# PREDICTION OF IMPACT AND HARMONIC FORCES ACTING ON ARBITRARY STRUCTURES: THEORETICAL FORMULATION

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Submitted

to

MECHANICAL SYSTEMS AND SIGNAL PROCESSING

Short running headline: IMPACT AND HARMONIC FORCE PREDICTION

Total number of pages: 33

Total number of tables: 1

Total number of figures: 5

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**ABSTRACT**

This work presents a new general approach in developing the force prediction model. Two types of unknown forces dealt with in this study are the impact and harmonic forces. Theoretical background of force prediction model is extensively reviewed and categorized into two approaches, the direct and optimization approaches. Both approaches can also be further divided into the time and frequency domain methods. The structural response due to the unknown forces is then derived. The new approach, an optimization approach, to predict the force magnitude and location simultaneously is developed for both time and frequency domain methods. The implementation of the prediction model is also discussed. Special concerns about the applications to engineering problems are addressed.

## 1. INTRODUCTION

Force prediction is one of interested inverse problems and important for engineering design and application [1]. The knowledge of the input force on structures is beneficial to the design and operation of the system. There are numerous examples showing the needs for the identification of applied forces to structures, such as the cutting forces of machine tools, reaction forces of engine mounts, and supporting forces of bearings [2]. In particular, the damage of composite structures subjected to impact forces is not easily visible; therefore, the prediction of the force location will cut down the effort in applying other inspection method all over the structures [3-7]. For rotating machines such as turbines and pumps, a direct measurement of excited forces due to imbalance or hydraulic flow is not practically possible. Vehoeven [8] used measured operational vibration data to calculate the total excitation forces of the rotating machine by an analytical modal analysis approach. Vyas and Wicks [9] presented a procedure to estimate turbine blade forces that have been a great concern in fatigue failure analysis.

The force prediction problem can be sketched as shown in Figure 1. When the structure is subject to an unknown force, the knowledge of mathematical model to represent the structure and the measured response due to the unknown force is essential so as to develop the force prediction model for determining the force contents. In general, the force contents can be the magnitude, direction and location. The external forces can be categorized into three forms. One is the spatial variant type such as point forces and distributed forces. Another is the time variant type such as the impact, harmonic, periodic and random forces. The time history or the frequency spectra of the force may be of interest. The other is the spatial and time variant type such as moving forces. Laws and his coworker presented a series of studies for force identification of moving forces on structures [10-16]. There are many literatures dealt with the force identification problems for different kinds of forces. There is no general model suitable for all kinds of problems in practice. This work do not intend to develop a general model to suit any type of force but to introduce the developed methods and to propose a new systematic approach in force identification for arbitrary structures subject to unknown forces. Two types of forces, i.e. the impact and harmonic forces, will be considered.

For impact force prediction problems, two categories of developed methods can be summarized as if the impactor is known or unknown. First, the impactor is generally presumed known in terms of size, shape, weight as well as its impact velocity. Hertzian contact is normally assumed during contact. Shivakumar et al. [5] considered the impactor an elastic or rigid ball with the known impact velocity onto a circular composite laminate. By the energy-balanced and spring-mass models including contact deformation as well as bending and shear deformations of the laminate, they predicted the time history of the impact force. McMillan et al. [17] also presented a similar approach to identify the impact force and structural bending stresses of the circular plate due to impact. The reality of the impact problem is that the impactor may not be well known in prior.

Another type of impact force prediction model does not account for the physical properties of the impactor but identify the impact force acting on the structure directly [3,4,6,7,18-31]. The interested force contents will be the amplitude, direction, location, and time history. Most works considered transverse normal impact to the structure and presumed known force locations. The force time history can then be predicted. Michaels and Pao [28] applied multiple Green's functions related a direction cosine of the oblique forces to determine the force time function and the direction cosine. Inoue et al. [25] decomposed the impact force into three directional components to predict its magnitude and direction. There are little literature dealt with the location prediction of impact force. Wu et al. [7] intended to identify the impact force location by comparing the reconstructed strain response among several candidate locations. Doyle [32] and his co-workers [33] presented the impact force location prediction. The basic scheme is based on the pattern match for the reconstructed force history. The solution of both the time history and location of impact forces are treated separately. Excessive computational effort will be required to solve the problem. Choi and Chang [34] introduced distributed piezoelectric sensors to detect the impact force time history and its location by comparing measured and estimated sensor outputs. Two loops in fitting process are involved. One is for predicting the force time history, and another is for predicting the force location. Turek and Kuperman [35] applied matched-field processing (MFP), a generalized procedure of array processing used in ocean acoustics to localize sources, to locate a point force on a vibrating beam without detail modeling of structures. This work will deal with the normal impact force on structures. With the assumption of ideal impact force, the amplitude and location of the impact force will be predicted simultaneously.

Force prediction problem dealt with harmonic force excitation is also of concern. D'Cruz et al. [36] used Gauss-Newton method to solve the least square optimization problem that is to minimize the errors between the predicted and measured response for the plate subject to a harmonic force. The location, amplitude and phase of the harmonic force were predicted through numerical simulations. Karlsson [37] assumed the force spatial distribution available a priori and predicted the complex amplitudes of harmonic forces. Ma and Lin [38] applied the Kalman filter with a recursive estimator to determine the harmonic forces of an equipment-isolator. Those prediction models normally assumed a prior known force location such that only the force amplitude and its time history were predicted. Moller [39] tentatively gave the spatial shape and position of the harmonic point load and applied Betti reciprocal theorem with a reference load case to calculate the magnitude and match the load location. This work will also develop the harmonic force prediction method for arbitrary structures in determining the force amplitude and location.

