# A FEASIBLE STUDY OF HYBRID STRUCTURAL VIBRATION CONTROL

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## ABSTRACT

This paper studies the feasibility of hybrid active and passive control of beam lateral vibration. A simply-supported beam is considered as the plant subjected to a harmonically excited point force disturbance input. Compact distributed actuators, i.e., piezoelectric actuators, are applied as active control forces to suppress the beam lateral vibration in conjunction with a passive control element, i.e., the concentrated mass attached to the beam, under the LMS feedforward control. The linear quadratic optimal control theory is used to obtain the optimal control voltages to be applied to piezoelectric actuators by minimizing a cost function. The cost function is defined as the sum of the least mean square of accelerations measured by a number of accelerometers located on the beam or an ideal distributed acceleration sensors over the beam. The performance of active piezoelectric actuator and passive control element (concentrated mass) on beam vibration control is discussed. The results show that the hybrid active and passive control of beam lateral vibration is convincing. This work leads to the design of intelligent material structure systems.


(keywords:beam, vibration, concentrated mass, LMS feedforward control, hybrid active and passive control, intelligent material structure systems)

## INTRODUCTION

Structural vibration and acoustic control is a great deal of interest. Passive control method, such as alternating the physical properties of structures by adding mass, increasing stiffness or applying damping materials, is frequently used for its low cost and easy implementation. For example, the block mass and isolators have been widely used to the control of mechanical vibration (Jones, 1984; Ungar and Dietrich, 1966); however, passive control is generally inadequate. Recently, due to the development of fast processible personal computer, active control becomes more feasible and practical than ever. Active control can achieve significant control effect and then play an important role in both structural vibration and acoustic control.

LMS adaptive feedforward control algorithms have been successfully applied to active structural sound and vibration control with a steady state sinusoidal input, such as (Elliott et al., 1987) and (Fuller et al., 1989). The algorithm is used to adjust the magnitude and phases of the sinusoidal inputs to the control actuators so as to minimize the sum of the mean square values

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which are measured by a set of error sensors. This paper will then apply the LMS control algorithm to study the hybrid active and passive control system.

Point force shakers are traditionally used as active control sources (Sinha, 1987; Fuller, 1988; Jones and Fuller, 1989); however, shakers have substantial disadvantages due to their large volume, large weight and requiring support. Compact distributed actuators, such as piezoceramic patches, have increasingly generated a great deal of interest because of their light weight and small volume. (Crawley and de Luis, 1987), (Wang et al., 1991) and (Wang and Fuller, 1991) have demonstrated the feasibility of using piezoelectric actuators for sound and vibration control. Piezoelectric actuators can essentially overcome the disadvantages of shakers.

Active control draws a great concern and becomes feasible not only the advent of fast processible CPU but also the evolution of actuator and sensor technologies. Therefore, active control can be economically implemented and achieve more efficient control than passive control. Nevertheless, it is also considerably advantageous to combine the active and passive control techniques. This paper will be concerned with the feasible study of hybrid active and passive control. A simply-supported beam is considered as the plant subjected to a harmonically excited point force disturbance. Piezoelectric actuators are applied as the active control sources in conjunction with a passive control element which is the concentrated mass attached to the beam. The linear quadratic optimal control theory (LQOCT) which is to simulate the LMS feedforward control algorithm is adopted to calculate the optimal control voltages to be applied to piezoelectric actuators by minimizing the cost function. The cost function which is a least mean square value depends on the use of the types of sensors. Both discrete acceleration sensors (accelerometers) and an ideal distributed acceleration sensors are considered. The lateral vibration of the beam with/without concentrated mass is first studied. The application of LQOCT for LMS feedforward vibration control is shown. Numerical results show that the hybrid active and passive control provide a feasible and practical control approach. This work may lead to a new design concept of intelligent material structural systems application to the structural sound and vibration control.

