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Optimal Placement of Piezoelectric Actuators for Active Structural Acoustic Control

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ABSTRACT: This paper presents a general formulation of the optimization problem for the placement and sizing of piezoelectric actuators in adaptive LMS control systems. The selection of objective function, design variables and physical constraints are separately discussed. A case study for the optimal placement of multiple fixed size piezoelectric actuators in sound radiation control is presented. A solution strategy is proposed to calculate the applied voltages to piezoelectric actuators with the use of linear quadratic optimal control theory which is to simulate the LMS feedforward control algorithm. The location of piezoelectric actuators is then determined by minimizing the objective function, which is defined as the sum of the mean square sound pressure measured by a number of error microphones. The optimal location of piezoelectric actuators for sound radiation control is determined and shown to be dependent on the excitation frequency. Particularly, the optimal placement of multiple piezoelectric actuators for on-resonance and off-resonance excitation is presented. The results show that the optimally located piezoelectric actuators perform far better sound radiation control than arbitrarily selected ones. This work leads to a design methodology for adaptive or intelligent material systems with highly integrated actuators and sensors. The optimization procedure also leads to a reduction in the number of control transducers.

INTRODUCTION

PIEZOELECTRIC actuators have been widely used in structural sound and vibration control. Wang and Rogers (1991a, 1991b) presented the theoretical analysis of the mechanics of piezoelectric actuators, and Wang et al. (1991) demonstrated their potential as transducers in structural sound control. However, the proper selection of number and location of piezoelectric actuators is critical to efficiently control structural sound radiation. Therefore, the determination of the optimal placement and number of piezoelectric actuators in sound radiation control is an important and interesting issue.

However, previous works on optimal placement of actuators are mostly concerned with vibration control and particularly for feedback control systems with the use of traditional force transducers, such as point force shakers (Norris and Skelton, 1989; Chang and Soong, 1980; Hamidi and Juang, 1981; Juang and Rodriguez, 1979; Chen and Seinfeld, 1975). For state feedback control, a state space equation is first constructed to represent the system model, and a performance index, which is a quadratic form in the state and control effort, can then be defined. Finally, the optimal location is to be determined by minimizing the performance index. Only a few literatures deal with the optimal location of distributed actuators, which are widely used in conjunction with so-called "smart" structures. Jia (1990) studied the optimal position of piezoelectric actuators for beam vibration control by adopting the independent modal space control approach (IMSC). Jia showed that the optimal location and size of piezoelectric actuators can be found by minimizing an objective function which can be either the structural response, control effort, residual response, spillover effect or combinations of all/any of these variables. However, Jia's work is limited to consider only one-dimensional vibration control.

Adaptive feedforward control, on the other hand, has been adopted for structural sound radiation control in recent years (Gibbs and Fuller, 1992; Burdisso and Fuller, 1992; Simpson et al., 1992). The control algorithm is flexible because it is not as crucial as feedback approaches to ac-

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curately model the system response. The adaptive feedforward controller can learn the system parameters by itself and converge to the optimal solution using various "training" approaches. However, little work has been discussed on the optimal placement of actuators, particularly distributed in nature, for feedforward control. This paper is thus concerned with the formulation of the optimization problem for the placement of piezoelectric actuators in feedforward control systems, in particular for the ASAC technique.

In this paper, a general formulation for the optimal placement of piezoelectric actuators in a feedforward control approach is first presented and then applied to a typical sound radiation control system using piezoelectric actuators and microphone sensors. A baffled, simply-supported, rectangular plate as shown in Figure 1 is considered as an idealized system. The plate is harmonically excited by a primary source (point force), and piezoelectric actuators are applied to control the plate vibration in order to reduce the associated sound radiation into a free field. The objective here is to determine the optimal location of piezoelectric actuators such that the sound pressure measured from the error microphones can be most efficiently reduced (i.e., with the lowest actuator power and/or number of actuators). A solution strategy is proposed to calculate the applied voltages to piezoelectric actuators with the use of linear quadratic optimal control theory (Wang, 1991). The location of the piezoelectric actuator(s) is then determined by minimizing the objective function, which is defined as the sum of the mean square sound pressure measured by a number of error microphones. The optimal locations for multiple piezoelectric actuators, up to three, were considered. The results show that the optimally placed actuators achieve a far better reduction of sound radiation than actuators whose positions are arbitrarily chosen.

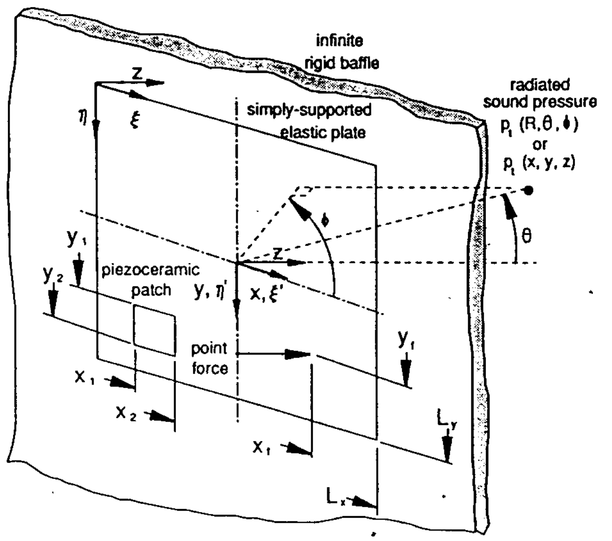

Figure 1. The arrangement and coordinates of the baffled simply-supported plate.

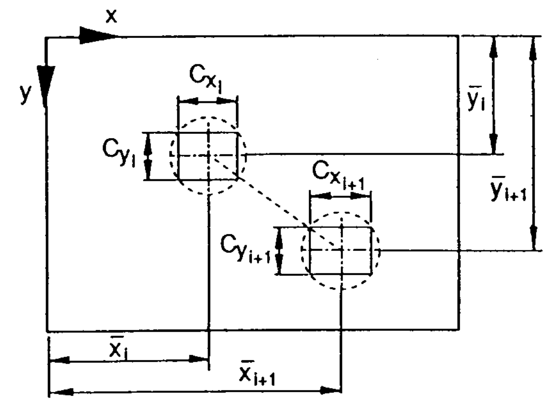

Figure 2. Illustration of design variables.