As mentioned previously, mathematically modeling the structural system is required in order to predict the force contents. Theoretically modeling and experimentally modeling or both can be adopted. The modeling process can be treated in different points of views. **First**, either the discrete or continuous system can be adopted depending on the characteristics of the structures. Huang et al. [24] simplified the vibratory mill into a rigid bar with two spring supports for the prediction of impact forces during mill collision. Lim and Pilkey [27] adopted the discrete system to model a ten-bay truss structure for the identification of dynamic force time function. Treated structures in force prediction problem include simple structures, such as bar [18], beams [19-21,34,37,40-45], plates [22,23,36,46], composite laminates [3-7,47] and frame structures [33,48]. A theoretical model can generally be well defined and solved. For complex structures like engine mounts [2], rotary machines [8,9] and computer enclosures [49], experimental models were generally adopted. This work will consider a general continuous system model suitable for arbitrary structures subject to unknown force excitation. **Second**, in terms of the representation of system response the time or frequency domain model can be adopted. For time domain approach, the convolution integral equation that correlates the input force and output response can generally be formulated. While Green's function is mainly for the propagation wave response, impulse response function (IRF) can be of interest for structural vibration. For frequency domain approach, frequency response function (FRF) that can be obtained by theoretical modal analysis (TMA) or experimental modal analysis (EMA) is required. In some circumstance, the modal domain approach, i.e. the system is expressed in terms of modal parameters, can also be developed to determine the system response. Kim and Kim [50] used modal model to construct the frequency response function. Therefore, the inverse of FRF matrix can be easily obtained for force prediction. Busby and Trujillo [19] performed TMA to obtain system modal parameters and so forth to estimate the system response in order to find the unknown force time history. This work will develop both time and frequency domain methods. System response will be derived as a function of system modal parameters and input force parameters. **Third,** the solution methods in resolving the system equation for the estimation of the predicted sensor response can be done by finite element method [39,47], state-space equation approach or dynamic programming [36,45,51,52], convolution and deconvolution method [3,30] and modal analysis [43,48,50]. This work presumes the known force function; therefore, the system vibrating response can be theoretically evaluated and expressed in a concise modal format.

To measure the system response due to the force excitation is also needed for force prediction models. Various kinds of sensors have been used, such as strain guages [6,7,20,21], accelerometers [53], slope sensors [42], laser vibrometer [54] and piezoelectric sensors [34]. The measured system response can be characterized as the concern of either the structural propagation wave [6,7,20,21,55] or the structural vibration [42]. Normally, the adoption of different types of sensors and mathematical models as well as the solution techniques will result in different approaches in developing the force prediction models. This work assumes the use of acceleration sensor to measure the structural vibration response. Since the displacement response is also obtained, applying different types of sensors for the force prediction model can also be manipulated to adapt the developed method.

The followings will introduce the theoretical background about force prediction problems in Section 2. The system analysis techniques in physical, time, frequency and modal domain are summarized. Force prediction models are classified into direct and optimization methods. The systematic development of force prediction model, which is one of the optimization methods, is then presented. Section 3 presents the derivation of system response for arbitrary structures subject to impact and harmonic force excitations. The force prediction models for both time and frequency domain methods can then be formulated in Section 4. Section 5 discusses the implementation of the developed force prediction model.

## 2. THEORETICAL BACKGROUND IN FORCE PREDICTION

### 2.1 SYSTEM ANALYSIS

The concept of force prediction can be theoretically simple. However, the practical implementation of the predictive algorithms can be complex and involved complicated tasks. To show the basic idea of force identification, the system block diagrams in different domains are first sketched as shown in Figure 2 and explained. Consider the equation of motion for an arbitrary structure over the domain $D$ as follows [56]:

$$L[w(P,t)]+\frac{\partial}{\partial t}C[w(P,t)]+M(P)\frac{\partial^2}{\partial t^2}[w(P,t)]= f(P,t) \tag{1}$$

where $L$ , $C$ are linear homogeneous self-adjoint differential operators consisting of derivatives through 2*p* with respect to the spatial coordinates *P* but not with respect to time *t*, containing information concerning the stiffness and damping functions. $M(P)$ is the mass distribution function of the system. $f(P,t)$ is the general force function. $w(P,t)$ is the structural response. For simplicity, the boundary conditions are assumed homogeneous so that at every point on the boundaries of domain *D* the following equation must be satisfied

$$B_i\left[w(P,t)\right]=0\ ,\quad i=1,2,\cdots,p \tag{2}$$

where $B_i$ is the linear homogeneous differential operator containing derivatives normal to the boundary and along the boundary of order through 2*p*-1. The corresponding physical domain block diagram is sketched as shown in Figure 2(a). *L*, *C*, and *M* contain physical properties of the system, while $f(P,t)$ and $w(P,t)$ designate the physical force and physical coordinate response, respectively.

The general solution of structural response in time and frequency domains will be derived in details later and can be expressed as follows, respectively:

$$w(P,t)=\int f(P,\tau)h(P,t-\tau)d\tau \tag{3}$$

$$W(P,\omega)=H(P,\omega)F(P,\omega) \tag{4}$$

where $w(P,t)$ and $W(P,\omega)$ denote the time and frequency response at coordinate *P*, respectively, and are Fourier transform pairs. $f(P,t)$ and $F(P,\omega)$ are the input force functions and also Fourier transform pairs. $h(P,t)$ is the impulse response function (IRF) or Green function, and $H(P,\omega)$ is the frequency response function (FRF). They are also Fourier transform pairs. Eqns. (3) and (4) satisfy the convolution theorem. The system block diagram for the time and frequency domains can be sketched as shown in Figure 2(b) and 2(c). Since the system equation can be solved by modal analysis or modal transform method, the modal domain block diagram can also be postulated in Figure 2(d). The system can also be represented by the modal parameters. $N_k(t)$ and $q_k(t)$ designate the *k*-th modal force and modal coordinate, respectively.

The mathematical model can be characterized in physical, time, frequency and modal domains as shown in Figure 2. In physical domain, the system is expressed as the system equation as shown in Eq. (1) in terms of physical parameters including structural material properties, geometry information, boundary conditions as well as physical inputs such as forces. For the representation of system in time domain, the system can be expressed in terms of IRF or Green's function. The convolution integral relations can be obtained as shown in Eq. (3). To characterize the structure system in frequency domain, the FRF between the output and input must be determined and can be obtained from either TMA or EMA. The system frequency response can be expressed as shown in Eq. (4). The system can also be characterized in modal domain by modal parameters as shown in Figure 2(d). $\omega_k$, $\phi_k$ and $\xi_k$ denote the *k*-th natural frequency, mode shape and damping ratio, respectively. The modal parameters can be obtained theoretically or experimentally. More details will be discussed later. Either one of the above to model and simulate the structure, model verification should be carried out to check the correctness of the mathematical model in order to be adopted for force prediction.

## 2.2 FORCE PREDICTION MODELS

For force prediction or force identification problem, the input or external force acting on the system is unknown. With the prior knowledge of the system, if the system response can be measured, the input force can be determined in either the time or frequency domain. The force prediction model can be categorized into two types as the following discussions.