# THEORETICAL ANALYSIS

## LATERAL VIBRATION OF UNIFORM BEAM

Consider a uniform simply-supported beam with length of L, as shown in Figure 1, the equation of motion can be obtained as follow:

$$E_b I \frac{\partial^4 y}{\partial x^4} + \rho_b b t_b \frac{\partial^2 y}{\partial x^2} = p(x,t) \qquad (1)$$

where $E_b$ : Young's modulus of the beam

$I$ : moment of inertia

$\rho_b$ : beam density

$t_b$ : beam thickness

$b$ : beam width

$p(x,t)$ : force function

Note that the damping effect is assumed small and can be neglected for simply application. The boundary conditions for a simply-supported beam are

$$M(0,t)=M(L,t)=E_bI\frac{\partial^2 y}{\partial x^2}=0 \tag{2}$$

$$y(0,t)=(L,t)=0 \tag{3}$$

For free vibration analysis, i.e., $p(x,t)=0$, the natural frequencies can be found to be

$$\omega_n=(n\pi)^2\sqrt{\frac{E_bI}{\rho_b bt_b L^4}} \tag{4}$$

The general form of beam lateral displacement, while the beam is subjected to harmonic force inputs, can be written as the follow:

$$y(x,t)=e^{i\omega t}\sum_{n=1}^{\infty} W_n\sin\alpha_n x \tag{5}$$

where

$$\alpha_n=\frac{n\pi}{L} \tag{6}$$

$$W_n=\frac{P_n}{\rho_b bt_b(\omega_n^2-\omega^2)} \tag{7}$$

Here $\omega$ is the excitation frequency; $\alpha_n$ is the modal number; $W_n$ is the modal amplitude; and $P_n$ is the modal force depending on the forms of external forces.

POINT FORCE EXCITATION:

For a harmonic point force with the amplitude of F located at $x_f$ acting on the beam, the force function, $p(x,t)$, can be written as follow:

$$p(x,t)=F\delta(x-x_f)e^{i\omega t} \tag{8}$$

The Delta function, $\delta(x)$, is employed to represent the location of the point force. The modal force, $P_n^f$, due to the point force excitation is given as follow:

$$P_n^f=\frac{2F}{L}\sin\alpha_n x_f \tag{9}$$

where the superscript f signify the point force.

PIEZOELECTRIC EXCITATION:

For an actuator consisting of two identical piezoceramic patches bonded symmetrically on the two opposite beam surfaces and activated 180° out-of- phase, the equivalent external forces can be derived as follow (Wang, 1991):

$$p(x,t)=M_{eq}[\delta'(x-x_1)-\delta'(x-x_2)]e^{i\omega t} \tag{10}$$

where

$$M_{eq}=C_0\Lambda=\frac{t_b^2E_b}{6}Kb_a\Lambda \tag{11}$$

$$K=\frac{6}{6+\Psi} \tag{12}$$

$$\Psi=\frac{E_bt_b}{E_at_a} \tag{13}$$

$$\Lambda = \frac{d_{31}}{t_a}V \tag{14}$$

$C_0\Lambda$ is the equivalent moment induced by the piezoelectric patches attached to the top and bottom of the beam and excited 180° out-of-phase. $\Lambda$ is the strain induced by an unconstrained piezoelectric layer of thickness, $t_a$ , when a voltage V is applied along its polarization direction, while $d_{31}$ is the piezoelectric dielectric strain constant. $\Psi$ is the effective stiffness ratio (Crawley and de Luis, 1990).. The resultant force is, in fact, the concentrated moments acting on the both edges of piezoelectric patches represented by the first derivative of Delta function. The corresponding expression of modal force for piezoelectric excitation, $P_n^c$ , can be derived (Wang, 1991) as follow:

$$P_n^c = \frac{2M_{eq}}{L}\alpha_n(\cos\alpha_n x_1 - \cos\alpha_n x_2) \tag{15}$$

where $x_1$ and $x_2$ are the coordinates of the piezoelectric actuator, and the superscript c signify the control force.

## LATERAL VIBRATION OF UNIFORM BEAM WITH CONCENTRATED MASS

As shown in Figure 1, a concentrated mass, M, is located at $x_M$ . The equation of motion can then be derived as follow:

$$E_b I\frac{\partial^4 y}{\partial x^4} + [\rho_b b t_b + M\delta(x-x_M)]\frac{\partial^2 y}{\partial t^2} = p(x,t) \tag{16}$$