MATHEMATICAL FORMULATION FOR OPTIMIZATION PROBLEM

Design Variable

As shown in Figure 2, the optimal placement of the jth piezoelectric actuator located inside the boundaries of the plate can contain five variables, $\bar{x}_i$, $\bar{y}_i$, C_{xi}, C_{yi} and V_i. The variables C_{xi} and C_{yi} denote the size of the ith piezoelectric actuator, while $\bar{x}_i$ and $\bar{y}_i$ denote the central location of the actuator, and V_i is the applied voltage to the piezoelectric actuator. If the primary source is known, and piezoelectric actuators are used as control sources, then the total radiated sound pressure into the free field can be shown as follows:

$$P_t = P_t(\bar{x}_i, \bar{y}_i, C_{xi}, C_{yi}, V_i) \quad i = 1, \ldots, N_c \tag{1}$$

The complete derivation of p_t is shown in Wang (1991) and omitted here. As discussed from previous works (Wang et al., 1991), V_i can be calculated from the linear quadratic optimal control theory (LQOCT). The total radiated sound pressure can then be written as follows:

$$P_t = P_t(\bar{x}_i, \bar{y}_i, C_{xi}, C_{yi}, V_i(\bar{x}_i, \bar{y}_i, C_{xi}, C_{yi})) \quad i = 1, \ldots, N_c \tag{2}$$

However, if the size of the piezoelectric actuators was first fixed, then the total radiated sound pressure becomes

$$p_t = p_t(\bar{x}_i, \bar{y}_i, v_i(\bar{x}_i, \bar{y}_i)) \quad i = 1, \ldots, N_c \tag{3}$$

The design variables, $\bar{x}_i$, $\bar{y}_i$, C_{xi}, C_{yi} and V_i can be properly selected based upon the concern of the size or location (or both) of the piezoelectric actuator and the control effort (i.e., the voltages or power required for piezoelectric actuators).

Objective Function

There are various choices for the objective function. Wang et al. (1991) chose the integral of the square of radiated sound pressure over a hemisphere of the radiating field as the cost function. However, such a cost function, in practice, is not useful. Wang and Fuller (1991) constructed a cost function which is the sum of the mean square radiated sound pressures measured by a limited number of microphones. The consideration of the above two types of objective functions is of particular interest in sound radiation control. Since the sound radiation is strongly coupled with the structural vibration, the objective function may also be chosen as the sum of the mean square plate acceleration measured by a limited number of accelerometers, or the integral of the square of the plate acceleration over the vibrating surface. The possible candidates for the objective function used in sound radiation control can be as follows:

1. Distributed pressure sensors

$$\Phi_p = \frac{1}{R^2}\int_s |p_t|^2 ds = \int_0^{2\pi}\int_0^{\pi/2} |p_t|^2 \sin\theta d\theta d\phi \quad (4)$$

2. Discrete pressure sensors

$$\Psi_p = \sum_{i=1}^{N_{mike}} |p_{t_i}(R_i,\theta_i,\phi_i)|^2 \quad (5)$$

6. Distributed accelerometer sensors

$$\Phi_w = \int_A |\ddot{w}_t|^2 dA = \int_0^{L_y}\int_0^{L_x} |\ddot{w}_t|^2 dxdy \quad (6)$$

4. Discrete accelerometer sensors

$$\Psi_w = \sum_{i=1}^{N_{acc}} |\ddot{w}_{t_i}(x_i,y_i)|^2 \quad (7)$$

where N_{mike} and N_{acc} are the number of microphones and accelerometers, respectively. It is noted that Φ_p and Φ_w are measured by ideal distributed sensors, which may not be practical in reality; however, Φ_p and Φ_w represent the power of sound radiation and energy density of out-of-plane structural vibration, respectively. They can be used as an index of control effectiveness. For practical applications, Ψ_p and Ψ_w are the alternative options. A reasonable number and location of sensors shall be selected to reflect the actual system response, such that an optimal solution can be found without losing the global nature of the problem. In effect, the discrete sensors should approach a form of numerical integration of the objective function associated with the distributed sensors to be truly global.

Design Constraints

The design constraints have to be specified to confine the design variables within a reasonable range. These design constraints are necessary for providing a reasonable result by maintaining the rectangular shape of the piezoelectric actuators, locating actuators inside the plate boundaries, avoiding overlapping between actuators, and operating actuators within the working voltage range. It is noted that the constraint set 3 below for avoiding overlapping is conceptually sketched in Figure 2. For the rectangular-shaped piezoelectric actuators, as shown in Figure 2, the constraint sets are listed as follows:

1. To maintain the piezoelectric actuator, a rectangular shape:

$$0 < C_{x_i} \le L_x/2 \qquad 0 < C_{y_i} \le L_y/2 \quad (8)$$

2. To maintain the piezoelectric actuator inside of the plate:

$$\bar{x}_i - C_{x_i}/2 \ge 0 \qquad \bar{x}_i + C_{x_i}/2 \le L_x \qquad \bar{y}_i - C_{y_i}/2 \ge 0 \qquad \bar{y}_i + C_{y_i}/2 \le L_y \quad (9)$$

3. To avoid overlapping between piezoelectric actuators:

$$\bar{x}_{i+1} - \bar{x}_i > 0$$

$$\bar{y}_{i+1} - \bar{y}_i > 0$$

$$[(\bar{x}_{i+1} - \bar{x}_i)^2 + (\bar{y}_{i+1} - \bar{y}_i)^2]^{1/2} - \frac{1}{2}[(C_{xi}^2 + C_{yi}^2)^{1/2} + (C_{x_{i+1}}^2 + C_{y_{i+1}}^2)^{1/2}] > 0 \quad (10)$$

4. To specify the working range of piezoelectric actuators:

$$|V_i| \le 150 \text{ (volt } p - p) \quad (11)$$

Note that the control power to the actuators is not an optimization variable. However, constraint 4 above ensures that the piezoelectric actuator is within a working range.

APPLICATION TO OPTIMAL PLACEMENT OF PIEZOELECTRIC ACTUATORS

For a simple application of the previous theoretical formulation to sound radiation control, the size of the piezoelectric actuators is assumed fixed, i.e., $C_{xi} = C_{yi}$ = constant. The applied voltage to the ith piezoelectric actuator, V_i, can be calculated from LQOCT (Wang, 1991).