### 2.2.1 *Direct method*

This approach is based on the fact of the frequency domain block diagram as shown in Figure 2(c). The schematic block diagram for the method is sketched in Figure 3. The major tasks can be as follows:

(1) measure structural response $\hat{w}(P,t)$ due to unknown force $\hat{f}(P,t)$

(2) take fast Fourier transform (FFT) on $\hat{w}(P,t)$ to get $\hat{W}(P,\omega)$

(3) measure structural frequency response function $\hat{H}(P,\omega)$

(4) use the relation shown in Eq. (4) to obtain $F(P,\omega)=[H(P,\omega)]^{-1}W(P,\omega)$

(5) perform inverse fast Fourier transform (IFFT) on $F(P,\omega)$ to get $f(P,t)$

For the measurement of system response in step (1), the proper selection of sensors that should be able to reflect the structural response due to the input force is necessary. The commonly used sensors include strain gauges and accelerometers as well as non-contact sensors like laser vibrometers. In step (2), FFT can be easily implemented by frequency spectrum analyzers or numerically evaluated. It is noted that proper windows must be selected to reduce leakages of the measured system response [57].

In step (3), experimental modal analysis (EMA) can be employed to obtain the structural FRFs by applying controlled excitation. The general form of experimentally measured FRFs is a matrix. From Eq. (3), in order to determine the input force the inversion of FRF matrix is required. The nature of FRF matrix is discrete in frequency and can be noisy due to the measurement noise. In step (4), the inversion operation of FRF matrix can be time-consuming and difficult due to the ill-conditioning matrix. There are two ways to obtain $[H(P,\omega)]^{-1}$. First, Pseudo inverse [8,40,41,43,50] and singular value decomposition (SVD) [25,37,46] techniques are frequently adopted to overcome the numerical difficulty for ill-conditioning matrices. The second approach is to apply general curve fitting algorithms to extract the system modal parameters, including natural frequencies, damping ratios, and mode shapes, from the measured FRFs. The inversion of FRF matrix can then be done by the operation of modal parameters. This approach is termed modal coordinate transform [43,48,50] and will largely reduce the computing time in the inversion operation. Therefore, the force frequency spectra can be obtained in step (4). If the force time function is interested, the inverse FFT can then be performed to get the force time history in step (5).

This method is a general approach and suitable for any time variant input force. In particular, most of random input force identification problems use this approach [40,41,44,46,49,53,58-60]. The system modeling may not be necessary, because the system information can be determined from the measured FRFs. An alternative for obtaining the system model is to get the theoretical FRFs instead of experimentally measured ones. However, model verification should be properly checked so as to validate the theoretical model. The drawback of this method is that the location of input force must be known in prior for obtaining the corresponding FRFs. The correctness of the force prediction strongly relies on the accurate measurement of FRFs.

It is also noted that every source of forces has an "inner mobility," because any attached technical source consists of masses as well as springs. Therefore only a certain part $\hat{f}(P,t)$ of the total generated force is acting on the structure, which is loaded by this source. Only this part $\hat{f}(P,t)$ forces the measurable response $\hat{w}(P,t)$ and is the force, which will be predicted with the proposed method. Measuring the structural mobility or the FRF in the frequency domain according to step (3), we have to secure to determine only the FRF $\hat{H}(P,\omega)$ of the structure. It means without the FRF of the attached source. Therefore we have to decouple this source for measuring the FRF.

2.2.2 *Optimization method*

The basic idea of this method is to correlate the estimated response from the theoretical model and measured sensor response due to the unknown force excitation. Formulating an optimization problem base on the least square error method to match the estimated and measured response, one can solve for the unknown force inputs. Both time and frequency domain approaches can be adopted and detailed as follows.

The basic scheme of the time domain optimization method is shown in Figure 4(a). The major procedures can be as follows:

(1) measure structural response $\hat{w}(P,t)$ or $\hat{a}(P,t)$ due to unknown force $\hat{f}(P,t)$

(2) do structural modeling to obtain mathematical model

(3) predict the structural response $w(P,t)$ or $a(P,t)$

(4) formulate the optimization problem

(5) solve the optimization problem for the input force $f(P,t)$

The measurement of structural response via proper sensors in step (1) is also required. The main difference from the direct method is that a parallel mathematical model corresponding to the real structure must be constructed. The system mathematical model should be able to characterize the physical phenomenon of the real structure. The mathematical model can be characterized in the time, frequency and modal domains as shown in Figure 2. As discussed for time domain representation, the system response can be expressed in terms of IRF or Green's function. The convolution integral relations can be obtained as shown in Eq. (3). To characterize the structure system in frequency domain, the FRF between the output and input must be determined from either TMA or EMA. To represent the structure system in modal domain, modal parameters including natural frequencies, mode shapes, and damping ratios must be extracted and can be obtained theoretically or experimentally. Either one of the above system representation to model and simulate the structure can lead to different force prediction approaches. However, the formulation of optimization problem is conceptually the same.

Once the mathematical model has been obtained in step (2), the system response $w(P,t)$ or $a(P,t)$ can then be theoretically determined and expressed as a function of input force contents, such as the magnitude and location. As discussed, there are many solution methods applied to solve for the structural response in step (3). Typically, the estimated structural response $w(P,t)$ or $a(P,t)$ is discretized in time and can be used directly or reconstructed as a vector.

In step (4), base on the least square error method the objective of the formulated optimization problem is to solve for the force contents such that the estimated response has the best match with the measured response. The objective function can be defined as the square errors between the predicted and measured response.

The general optimization algorithms, such as Gauss-Newton method [36], gradient projection method [6] and genetic algorithm (GA) [32,33], can be applied to resolve the optimization problem. Another frequently adopted method to obtain the optimum solution is solved by Bellman's principle of optimality [19], if the response is expressed as a state vector.

Similarly, the frequency domain optimization method can also be developed. Figure 4(b) shows the schematics of this method. The major procedures can be as follows:

(1) measure structural response $\hat{w}(P,t)$ or $\hat{a}(P,t)$ due to unknown force $\hat{f}(P,t)$

(2) take fast Fourier transform (FFT) on $\hat{w}(P,t)$ or $\hat{a}(P,t)$ to get

(3) do structural modeling to obtain mathematical model

(4) predict the structural response $w(P,t)$ or $a(P,t)$

(5) take fast Fourier transform (FFT) on structural response to get $W(P,\omega)$ or $A(P,\omega)$

(6) formulate the optimization problem

(7) solve the optimization problem for the input force $f(P,t)$

This approach is nearly the same as the time domain approach except that the optimization problem is formulated base on the frequency domain response. FFT and IFFT procedures are needed to perform time and frequency transformation vice versa.

