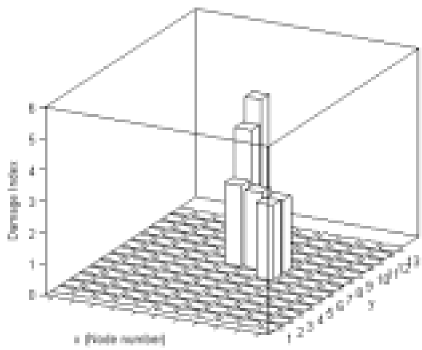


Figure 6: After truncation (1st EMA)

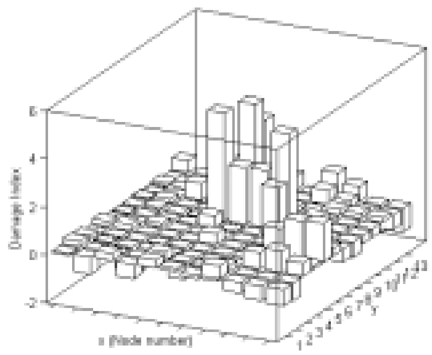


Figure 7: Damage index (2nd EMA)

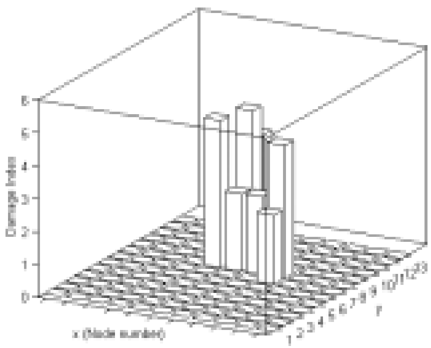


Figure 8: After truncation (2nd EMA)

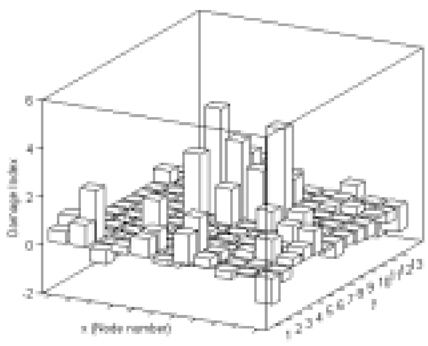


Figure 9: Damage index (3rd EMA)

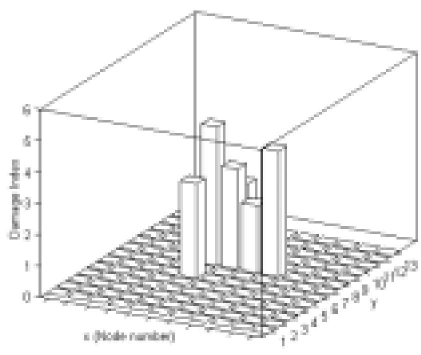


Figure 10: After truncation (3rd EMA)

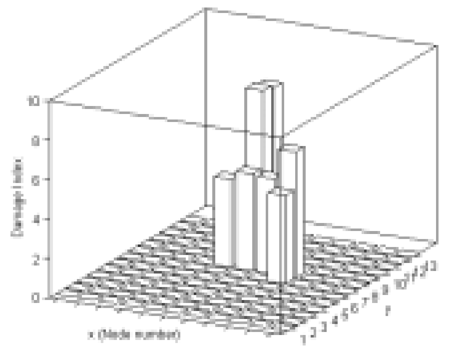


Figure 11: Damage index ($1^{st}+2^{nd}$ EMA)

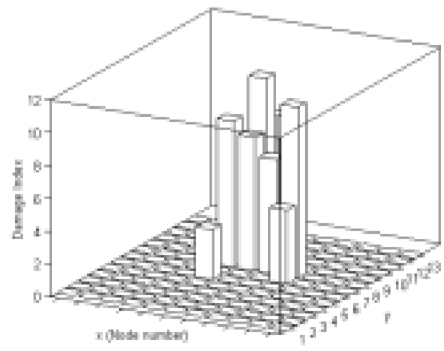


Figure 12: Damage index ($1^{st}+2^{nd}+3^{rd}$ EMA)

Conclusions

Damage index using modal analysis and strain energy methods is developed to detect a surface crack in composite quasi-isotropic laminate in this paper. This method only requires a few mode shapes of the plate before and after damage. Both FEA and EMA results successfully locate the surface crack in test plate. DQM provides us an accurate approach to compute strain energy using only a few grid points in test plate. However, limited by grid point number, challenge still lies in the mode shape measurement. Future work will focus on the study of various types of damage.

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