Only the optimal location of piezoelectric actuators $\bar{x}_i$ and $\bar{y}_i$ will be determined. The objective function is chosen as the sum of the mean square sound pressure measured by a number of microphones in the far-field. Therefore, the optimization problem can be written as:

Objective function:

$$\Psi_p = \Psi_p(\bar{x}_i, \bar{y}_i, V_i(\bar{x}_i, \bar{y}_i)) = \sum_{j=1}^{N_{mike}} |P_{t_j}(R_j, \theta_j, \phi_j)|^2$$

$$i = 1, \ldots, N_c \quad (12)$$

design variables:

$$(\bar{x}_i, \bar{y}_i) \quad i = 1, \ldots, N_c \quad (13)$$

design constraints:

constraint sets 2, 3 and 4 as shown in Equations (9–11).

The design variables are to be determined by minimizing the objective function subjected to a set of design constraints. Now, a suitable optimization algorithm must be adopted to solve the optimal solution.

OPTIMIZATION ALGORITHM

An IMSL subroutine N0ONF (IMSL, 1989) for solving a general nonlinear programming problem using the successive quadratic programming algorithm and a finite difference gradient technique was adopted to calculate the optimal solution. The algorithm requires a high accuracy arithmetic in estimating the gradient. The central finite difference method was then applied to approximate the gradient by adopting the IMSL CDGRD subroutine (IMSL, 1989).

SOLUTION STRATEGY

To solve the above optimization problem, a solution strategy was developed. The flow chart of solution strategy is shown in Figure 3. The procedure to solve the problem is to first set up the initial guess of the optimal central location of the ith actuator, $(\bar{x}_i)_k$, $(\bar{y}_i)_k$, where k denotes the number of iterations. The following steps are then performed:

1. Utilize the linear quadratic optimal control theory (LQOCT) (Wang, 1991) to obtain the applied voltages, $(V_i)_k$, to actuators at the current location, $(\bar{x}_i)_k$, $(\bar{y}_i)_k$.
2. Evaluate the objective function and constraints at the current location, $(\bar{x}_i)_k$, $(\bar{y}_i)_k$.
3. Evaluate the gradients of the objective function and constraints at the current location, $(\bar{x}_i)_k$, $(\bar{y}_i)_k$.
4. Employ an optimization algorithm, N0ONF, to update the optimal location, $(\bar{x}_i)_{k+1}$, $(\bar{y}_i)_{k+1}$.

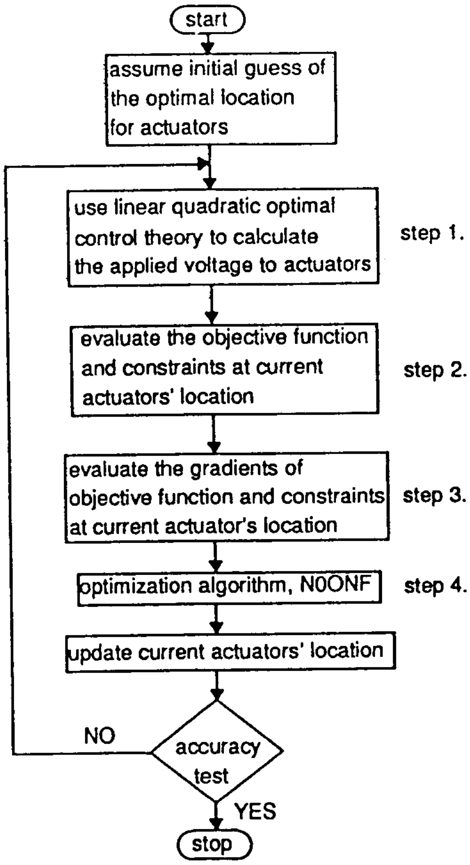

Figure 3. Flow chart of solution strategy.

5. Stop the procedure if the results pass the accuracy test; otherwise, update the current optimal location of actuators and repeat the above steps.

It is noted that the design variables, the central location of the piezoelectric actuators $(\bar{x}_i, \bar{y}_i)$, were normalized by the plate length and width (L_x, L_y), respectively, such that the design variables will be relocated between zero and one. This normalization process will benefit the solution process of the optimization problem.

LQOCT FOR SOLVING APPLIED VOLTAGES TO ACTUATORS

Lester and Fuller (1990) presented an optimization algorithm to obtain the minimum for a linear quadratic function. Wang et al. (1991) had shown the use of linear quadratic optimal control theory (LQOCT) to determine the applied voltages to minimize the selected objective function, which is quadratic. Here, the LQOCT is adopted to solve the voltages independently. One of the advantages is that the optimal voltages can be always determined whenever the location of the actuators is known. The other reason to evaluate the optimal voltage separately is that the order of voltage and the central location of the piezoelectric actuator is

Table 1. Plate specification.

$E = 207 \times 10^9$ (N/m²)	$\nu = 0.292$	$L_x = 0.38$ (m)
$\rho_p = 7870$ (kg/m³)	$h = 1.5875$ (mm)	$L_y = 0.30$ (m)

not consistent mathematically, even after the normalization process. Hence, upon consideration of the numerical difficulty and the number of design variables, it is beneficial to obtain the optimal voltages using LQOCT separately from solving the optimization problem.

ANALYTICAL RESULTS

Table 1 shows the physical properties of the simply-supported plate used for the following simulations. The structural disturbance was assumed to be a point force with magnitude of $F_1 = 1N$ and located at $x_{f1} = 0.08$ m, $y_{f1} = 0.08$ m.

Nine error microphone sensors, whose locations are tabulated in Table 2 and shown in Figure 4, were used; therefore, the objective function defined in Equation (12), which is the sum of mean square measured pressure, can be constructed. This number of microphones is clearly based on the consideration of computing time and a reasonable approximation to the continuous integral of pressure over the complete radiation hemisphere. Too few microphones will not reveal the actual system global radiation response. On the other hand, too many microphones will require excessive computing effort to solve the optimization problem. The microphones located in the far-field are arranged five in a row across the central line of the plate in both the x- and y-directions, as shown in Figure 4. The size of the piezoelectric actuators is fixed, $C_{xi} = 0.06$ m and $C_{yi} = 0.04$ m. The location and applied voltages of the piezoelectric actuators are to be determined.