## 3. RESPONSE ANALYSIS FOR ARBITRARY STRUCTURES

As discussed for the optimization method in force prediction problem, the system response must be estimated with a proper mathematical model. This section will derive the system response of arbitrary structures subject to the impact and harmonic force excitation, respectively. Both modal analysis and response analysis will be presented.

### 3.1 MODAL ANALYSIS

To perform modal analysis for the arbitrary structure as shown in Eq. (1), the damping term and the general force function can be neglected. The solution of $w(P,t)$ is assumed to be separable in time and of the form

$$w(P,t) = \phi(P)q(t) \tag{5}$$

By the substitution of above equation into Eq. (1), one can get the following variable separated equations

$$\ddot{q}(t) + \omega^2 q(t) = 0 \tag{6}$$

$$L[\phi(P)] = \omega^2 M(P)\phi(P) \tag{7}$$

The general solution of Eq. (6) can be obtained

$$q(t) = Qe^{i(\omega t - \theta)} \tag{8}$$

where $Q$ and $\theta$ are some constants and represent the amplitude and phase angle, respectively. Eq. (7) can be observed as the eigenvalue problem consisting of the differential equations and must be satisfied over domain $D$. The eigenvalue problem of Eq. (7) can be solved to obtain an infinite set of natural frequencies $\omega_k$ and their corresponding eigenfunctions $\phi_k(P)$. If the proportional damping is assumed, and the orthonormal relations of eigenfunctions can be obtained as follows [56]:

$$\int_D M(P)\phi_k(P)\phi_l(P)dD(P) = \delta_{kl} \tag{9}$$

$$\int_D \phi_k(P)L[\phi_l(P)]dD(P) = \omega_k^2 \delta_{kl} \tag{10}$$

$$\int_D \phi_k(P)C[\phi_l(P)]dD(P) = 2\xi_k \omega_k \delta_{kl} \tag{11}$$

where

$$\delta_{kl} = \begin{cases} 0 , & k \neq l \\ 1 & k = l \end{cases} \tag{12}$$

is the Kronecker delta. $\xi_k$ is the modal damping ratio.

## 3.2 RESPONSE ANALYSIS FOR IMPACT FORCE EXCITATION

Assume an ideal impact force acting at $P = P_j$ with amplitude of $F_j$ at $t$=0. The force function can be expressed as follows:

$$f(P,t) = F_j \delta(t)\delta(P - P_j) \tag{13}$$

where $\delta(P - P_j)$ is the unit impulsive function or Dirac's delta function. From expansion theorem [56], the system response can be assumed

$$w(P,t) = \sum_{k=1}^{\infty} \phi_k(P) q_k(t) \tag{14}$$

where $\phi_k(P)$ is known as the structural mode shape function. $q_k(t)$ is called normal coordinates or modal coordinates. By the substitution of Eqns. (13) and (14) into Eq. (1) and with the adoption of the orthonormal relations of eigenfunctions, the equation of motion can be simplified to an infinite set of independent differential equations as follows:

$$\ddot{q}_k(t) + 2\xi_k\omega_k\dot{q}_k(t) + \omega_k^2 q_k(t) = N_k(t)\,, \quad k = 1,2,\cdots \tag{15}$$

where

$$N_k(t) = \int_D \phi_k(P)F_j\delta(t)\delta(P-P_j)dD(P) = F_j\delta(t)\phi_k(P_j) \tag{16}$$

It is noted that Eq. (15) is the modal domain equation, which corresponding system block diagram is shown in Figure 2(d). The solution of modal coordinate $q_k(t)$ in Eq. (15) can be derived

$$q_k(t) = \int_0^t N_k(\tau)h_k(t-\tau)d\tau + e^{-\xi_k\omega_k t}\left(q_k(0)\cos\omega_{d_k}t + \frac{\dot{q}_k(0) + \xi_k\omega_k q_k(0)}{\omega_{d_k}}\sin\omega_{d_k}t\right)\,,\ k = 1,2,\cdots \tag{17}$$

where

$$h_k(t) = \frac{1}{\omega_{d_k}}e^{-\xi_k\omega_k t}\sin\omega_{d_k}t \tag{18}$$

$$\omega_{d_k} = \sqrt{1-\xi_k^2}\quad\omega_k \tag{19}$$

$$q_k(0) = \int_D M(P)\phi_k(P)w(P,0)dD(P) \tag{20}$$

$$\dot{q}_k(0) = \int_D M(P)\phi_k(P)\dot{w}(P,0)dD(P) \tag{21}$$

Eq. (17), in which $h_k(t)$ is the IRF, is the convolution integral equation for modal domain. $w(P,0)$ and $\dot{w}(P,0)$ are the initial displacement and initial velocity of the spatial coordinate $w(P,t)$. For zero initial conditions,

$$q_k(t) = \frac{1}{\omega_{d_k}}\int_0^t N_k(\tau)e^{-\xi_k\omega_k(t-\tau)}\sin\omega_{d_k}(t-\tau)d\tau = \frac{F_j\phi_k(P_j)}{\omega_{k_d}}e^{-\xi_k\omega_k t}\sin\omega_{d_k}t \tag{22}$$

By the substitution of above equation into Eq. (14), the system response at $P = P_i$ can be derived:

$$w(P_i,t) = \sum_{k=1}^{\infty}\frac{\phi_k(P_i)\phi_k(P_j)F_j}{\omega_{d_k}}e^{-\xi_k\omega_k t}\sin\omega_{d_k}t \tag{23}$$

One can also obtain the acceleration response as follows by differentiating $w(P_i,t)$ with respect to time twice

$$a(P_i,t)=\sum_{k=1}^{\infty}\frac{\phi_k(P_i)\phi_k(P_j)F_j}{\omega_{d_k}}e^{-\xi_k\omega_k t}\left[\left(2\xi_k^2\omega_k^2-\omega_k^2\right)\sin\omega_{d_k}t-2\xi_k\omega_k\omega_{d_k}\cos\omega_{d_k}t\right] \tag{24}$$

Taking Fourier Transform upon the above equation, one can get acceleration frequency response as follows:

$$A(P_i,\omega)=\sum_{k=1}^{\infty}\frac{\phi_k(P_i)\phi_k(P_j)F_j}{\left(\omega_k^2-\omega^2\right)+i\left(2\xi_k\omega_k\omega\right)}\left(-\omega_k^2-i2\xi_k\omega_k\omega\right) \tag{25}$$

From Equations (24) and (25), one can observe that the response is functions of system modal parameters as well as the amplitude and location of the impact force.