Here the point force was applied as the disturbance input and piezoelectric actuator was applied as the control input. Therefore, by superposition method the force function, p(x,t) , can be considered as the sum of Equations (9) and (15). To solve the above equation, Fourier Analysis with the eigenfunctions of homogeneous beam vibration forming the basis for spatial expansion will be performed (Sandman, 1977). The beam lateral displacement is first assumed as follow:

$$y(x,t) = e^{i\omega t}\sum_{n=1}^{\infty} W_n \sin\alpha_n x \tag{17}$$

By substituting the spatial expansion of beam lateral displacement Equation (17) into Equation (16), the equation of motion is multiplied by $\sum_{m=1}^{\infty} W_n \sin\alpha_m x$ and integrated over the beam, then the equation of motion become

$$\begin{aligned}
&\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\{W_n\alpha_n^4\int_0^L \sin\alpha_n x\sin\alpha_m x dx \\
&-\frac{\rho_b b t_b\omega^2}{E_b I}W_n\int_0^L \sin\alpha_n x\sin\alpha_m x dx \\
&-\frac{M\omega^2}{E_b I}W_n\int_0^L \delta(x-x_M)\sin\alpha_n x\sin\alpha_m x dx\} \\
&=\sum_{n=1}^{\infty}\{\frac{F}{E_b I}\int_0^L \delta(x-x_f)\sin\alpha_m x dx \\
&+\frac{C_0\Lambda}{E_b I}\int_0^L[\delta'(x-x_1)-\delta'(x-x_2)]\sin\alpha_m x dx
\end{aligned} \tag{18}$$

By applying the orthogonal properties of the eigenfunctions, the above equation can be simplified as follow:

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\{W_n(\alpha_n^4-\frac{\rho_b b t_b}{E_b I}\omega^2)\frac{L}{2}\delta_{nm}$$
$$-\frac{M\omega^2}{E_b I}W_n \sin\alpha_n x_M \sin\alpha_m x_M\} \tag{19}$$
$$=\sum_{m=1}^{\infty}\{\frac{F}{E_b I}\sin\alpha_m x_f+\frac{C_0\Lambda}{E_b I}\alpha_m(\cos\alpha_m x_1-\cos\alpha_m x_2)\}$$

The above equation can be rewritten in matrix form for the numerical solution purpose.

$$[R]_{m\times n}\{w\}_{n\times 1}=\{P\}_{m\times 1} \tag{20}$$

where [R], if m=n, is a symmetric modal coefficient matrix including both the beam stiffness and mass effect; {w} is the modal amplitude vector depending on the form of force function; {p} is the modal force vector which consists of the modal component of the point force disturbance and piezoelectric actuator. A typical element of [R], {w} and {P} can be shown as follow:

$$R_{nm}=A_{nm}\omega^2 B_{nm} \tag{21}$$

$$A_{nm}=\frac{L}{2}\alpha_n^4\delta_{nm} \tag{22}$$

$$B_{nm}=\frac{M}{E_b I}[\frac{L}{2}\delta_{nm}+\sin\alpha_n x_M \sin\alpha_m x_M] \tag{23}$$

$$w_n=W_n \tag{24}$$

$$P_m=\frac{L}{2E_b I}(P_m^f+P_m^c) \tag{25}$$

where $P_m^f$ and $P_m^c$ are given in Equations (9) and (15) respectively. For free vibration analysis, Equation (20) can be reduced to the following form:

$$[A]\{w\}=\omega^2[B]\{w\} \tag{26}$$

One can easily solve the above generalized eigenvalue problem. Therefore, the natural frequencies of the beam with the concentrated mass can be obtained. To numerically solve Equation (20), Equation (20) can be decomposed into two independent linear system equations for both point force disturbance and piezoelectric actuator respectively as follow:

$$[R]\{w^f\}=\{p^f\} \tag{27}$$

$$[R]\{w^c\}=\{p^c\} \tag{28}$$

where $\{w^f\}$ and $\{w^c\}$ are the modal amplitude vectors for the point force and piezoelectric actuator which are to be determined; the n-th elements of $\{p^f\}$ and $\{p^c\}$ , which are the modal force vectors for the point force and piezoelectric actuator, is given as follow:

$$\{p^f\}=\frac{L}{2E_b I}\begin{bmatrix}p_1^f\\p_2^f\\\vdots\\\vdots\\p_m^f\end{bmatrix} \tag{29}$$

$$\{p^c\} = \frac{L}{2E_bI}\begin{bmatrix} p_1^c \\ p_2^c \\ \vdots \\ \vdots \\ p_m^c \end{bmatrix} \tag{30}$$

The modal amplitude vectors can then be determined by solving the above linear system equations (27) and (28). The beam lateral displacement due to the excitation of the point force and piezoelectric actuator can be obtained by substituting the corresponding modal amplitude vectors into Equation (17). Finally, the lateral displacement of the beam with a concentrated mass can be obtained by superposition method.