SUB-REGION SEARCH METHOD

The determination of the optimal location of piezoelectric actuators is dependent on the excitation frequency. A different excitation frequency will lead to a different optimal location. For a particular frequency of excitation, all of the plate modes can be excited; however, only the plate modes near the excitation frequency will contribute significantly to the plate response as well as the sound radiation. It is clear that if the plate was excited near the (1,1) mode, the plate response will be shown as a convex surface. Similar characteristics can be found for the objective function. If the plate was excited at 87 Hz near the (1,1) mode, and the central location of the piezoelectric actuator was varied and moved around the plate, then the objective function and the applied voltage could be calculated from LQOCT and plotted, as shown in Figure 5. Because the objective function is shown as a convex surface, an optimal location for the piezoelectric actuator can always be found to guarantee global minimum. It is also noted from Figure 5 that high control voltages are required for the actuator located near the corner of the plate to achieve sound radiation control. The actuator with the minimum control effort is located at about the same position as the actuator with the minimum objective function.

Table 2. Location of error microphones.

The *i*th Microphone	(R, θ, ϕ)
1	(1.8, 75°, 180°)
2	(1.8, 45°, 180°)
3	(1.8, 0°, 0°)
4	(1.8, 45°, 0°)
5	(1.8, 75°, 0°)
6	(1.8, 75°, 90°)
7	(1.8, 45°, 90°)
8	(1.8, 45°, 270°)
9	(1.8, 75°, 270°)

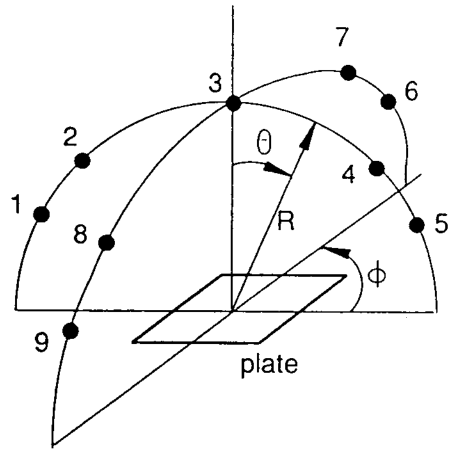

Figure 4. *Location of error microphones.*

As shown in Figure 6, for an excitation frequency $f = 357$ Hz near the (3,1) mode, the objective function and the applied voltage reveal a shape close to the (3,1) mode distribution. There is more than one minimum for the objective function; in fact, there is one local minimum at each division separated by nodal lines. This characteristic, related to the plate mode shapes, is similar to what has been shown in Figure 5. If the actuator is located near the nodal line of the plate mode, then sound radiation control is not effective—at least for on-resonance excitation—because of high control voltage and small control authority. The above discussions seem to be contrary to the previous work by Dimitriadis et al. (1991). They claimed that the optimum boundary of the piezoelectric actuators may be along the nodal lines of selected modes to be excited. In the sense of vibration excitation and the characteristic of induced moments by the piezoelectric actuators, their statement may be intuitive. Jia (1990) had also shown that optimal location

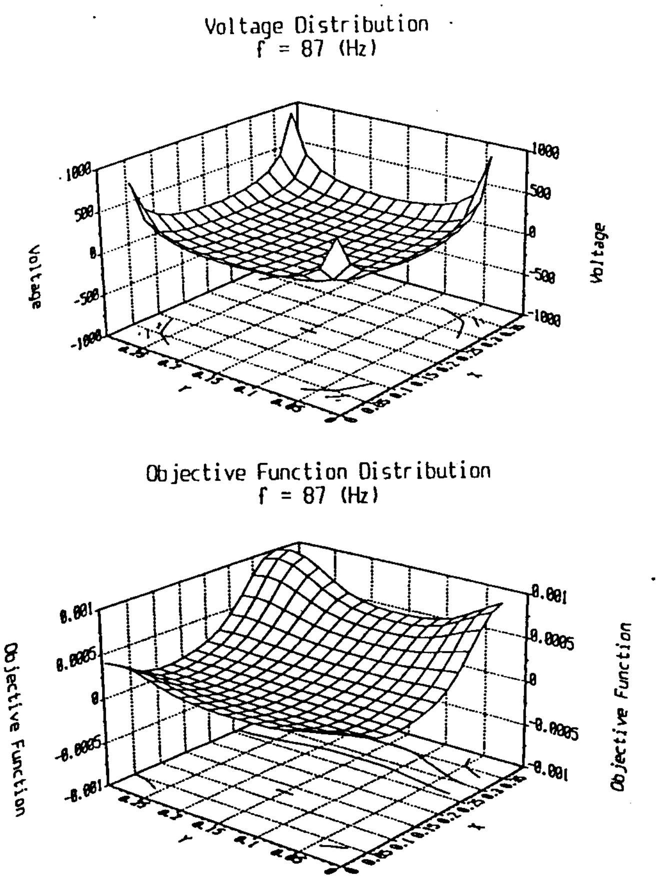

Figure 5. Distribution of objective function and control voltage for f = 87 Hz.

and dimension of piezoelectric actuators to effectively suppress the single mode response of beam vibration is to maximize the length of the piezoelectric actuator covering a whole lobe. Here, in our work, in terms of the attenuation of total sound radiation, the optimal location of "finite" length of piezoelectric actuators is determined upon a compromise to equally reduce several plate modes instead of just one. Therefore, the optimal location of the actuator is found to be away from the nodal line.

Figure 7 shows plots similar to Figures 5 and 6 except that the excitation frequency, $f = 272$ Hz, is between the (2,1) and (3,1) modes. One can see that those distributions become complex and result from the combination of several plate modal responses. Again, there are multiple minima; this makes the global minimum difficult to find using the optimization procedure. However, according to the characteristics shown in Figures 5 and 6, a sub-region search method, which comes from the nature of the objective function distribution similar to that of the plate mode shapes, can be proposed. This search technique involves subdividing the plate into several cells based on the nodal lines of the plate mode shapes, which are set up to be the bound of the locations for the actuators. In other words, in addition to the design constraints illustrated previously, the upper and lower bounds of the locations for actuators can also be specified according to the nodal lines associated with the plate mode shapes.