## 3.3 RESPONSE ANALYSIS FOR HARMONIC FORCE EXCITATION

Consider the arbitrary structure as shown in Eq. (1) and assume that a harmonic point force with amplitude $F_j$ acting on $P=P_j$ at $t=0$. The force function can be expressed as follows:

$$F(P,t)=F_j\delta(P-P_j)e^{i\omega t} \tag{26}$$

where $\omega$ is the excitation frequency. The steady state response of the structure will also be harmonic. From expansion theorem, the displacement response can be assumed:

$$w(P,t)=e^{i\omega t}\sum_{k=1}^{\infty}\phi_k(P)q_k(\omega), \tag{27}$$

By the substitution of Eqns. (26) and (27) into Eq. (1) and with the adoption of the orthonormal relations of eigenfunctions, the modal response can be derived as follows:

$$q_k(\omega)=\frac{F_j\phi_k(P_j)}{\left(\omega_k^2-\omega^2\right)+i\left(2\xi_k\omega_k\omega\right)}\quad,\quad k=1,2,\cdots \tag{28}$$

By the substitution of the above equation into Eq. (27), the system time domain response at $P=P_i$ can be obtained

$$w(P_i,t)=e^{i\omega t}\sum_{k=1}^{\infty}\frac{F_j\phi_k(P_j)\phi_k(P_i)}{\left(\omega_k^2-\omega^2\right)+i\left(2\xi_k\omega_k\omega\right)} \tag{29}$$

By differentiating the above equation twice with respect to time, one can get the acceleration response as follows:

$$a(P_i,t)=-\omega^2e^{i\omega t}\sum_{k=1}^{\infty}\frac{F_j\phi_k(P_j)\phi_k(P_i)}{\left(\omega_k^2-\omega^2\right)+i\left(2\xi_k\omega_k\omega\right)} \tag{30}$$

The acceleration frequency response can also be obtained by the operation of Fourier Transform

$$A(P_i,\omega) = -\omega^2 \sum_{k=1}^{\infty} \frac{F_j \phi_k(P_j)\phi_k(P_i)}{(\omega_k^2 - \omega^2) + i(2\xi_k \omega_k \omega)} \tag{31}$$

Similar to Eqns. (24) and (25) the acceleration response for the impact force excitation, Eqns. (30) and (31) are the acceleration time and frequency domain response for the harmonic force excitation.

## 4. FORCE PREDICTION FORMULATION

This section will present the development of the new approach for the force prediction problem. One can observe that both the time and frequency domain response as shown in Eqns. (24), (25), (30) and (31) reveal a similar format for both the impact and harmonic force excitations. The response is functions of system modal parameters as well as the amplitude and location of the force. The following derivation is for the impact force prediction and can be easily extended to the harmonic force prediction omitted here for brevity.

### 4.1 TIME DOMAIN METHOD

For the proportionally damped structure as discussed subject to an unknown impact force excitation, the structural acceleration response at $P = P_i$ can be measured and denoted as $\hat{a}_i(t)$. The corresponding theoretical acceleration response can be approximated and obtained from Eq. (24).

$$a_i(t) = a(P_i,t) = \sum_{k=1}^{n} \frac{\phi_{k,i}\phi_{k,j}F_j}{\omega_{d_k}} e^{-\xi_k\omega_k t}\left[(2\xi_k^2\omega_k^2 - \omega_k^2)\sin\omega_{d_k} t - 2\xi_k\omega_k\omega_{d_k}\cos\omega_{d_k} t\right] \tag{32}$$

where $\phi_{k,i} = \phi_k(P_i)$ represents the value of mode shape function at $P_i$ location. Although the mode shape function is a continuous function of spatial coordinates, it can be approximated as the mode shape vector with dimension $m \times 1$. $m$ is the number of measurement points in experiments or the discretized points in numerical simulation. $n$ is the number of modes considered in simulation. Therefore, $\phi_{k,i}$ denotes the $i$-th component of the $k$-th mode shape vector. In Equation (32), $\phi_{k,i}$, $\omega_k$, $\xi_k$ and $\omega_{d_k}$, which are known as structural modal parameters, can be determined from experimental modal analysis (EMA) or theoretical modal analysis (TMA). $F_j$ and $P_j$, which are the amplitude and location of the impact force respectively, are unknown parameters to be determined. $\phi_{k,j}$ is related to the force location $j$. Although all of mode shape components are known, index $j$ designating the force location is unknown.

In order to determine the impact force amplitude and its location, the following optimization problem can be defined:

**Objective function:**

$$Q_t = \sum_{r=1}^{N_t} \left[a_i(t_r) - \hat{a}_i(t_r)\right]^2$$

$$= \sum_{r=1}^{N_t} \left[ \left( \sum_{k=1}^{n} \frac{\phi_{k,i}\phi_{k,j}F_j}{\omega_{d_k}} e^{-\xi_k \omega_k t_r} \left[ \left(2\xi_k^2\omega_k^2 - \omega_k^2\right)\sin\omega_{d_k} t_r - 2\xi_k\omega_k\omega_{d_k} \cos\omega_{d_k} t_r \right] \right) - \hat{a}_i(t_r) \right]^2 \quad (33)$$

**Design variables:** $F_j$ , $\phi_{k,j}$ , $k$=1,2,…,$n$ (34)

where $Q_t$ is defined as the sum of square errors between the theoretically estimated $a_i(t)$ and the experimentally measured $\hat{a}_i(t)$ for $N_t$ time steps. The objective is to find $F_j$ and $\phi_{k,j}$ such that the objective function is zero or minimum. Therefore, the amplitude of the impact force $F_j$ and a set of modal vector $\{\hat{D}\}_j$ can be obtained after the resolution of the optimization problem. $\{\hat{D}\}_j$ is defined as follows:

$$\{\hat{D}\}_j = \begin{bmatrix}\phi_{1,j} & \phi_{2,j} & \cdots & \phi_{n,j}\end{bmatrix}^T \quad (35)$$