Note that although the above analysis is particular for one point force disturbance, one piezoelectric actuator and only one concentrated mass, the derivation and solution techniques for multiple disturbances, actuators and masses are essentially the same. One can easily perform the analysis under the assumption of superposition.

By superposition, the total beam dynamic response by a number of point force disturbances can be evaluated by the composite of individual response. For $N_f$ primary (disturbance) sources or $N_c$ control sources, the beam lateral displacement can be derived as follows:
for primary sources (point forces):

$$y_f(x,t) = e^{i\omega t}\sum_{j=1}^{N_f}\sum_{n=1}^{\infty} W_{nj}^f \sin\alpha_n x \tag{31}$$

for control sources (piezoelectric actuators):

$$y_c(x,t) = e^{i\omega t}\sum_{j=1}^{N_f}\sum_{n=1}^{\infty} W_{nj}^c \sin\alpha_n x \tag{32}$$

If the primary and control sources act simultaneously, the resultant beam response can be viewed as a superposition of Equations (31) and (32) for steady-state harmonic excitation. The total beam lateral displacement can then be written as

$$y_t = y_f + y_c \sum_{j=1}^{N_f} F_iB_j + \sum_{j=1}^{N_c} (C_0\Lambda)_j A_j \tag{33}$$

where $B_j$ and $A_j$ are the beam lateral distribution functions for the j-th primary source and the j-th control source respectively, given by

$$B_j(x,t) = e^{i\omega t}\sum_{n=1}^{\infty} Q_{nj}^f \sin\alpha_n x \tag{34}$$

$$A_j(x,t) = e^{i\omega t}\sum_{n=1}^{\infty} Q_{nj}^c \sin\alpha_n x \tag{35}$$

where

$$Q_{nj}^f = \frac{W_{mj}^f}{F_j} \tag{36}$$

$$Q_{nj}^f = \frac{W_{mj}^f}{(C_0\Lambda)_j} \tag{37}$$

Note that Equations (33)-(37) for the total beam dynamic response is valid for beam with or without concentrated mass. Only the derivation of modal amplitude which is clearly shown in this section is different.

## LINEAR QUADRATIC OPTIMAL CONTROL THEORY

For vibration control, the most general cost function can be formed as the integral of the mean squared beam lateral acceleration over the beam. Such a cost function gives a global sense of vibration level attenuation (in particular, it is proportional to the total vibrational energy density) ; however, in practical application, it is difficult to measure a distributed acceleration. Instead, a finite number of accelerometers can be imposed to the beam to measure the accelerations. The objective here is to apply an minimization procedure for a quadratic function developed by Lester and Fuller (1990) using tensor calculus, and to calculate the input amplitude of the control source such that a selected cost function can be minimized.

### DISTRIBUTED ACCELERATION SENSOR

The cost function corresponding to the distributed acceleration sensor can be defined as follow:

$$\Phi_y = \int_0^L |\ddot{y}_t|^2 dx \tag{38}$$

### DISCRETE ACCELERATION SENSORS

If a finite number of accelerometers located on the beam serve as error sensors, then the cost function can be defined as the sum of the mean square acceleration:

$$\Psi_y = \sum_{i=1}^{N_{acc}} |\ddot{y}_t|^2 \tag{39}$$

In summary, $\Phi_y$ is measured by ideal sensors, which may not be practical in reality; however, $\Phi_y$ represents the energy density of out-of-plane structural vibration. Therefore, $\Phi_y$ can be used as an index of control effectiveness. For practical application, $\Psi_y$ is the alternative option. A reasonable number and location of sensors shall be selected to estimate the actual system distributed response, such that an optimal solution can be found without losing general nature of the response. When the expression of $y_t$ from Equation (33) is substituted into Equation (38), the cost function is obviously quadratic and positive definite and possesses a unique minimum. A minimization procedure (Lester and Fuller, 1990) for the quadratic function was employed to calculate the optimal control parameters. The complete derivations is shown in (Wang, 1992) and omitted here for brevity.