OPTIMAL LOCATION OF ONE ACTUATOR FOR DIFFERENT EXCITATION

As discussed previously, the optimal location of piezoelectric actuators is dependent on the excitation frequency due to the variation of the modal transfer function in frequencies. The optimal location of the actuator has been of interest for many concerns. As discussed by Juang and Rodriguez (1979), to control a single mode of beam vibration, there are multiple optimal locations for one actuator in high-mode control. If several modes contribute to the response simultaneously, and only a few actuators are applied, then the optimal location will be much different from that for single mode control. The feedforward control approach adopted here is to minimize the objective function, which is the mean square of sound pressure measured by error microphones, and thus to control all of the modal contributions at the same time. Therefore, the optimal location and applied voltage of the actuator are solved under a compromise to eliminate the significant modal responses; however, this compromise will probably incur spillover to other

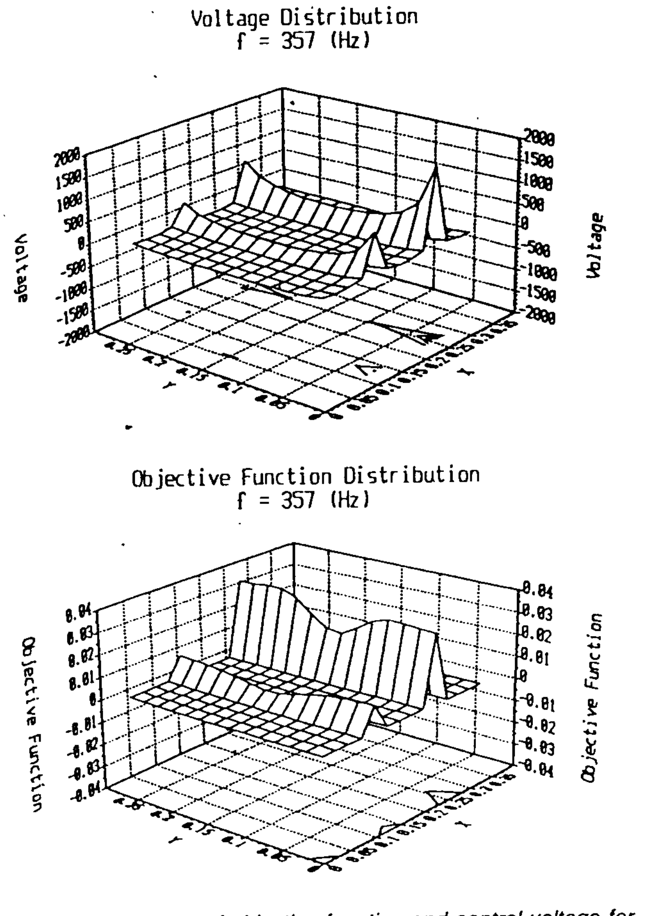

Figure 6. Distribution of objective function and control voltage for f = 357 Hz.

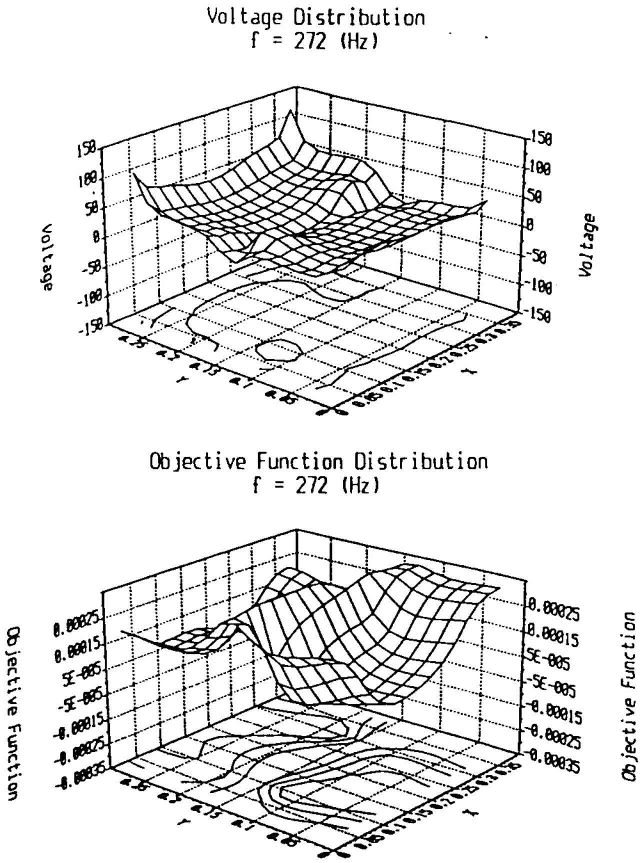

Figure 7. Distribution of objective function and control voltage for f = 272 Hz.

higher modes, which do not radiate efficiently to the error microphones, causing an increase in plate response.

When one piezoelectric actuator is considered, the normalized optimal central location of the piezoelectric actuator and the applied voltage to the actuator can be tabulated (Table 3), as well as the reduction of the objective function, radiated power and pressure modal amplitude. Table 3 is shown for different excitation frequencies varying from 87 Hz to 357 Hz, i.e., between the (1,1) and (3,1) modes. The underlined values are for on-resonance excitation, such as 87 Hz near the (1,1) mode, 190 Hz near the (2,1) mode, and 357 Hz near the (3,1) mode. One can see that the optimal central location of the actuator is located at about one-third of the plate length and width, in the left-bottom quadrant of the plate, i.e., the same quadrant where the point force disturbance is located. The results match well those found in sub-region search methods; therefore, the optimization procedure is suitable to search for the optimal location of piezoelectric actuators. As the excitation frequency increases, the optimal central location of the actuator moves toward the corner of the plate. This can be understood by realizing that when the excitation frequency increases, the contribution of higher modes becomes significant, and thus the optimal location of the actuator is placed where it can couple into all higher mode responses. In applying one actuator—for example, $f = 87$ Hz near the resonance of the (1,1) mode—the actuator attempts to control all of the significant radiating mode responses, including the (1,1) and (2,1) modes, instead of just the (1,1) mode. Therefore, the optimal location is determined under a compromise to eliminate the significant modes; however, as one can see in Table 3, there is spillover to higher modes, such as (3,1), (4,1) and (5,1). This result indicates that the optimal location of a single actuator is such that it eliminates the significant modal response near the excitation frequency; however, this will result in spillover to higher modes, which ultimately limits the amount of attenuation.