$\{\hat{D}\}_j$ is the vector containing the $j$-th components of all mode shape vectors at location $P_j$. The modal matrix of the structure is assumed known and can be expressed as follows:

$$[\Phi] = \begin{bmatrix}\{\phi\}_1 & \{\phi\}_2 & \cdots & \{\phi\}_n\end{bmatrix}$$

$$= \begin{bmatrix} \phi_{1,1} & \phi_{2,1} & \cdots & \phi_{n,1} \\ \phi_{1,2} & \phi_{2,2} & \cdots & \phi_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1,m} & \phi_{2,m} & \cdots & \phi_{n,m} \end{bmatrix} = \begin{bmatrix} [G]_1 \\ [G]_2 \\ \vdots \\ [G]_m \end{bmatrix} \quad (36)$$

where

$$[G]_l = \begin{bmatrix}\phi_{1,l} & \phi_{2,l} & \cdots & \phi_{n,l}\end{bmatrix} = \{D\}_l^T, \quad l = 1,2\cdots,m \quad (37)$$

MAC (Modal Assurance Criterion) [57] is usually adopted to evaluate the correlation between the theoretical and experimental mode shape vectors. Here, $MAC_{jl}$ is used to determine the correlation between $\{\hat{D}\}_j$ and $\{D\}_l$ and defined as follows:

$$MAC_{jl} = MAC\left(\{\hat{D}\}_j, \{D\}_l\right) = \frac{\left|\{\hat{D}\}_j^T \{D\}_l^T\right|^2}{\left(\{\hat{D}\}_j^T \{\hat{D}\}_j^*\right)\left(\{D\}_l^T \{D\}_l^*\right)}, \quad l = 1,2\cdots,m \quad (38)$$

When $MAC_{jl}$ is equal or close to 1, both $\{\hat{D}\}_j$ and $\{D\}_l$ have very good correlation, i.e., $P_j = P_l$. Therefore, the location of impact force can be determined at $P_l$.

## 4.2 FREQUENCY DOMAIN METHOD

From Equation (25), the acceleration frequency response at location $P = P_i$ due to the impact force can be theoretically estimated as follows:

$$A_i(\omega) = A(P_i, \omega) = \sum_{k=1}^{n} \frac{\phi_{k,i}\phi_{k,j}F_j}{\left(\omega_k^2 - \omega^2\right) + i\left(2\xi_k\omega_k\omega\right)}\left(-\omega_k^2 - i2\xi_k\omega_k\omega\right) \tag{39}$$

Let $\hat{A}_i(\omega)$ denote the measured acceleration frequency response. Similar to the derivation of the time domain method, the optimization problem can be defined as follows:

**Objective function:**

$$\begin{aligned} Q_\omega &= \sum_{r=1}^{N_\omega}\left[A_i(\omega_r) - \hat{A}_i(\omega_r)\right]^2 \\ &= \sum_{r=1}^{N_\omega}\left[\left(\sum_{k=1}^{n} \frac{\phi_{k,i}\phi_{k,j}F_j}{\left(\omega_k^2 - \omega_r^2\right) + i\left(2\xi_k\omega_k\omega_r^2\right)}\left(-\omega_k^2 - i2\xi_k\omega_k\omega_r^2\right)\right) - \hat{A}_i(\omega_r)\right]^2 \end{aligned} \tag{40}$$

**Design variables:** $F_j$, $\phi_{k,j}$, $k=1,2,\ldots,n$ (41)

where $Q_\omega$ is defined as the sum of square errors between $A_i(\omega)$ and $\hat{A}_i(\omega)$ for $N_\omega$ frequency points. After the resolution of the optimization problem, the amplitude of the impact force $F_j$ and a set of modal vector $\{\hat{D}\}_j$ can be obtained. Similar to the definition of Equation (38) $MAC_{jl}$ can be obtained to determine the location of the impact force.

## 5. IMPLEMENTATION OF FORCE PREDICTION APPLICATIONS

To implement the proposed force prediction model, a computer program can be developed with different high-level computer languages or numerical softwares. This section highlights the programming flow chart and those required mathematical tools. As shown in Figures 4(a) and 4(b), the input data for the force prediction model includes the measured structural response $\hat{a}_i(P,t)$ or $\hat{A}_i(P,\omega)$ and the representation of the mathematical model. The proposed method uses the system modal parameters to represent the structure, i.e. the modal domain modeling approach. Therefore, natural frequencies, modal damping ratios and mode shapes should also be the input data to the program. In order to determine the force amplitude and location, the formulated optimization problems as shown in previous section must be solved. A general-purpose optimization solver is necessary. Most of numerical softwares provide with optimization tools. The prediction program requires two major tasks: (1) to estimate the predicted response and (2) to formulate and solve the optimization problem. The program flowchart is outlined in Figure 5 and briefly summarized as follows:

1. Setup the initial guesses of design variables. The initial values of the impact force amplitude $F_j$ and mode shape information $\phi_{k,j}, k = 1,2,...,n$ must be specified.
2. Construct the predicted response $a_i(t)$ or $A_i(\omega)$. Sections 3.2 and 3.3 detail the necessary equations to predict the acceleration response in the time and frequency domains for both the impact and harmonic forces, respectively.
3. Read experimentally measured response $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$. This work assumes that the accelerometer is applied as the sensor located at $P = P_i$. The acceleration response due to the unknown impact or harmonic force can be measured and input to the prediction program.
4. Formulate the optimization problem. The objective functions as shown in Eqns. (33) and (40) for both the time and frequency domain methods can be formulated and expressed as a function of the force amplitude and location associated with mode shape components.
5. Solve the optimization problem. The force amplitude $F_j$ and mode shape information $\{\hat{D}\}_j$ can be determined.
6. Compare $MAC_{jl}$. With the use of Eq. (38), $MAC_{jl}$ value can be determined and so forth the force location can also be predicted.
7. Print out the prediction results.

It is noted that in solving the optimization problem several factors can affect the effectiveness of optimum solution. They are the number of modes, the number of measurement points, the selection of initial guess of design variables, the number of data points $N_t$ in Eq. (33) or $N_\omega$ in Eq. (40) and the distribution of those data points. The number of modes $n$ should be sufficient to reveal the characteristics of system response such that the predicted response can match the measured response. The number of measurement points $m$ provides the spatial resolution to identify the force location. The initial guess of design variables should be properly selected to avoid finding the local minimum. The general rule in selecting the data points is that as more as data points and as wider as their distribution will be beneficial to accommodate in solving for the optimum. Upon the consideration of solution accuracy and efficiency, the above factors may need to be justified. Besides, a robust optimization solver that can efficiently acquire the accurate optimal solution is needed. The issue can be out of content of this work.