On the other hand, when the excitation frequency increases, for example, $f = 357$ Hz near the (3,1) mode excitation, the radiation from the (3,1) mode is controlled, as well as the (1,1) and (2,1) modes, but with less reduction.

Table 3. Results for one actuator with different excitation frequencies.

Excitation Frequency f (Hz)	Normalized Optimal Central Location of Actuator		Optimal Voltage (volt) V	Reduction of Objective Function Ψ_p (dB)	Reduction of Radiated Power Φ_p (dB)	Reduction of Pressure Modal Amplitude (dB)				
	$\bar{x}/L_x$	$\bar{y}/L_y$				(1,1)	(2,1)	(3,1)	(4,1)	(5,1)
87	0.3456	0.3933	51.15	171.22	101.51	56:55	12.61	−2.47	−18.94	−27.62
100	0.3442	0.3919	49.89	147.27	59.61	30.94	17.55	−3.53	−18.54	−25.50
120	0.3413	0.3892	47.73	134.37	61.10	22.34	15.54	−1.58	−18.10	−27.45
140	0.3378	0.3861	45.37	128.43	53.33	17.69	18.36	−0.88	−17.63	−27.31
165	0.3324	0.3813	42.26	120.65	46.96	13.87	24.55	0.16	−16.77	−25.24
190	0.3255	0.3757	39.13	138.03	65.11	11.17	50.18	1.47	−15.74	−26.82
220	0.3150	0.3679	35.60	111.69	38.47	8.83	22.61	3.45	−14.52	−26.32
245	0.3043	0.3609	33.08	119.53	46.54	7.41	18.04	5.60	−12.66	−25.83
270	0.2920	0.3541	31.11	99.02	34.11	6.36	15.61	8.46	−10.76	−25.13
300	0.2765	0.3477	29.55	99.69	26.84	5.50	14.02	13.44	−7.94	−23.99
330	0.2631	0.3456	28.67	100.46	29.91	4.99	13.27	21.63	−4.74	−22.68
357	0.2548	0.3416	53.34	148.75	78.26	4.62	12.51	70.28	−1.22	−21.23

Table 4. On-resonance excitation, f = 357 (Hz), near (3,1) mode.

Case	The *i*th Actuator	Optimal Location		Optimal Voltage V_i (volt)	Reduction of Objective Function Ψ_p (dB)	Reduction of Radiated Power Φ_p (dB)
		$\bar{x}/L_x$	$\bar{y}_i/L_y$			
One Actuator	(1)	0.2548	0.3416	53.34	148.75	78.26
Two Actuators	(1)	0.2511	0.2852	100.54	189.06	66.70
	(2)	0.5504	0.6143	27.97		
Three Actuators	(1)	0.2499	0.2846	56.65	192.78	66.56
	(2)	0.5419	0.6052	50.61		
	(3)	0.8223	0.2198	61.09		
Three Actuators (Lab)	(1)	0.167	0.5	24.11	61.60	60.28
	(2)	0.5	0.833	25.73		
	(3)	0.833	0.167	10.78		

There is still spillover to the higher modes, but there is less than at the lower frequency excitations. This is due to the fact that the (1,1) and (2,1) modes, having high radiation efficiency, can contribute a larger amount of sound radiation to the far-field, even though the (3,1) mode is dominant on the plate due to the excitation frequency. Therefore, the optimal location is determined from a result of compromise to efficiently eliminate the most significant radiating modes, i.e., the (1,1), (2,1) and (3,1) modes in this case. However, this effort causes spillover to higher modes, such as (4,1) and (5,1) modes, which have lower radiation efficiency (Wallace, 1972). It is also noted from Table 3 that for on-resonance excitation, the reduction of radiated power is generally larger, and the control effort (voltage) is higher than for those cases with off-resonance excitation. This result is due to the fact that modes on resonance always contribute considerably more to the modal response and thus require more control effort.

OPTIMAL LOCATION OF MULTIPLE ACTUATORS FOR DIFFERENT EXCITATION FREQUENCIES

On-Resonance Excitation, $f = 357$ Hz, Near (3,1) Resonant Mode

Table 4 shows the optimal central location and applied voltages of piezoelectric actuators, as well as the reduction of the objective function and radiated power for an excitation frequency of $f = 357$ Hz. As one can see, although the reduction of objective function increases when more actuators are applied, the amount of attenuation of radiated power is not always increased. This means that the optimization algorithm does work to find a better solution. However, the minimization of the objective function, which is the sum of mean square pressures measured by error microphones, will not guarantee the reduction of radiated power due to the spillover of sound pressure to locations other than the position of error microphones. In terms of the attenuation of radiated power, to properly locate one actuator in controlling sound radiation is more effective than to use two or three actuators when a set of microphones are used as error sensors, as it reduces unnecessary spillover.

Figure 8 shows the radiation directivity pattern for the excitation frequency $f = 357$ Hz. The point force disturbance input and piezoelectric actuator patches are sketched to scale at the top of Figure 8. The disturbance response, denoted by a solid line, indicates a monopole-like response but is nonuniform, and evidently shows the existence of the (3,1) mode and a significant (1,1) modal contribution. The optimal location of one actuator is at the left-bottom quadrant of the plate, similar to the primary source. The residual pressure field is shown to be a combination of the (3,1) and (1,1) modes.