There can be three options based on the selection of theoretical response of $a_i(t)$ or $A_i(\omega)$ and the experimental response of $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$. Both the estimated response and measured response in the objective functions shown in Eqns. (33) and (40) for the optimization problems are needed in order to predict the unknown force amplitude and location. The three options are tabulated in Table 1. The purpose and their applications of the three kinds of combinations are explained as follows:

**Option I:** Use theoretical modal parameters to estimate $a_i(t)$ or $A_i(\omega)$ incorporated with the theoretical response to represent $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$ instead of experimentally measured ones. This approach is simply numerical simulation and mainly for the validation of the developed force prediction methods.

**Option II:** Use theoretical modal parameters to estimate $a_i(t)$ or $A_i(\omega)$ incorporated with the experimentally measured response $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$. The idea of this approach can be applied to complex structures that are not easy or feasible to experimentally determine the modal parameters of the structure. The modal parameters used in Eqns. (33) and (40) are derived from theoretical modal analysis.

**Option III:** Use the experimentally extracted modal parameters to estimate response $a_i(t)$ or $A_i(\omega)$ incorporated with the experimentally measured response $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$. This option is suitable for the case that the experimental modal parameters can be extracted experimentally or that the theoretical modal analysis of structures is not possible.

The proposed method has been numerically and experimentally shown for its feasibility in predicting the unknown impact force acting on a beam structure subject to the impact force [61]. They showed that the impact force amplitude and location can be reasonably predicted.

## 6. CONCLUSIONS

This work proposes a new systematic approach to predict the unknown force amplitude and location simultaneuosly for an arbirary structure subject to the impact and harmonic forces. The proposed approach can be categorized as one of the optimization method. The accelerometer is assumed as the sensor to detect the structural response due to the unknown forces. For other types of sensors applications, the developed systematic approach is also suitable and can be adopted accordingly to develop the prediction method. The system modeling is base on continuous system formulation. The modal domain approach is adopted to represent the system model. Therefore, the mode shape components associated with the force location can be utilized to predict the force location. The developed systematic approach in predicting the force contents is generic. The detailed theoretical formulation of the prediction model is provided and useful for force prediction problems. The developed methodology can not only enhence the force prediction problems for arbitrary structures subject to unknown impact and harmonic forces but also leads to potential applications for other types of forces as well.

## ACKNOWLEDGEMENT

The author greatly thanks the partial supports for this work from National Science Council of Republic of China under the contract number: NSC89-2212-E-020-009.

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Table 1. Three options in evaluating the objective functions

| Option \ response | $a_i(t)$ or $A_i(\omega)$ | $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$ |
| --- | --- | --- |
| Option I | Use theoretical modal parameters to estimate the response $a_i(t)$ or $A_i(\omega)$ | Use theoretical response $a_i(t)$ or $A_i(\omega)$ to represent $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$ |
| Option II | Use theoretical modal parameters to estimate the response $a_i(t)$ or $A_i(\omega)$ | Use experimentally measured response $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$ |
| Option III | Use experimentally extracted modal parameters to estimate $a_i(t)$ or $A_i(\omega)$ | Use experimentally measured response $\hat{a}_i(t)$ or $\hat{A}_i(\omega)$ |

FIGURE LIST:

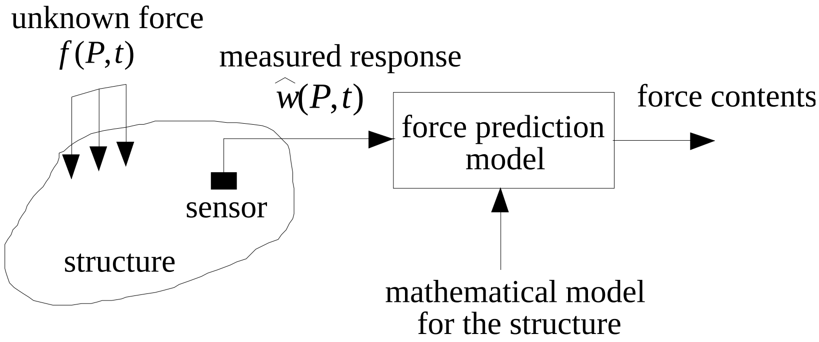



Figure 1. Basic ideas of force prediction problem

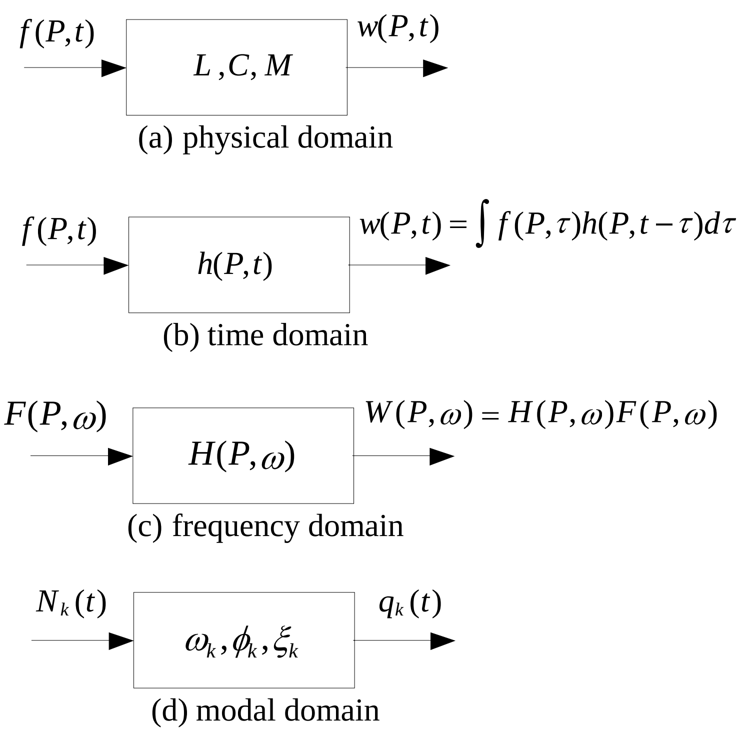



Figure 2. System block diagrams in different domains

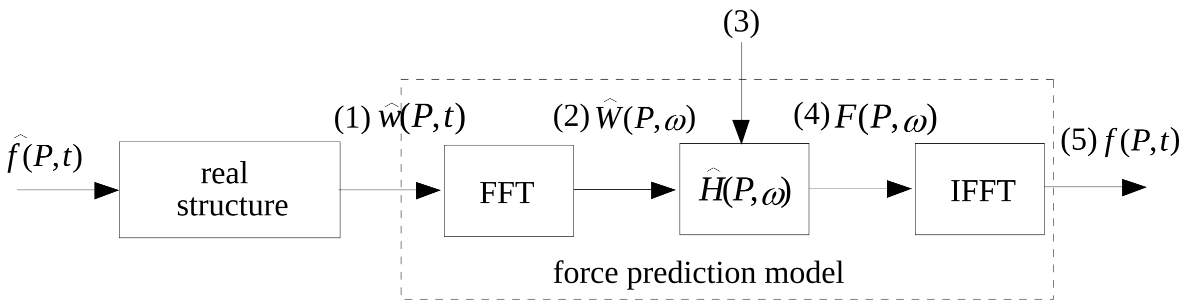
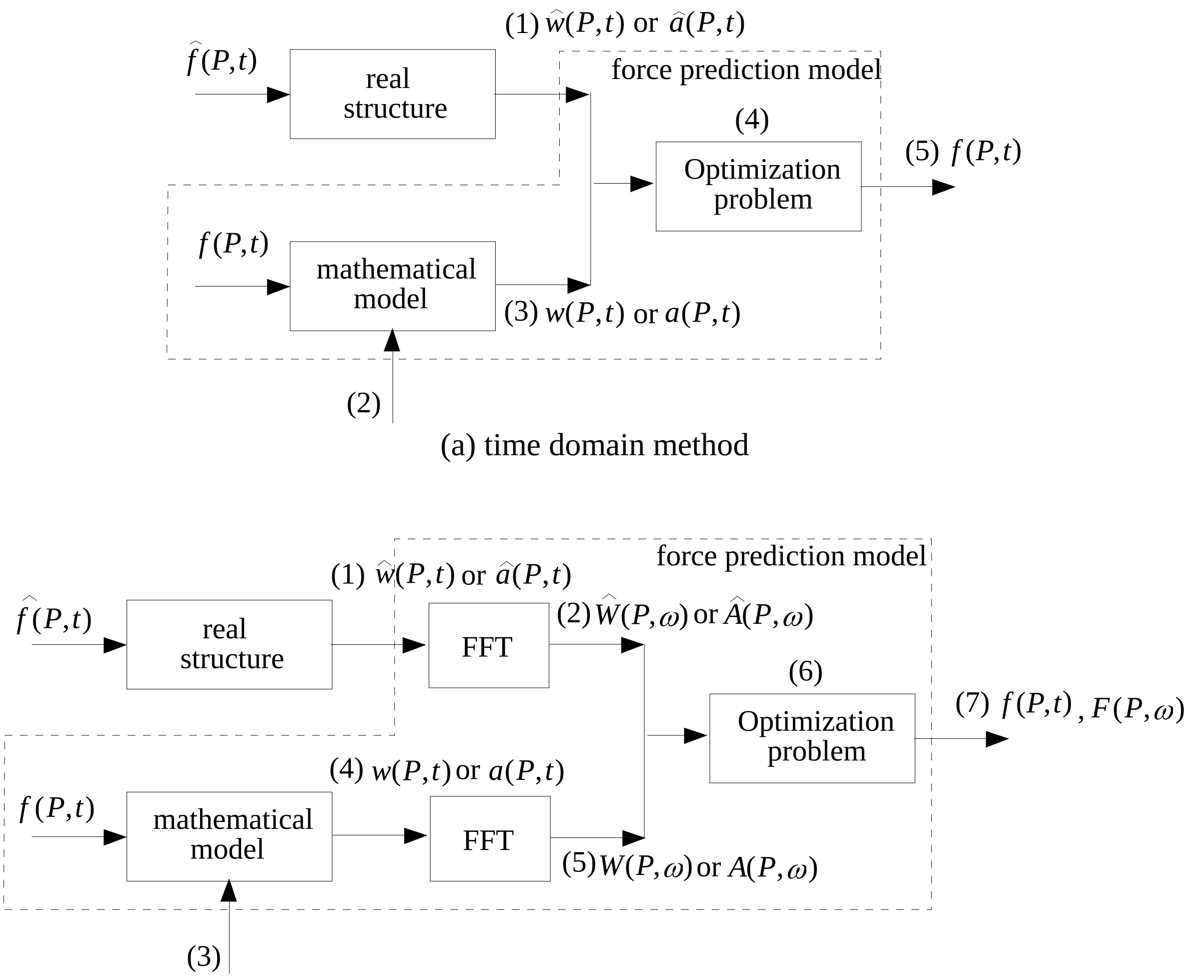



Figure 3. Schematics of force prediction model for direct method




(a) time domain method



(b) frequency domain method

Figure 4. Schematics of force prediction model for optimization method

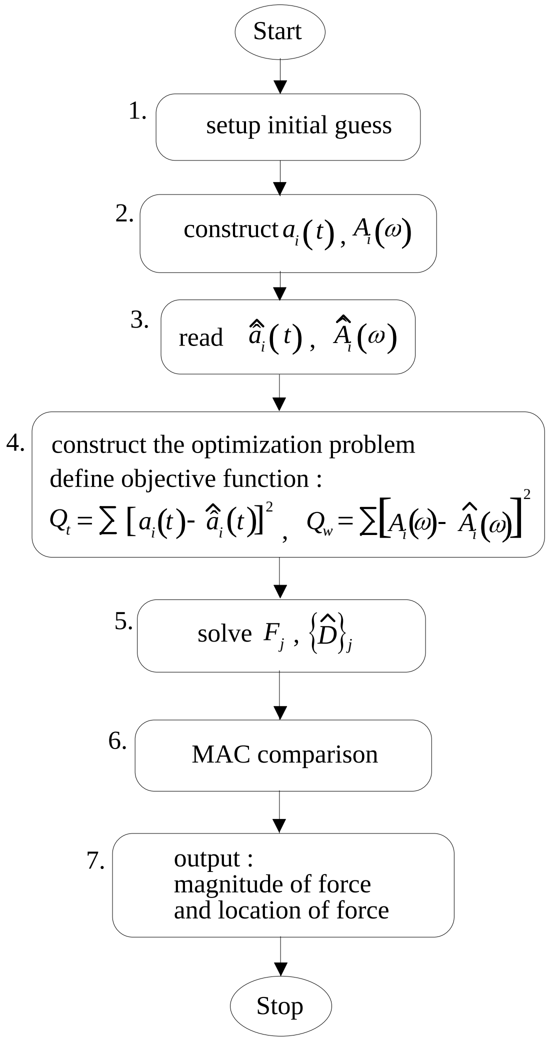



Figure 5. Program flow chart