For two-actuator control as shown at the top of Figure 8, the first optimally located actuator is somewhat near the optimal location for one-actuator control, and the second one

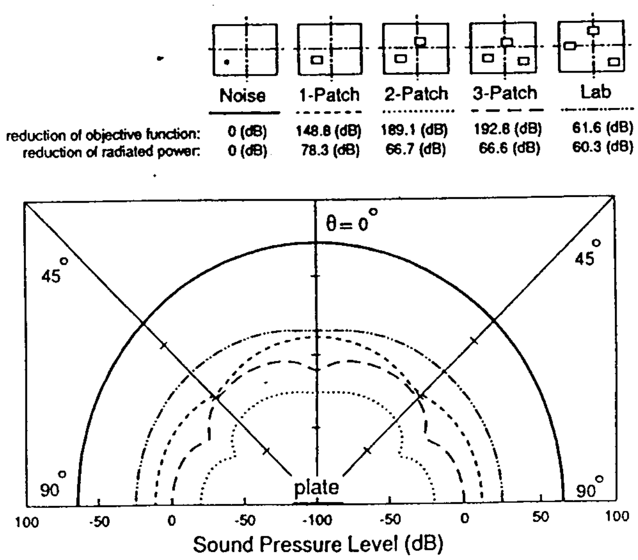

Figure 8. Radiation directivity pattern for f = 357 Hz.

is located at the upper-right quadrant of the plate near the central line. As shown in Table 4, the reduction of objective function is increased, but the reduction of radiated power is decreased. A result such as this implies that more error microphones need to be used due to spillover effects to unobserved radiation points. Nevertheless, the sound pressure level along the central line of the plate in both the x- and y-direction is less than that using one actuator, and it exhibits a combination of the (4,1) and (1,1) modes. It can be seen that there are dips at $\theta = 0°, 75°$ for $\phi = 0°$, and $75°$ for $\phi = 180°$, where the error microphones are located. For three-actuator control, the optimal locations of the first two actuators close to those of two-actuator control, and the third one is located at the bottom-right quadrant of the plate. Again, the objective function has been further minimized. As shown in Figure 8, the dips at $\theta = 0°, 75°$ for $\phi = 0°$ and $75°$ for $\phi = 180°$ are enhanced, but the reduction of radiated power has not increased.

An interesting feature can be observed from the above results, indicating that a one-by-one search method may be used to solve for the location of the successive actuator. The idea is to first find an optimal location for one-actuator control, and then to find a second optimal location for two-actuator control with the same location for the first actuator, and so on. With this searching technique, the computing time can be largely reduced since it costs less to optimize a reduced-parameter problem than a full-parameter problem. This method was attempted; however, the results were not encouraging since the selected objective function cannot be attenuated further due to numerical difficulty (even though a double precision number was used in the program) while an additional actuator was considered. The authors believe that if the objective function is reconstructed as the radiated power rather than the mean square pressure, the one-by-one search method would be appropriate and could reduce significant computing effort for multiple actuator control. Also, it appears that each actuator is optimally configured for separate modes, since their locations stay the same. Hence, an independent optimization procedure for each mode and its associated radiation might be attempted.

Also shown in Figure 8 is an arbitrary selection of multiple actuators located at one-sixth of the plate length or width (denoted lab arrangement). This arrangement is assigned to control the low modal number excitation based upon the nature of the plate mode shapes and was used in companion experiments (Clark and Fuller, 1992). The results show that optimally configured one-, two- or three-actuator control is superior to the arbitrarily chosen actuators for the on-resonance excitation in terms of both objective function and radiated power.

Figure 9 shows the plate displacement distribution along the x-direction at $y = 0$ corresponding to the cases of Figure 8. The solid line depicts the disturbance response and reveals that the (3,1) mode is dominant. With control, the plate displacement has been reduced globally and exhibits a more complex pattern, and the (3,1) mode has been at-

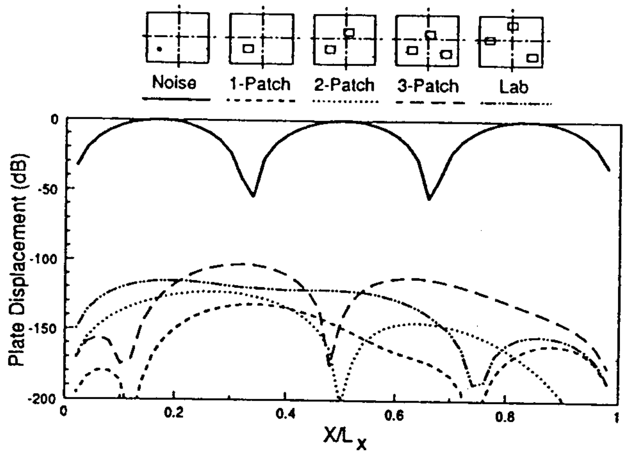

Figure 9. Plate displacement distribution for f = 357 Hz.

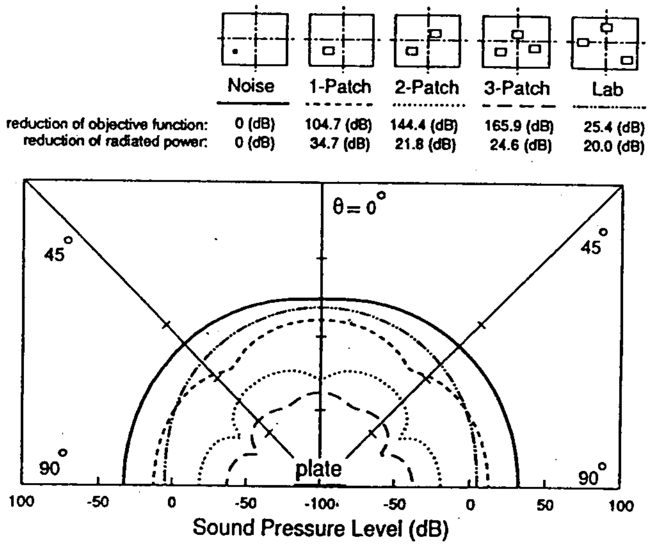

Figure 10. Radiation directivity pattern for f = 272 Hz.

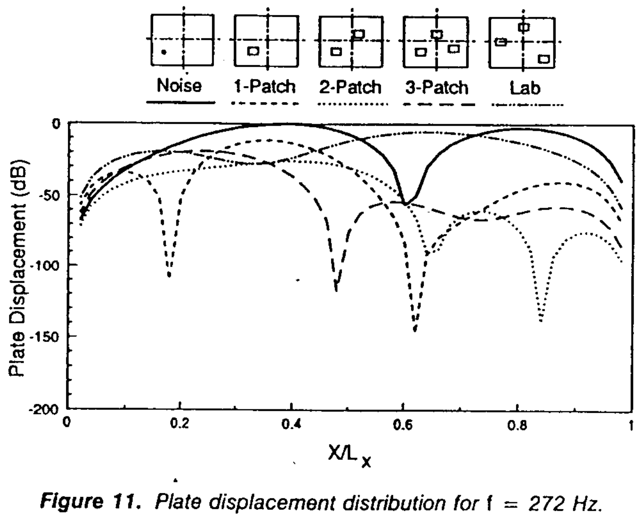

Figure 11. Plate displacement distribution for f = 272 Hz.

Table 5. Off-resonance excitation, f = 272 (Hz), between (2,1) and (3,1) modes.

Case	The *i*th Actuator	Optimal Location $\bar{x}/L_x$	Optimal Location $\bar{y}_i/L_y$	Optimal Voltage V_i (volt)	Reduction of Objective Function Ψ_p (dB)	Reduction of Radiated Power Φ_p (dB)
One Actuator	(1)	0.2945	0.3533	59.68	104.73	34.66
Two Actuators	(1)	0.2554	0.2978	100.56	144.38	21.77
	(2)	0.6031	0.6803	33.17		
Three Actuators	(1)	0.2387	0.2762	94.25	165.90	24.55
	(2)	0.5036	0.6353	32.14		
	(3)	0.7701	0.4044	6.70		
Three Actuators (Lab)	(1)	0.167	0.5	39.81	25.43	19.95
	(2)	0.5	0.833	98.24		
	(3)	0.833	0.167	122.64		

tenuated considerably. Further comments on the behavior are as in the previous section.

Off-Resonance Excitation, $f = 272$ Hz, between (2,1) and (3,1) Resonant

Table 5 shows the optimal central location and applied voltages of piezoelectric actuators, as well as the reduction of objective function and radiated power for an excitation frequency $f = 272$ Hz between the (2,1) and (3,1) modes. The control effort (i.e., the control voltages) is not necessarily smaller than that for on-resonance excitation, unlike the previous observation for one-actuator control. In fact, either one of the actuators may require extremely high control voltages; however, others may simultaneously need only a small voltage. The optimal location and required control voltages for multiple actuators are determined in such a way as to not only suppress the disturbance response, but also reduce the interactive spillover effects due to the actuators themselves.

Figures 10 and 11 show the radiation directivity pattern and the plate displacement distribution, respectively, corresponding to the case in Table 5 for the off-resonance excitation. The optimal locations of the piezoelectric actuators, sketched at the top of Figure 10, are very similar to those of the on-resonance excitation. From Figure 10, the primary radiated sound denoted by a solid line shows a small dip at $\theta = 0°$, indicating the strong response of the (2,1) mode. In applying one actuator, the residual response shows a combination of the (3,1) and (1,1) modes, and there are no dips at any location of the error microphones. In applying two and three actuators, the residual response reveals a more complex pattern similar to the (4,1) and (5,1) modes, respectively. Dips can now be seen located at the error microphone locations. It is again shown that increased actuators can further attenuate the pressures at error microphone position; however, the overall radiated power is not necessarily reduced because of spillovers in sound pressure into locations other than the position of the error microphones.

The plate displacement distribution for the case of disturbance, as shown in Figure 11, exhibits the (2,1) mode characteristic shape. With control, the residual plate response reveals a more complex pattern and is not attenuated globally, as was seen in Figure 11 for resonance excitation. This phenomenon is referred to as "modal restructuring" for the off-resonance excitation and "modal suppression" for the on-resonance excitation (Fuller, Hansen and Snyder, 1991).

COMPUTING TIME ANALYSIS

In an optimization procedure, to find the gradient of the objective function is generally the most difficult and the most CPU time-consuming task. Table 6 shows the percentage of CPU time consumed for each step in the optimization procedure. It takes about 20% of CPU time for steps 1 and 2, i.e., the evaluation of the objective function and the applied voltages to actuators, and over 70% (up to 90% for

Table 6. Typical example of CPU time for optimization.

Case	Number of Iteration	CPU Time (sec)	Percentage of Main Program (%) Step 1,2	Step 3	Step 4	Percentage of Optimization Program (%) Step 1,2	Step 3	Step 4
One Actuator	7	181	10.21	28.45	0.004	26.41	73.57	0.01
Two Actuators	27	2787	17.52	65.25	0.003	21.17	78.83	0.003
Three Actuators	7	2285	4.79	57.24	0.001	7.72	92.28	0.002

three actuators) of CPU time for step 3, i.e., the evaluation of the gradients of the objective function and constraints in the optimization procedure. However, it takes only a small percentage of time for step 4 in calculating the update actuator's location in the optimization subroutine. In order to efficiently solve the optimization problem, it is necessary to do a sensitivity analysis. For future work, it could be beneficial to apply an analytical or semi-analytical method rather than the finite difference method to evaluate the gradients so that CPU time can be reduced for solving the optimization problem. Furthermore, the acoustic radiated power could also be considered as the objective function to solve the optimal location of piezoelectric actuators.

SUMMARY

This paper has presented the mathematical formulation for the optimization problem of the placement of piezoelectric actuators in a feedforward control implementation of ASAC. The analysis is applied to an example problem to obtain preliminary information on how the optimization procedure performs. Four different forms of objective functions, which are differentiated by discrete or distributed and by vibrational or pressure sensor, are discussed for sound radiation control. An objective function, which is constructed based on the use of a number of discrete pressure sensors, is applied to the example of sound radiation control. Some significant observations may be summarized as follows:

1. Different excitation frequencies will result in different optimal locations of piezoelectric actuators.
2. The optimally located piezoelectric actuators can provide a large amount of reduction of sound radiated power and are observed to perform better than arbitrarily chosen locations. Properly locating one piezoelectric actuator generally gives a higher reduction of radiated acoustic power than using two or three actuators for the selected objective function, which is the sum of mean square sound pressure measured by a limited number of microphones. This is due to control spillover resulting from driving down the error signals at all error microphones. An alternative would be to limit the attainable attenuation achieved at each error microphone or to use the total radiated power as the objective function.
3. A computing time analysis shows that the evaluation of the gradients of the objective function and constraints consumes most of the CPU time. Sensitivity analysis, which can be used to analytically or semianalytically evaluate the gradients, is required for future research.
4. This work, which lays out the theory for optimal location of piezoelectric actuators, will be the basis for the design of "smart" structures for ASAC with distributed actuators and sensors.

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