## ANALYTICAL RESULTS AND DISCUSSIONS

To study the beam vibration characteristics, several fundamental mode shapes of simply-supported beam are illustrated in Figure 2 for future references. Table 1 shows the specifications of the steel beam used in the simulations. Natural frequencies are tabulated in Table 2. It is noted that no damping was included in the following analysis. In order to calculate the beam response, it was necessary to truncate the modal sums in Equations (31) and (32). Upon consideration of computing time and accuracy, the first 20 modes were considered, and it was found to provide sufficient convergence of series.

The following results consist of the displacement distribution of beam vibrational amplitude plotted along the beam length. The results are normalized by the largest amplitude obtained in each case. The amplitude of the point force disturbance input is fixed at F=0.1N located at $x_f$ =0.067m. The physical properties of piezoceramic patch (G-1195) to be used as actuators are shown in Table 3. A concentrated mass about one twentieth of the beam weight 0.05 Kg is used as the passive control element.

## EFFECT OF CONCENTRATED MASS ON BEAM RESPONSE

Figure 3 presents the first few natural frequencies while the beam is attached a concentrated mass on various beam locations. The horizontal axis is the beam length, and the vertical axis is the frequency. The horizontal solid lines at 32, 128, 289 and 515 Hz indicate the first four natural frequencies of the beam without attached masses. When a concentrated mass is attached to the beam, the beam natural frequencies have been reduced in general; however, the natural frequency remains unchanged while the concentrated mass located at the nodal point of the corresponding mode. For example, if the mass is located at the center of the beam where is the nodal point of the second and fourth modes, the second and fourth mode natural frequencies are not affected. When the concentrated mass is located at the antinode of a mode, the corresponding natural frequency is reduced most. As shown in Figure 3, the deviation of beam natural frequency due to the different location of the concentrated mass is somewhat like the modeshape of the beam.

Figure 4 shows the displacement distribution of the beam with a concentrated mass at different locations for harmonically excited disturbance (point force) at 290 Hz, i.e., near the third resonance mode of the "raw" beam (without attached masses) under active control. As expected, the solid line which indicates the beam response due to the disturbance reveals the third mode response referred to Figure 2. The dashed line labeled "Mass" represents the displacement distribution of the beam with a concentrated mass located at $x_M$=0.19m without active control.
It is noted that the beam lateral displacement has been globally reduced about 10 dB due to the passive control element. The other lines show the displacement distribution of the beam with the concentrated mass under the active control of a piezoelectric actuator with the use of an accelerometer as the error sensor. The location of disturbance, mass, sensor and actuator are graphically shown on the top of Figure 4. The case of M10 means the concentrated mass is located at $x_M$=0.10m, and so on. Several interesting observations are made. The beam displacement after control is globally reduced. Applying active control in conjunction with passive control element generally achieve better reduction of vibration level than using passive control element alone. As shown in Figure 4, due to the LMS control effect the beam response is driven to zero at x=0.1m where is the accelerometer's location. Since the location of the concentrated mass do affect the control effectiveness, the residual beam displacement distributions are somewhat different from each others due to the different location of the concentrated mass.
In summary, the hybrid control (i.e., active piezoelectric actuator in conjunction with the passive control element, a concentrated mass) of structural vibration is feasible.

The location of the concentrated mass is shown a key factor to achieve effective control. Now the question is how the mass weight affect the beam vibration control. When the beam is subjected to a harmonically excited point force at f=33Hz, i.e., near the first mode, Figure 5 shows the beam displacement distribution for the fixed location of the concentrated mass at x=0.19m by varying the mass weight. The mass weight is 10-50 grams (denoted M10-M50), i.e., about 4-20% of the beam weight. The cost function is chosen as the ideal distributed accelerometer sensor. Again, the solid line represents the displacement distribution due to the disturbance alone. For passive control, the residual response denoted "Mass" as shown in Figure 5 is still revealed the first modeshape. The reason is that the effect of adding the passive control element, the concentrated mass, is to offset the natural frequency only; however, the significant mode (the first mode) still dominates the total beam response. On the other hand, for hybrid active and passive controls the residual response exhibits the second modeshape, because the most significant mode (the first mode) has been fully controlled by the active control actuator. The effect of mass weight to the beam lateral vibration is not significant. As shown in Figure 5, the residual response is pretty similar except a little distortion of dip position due to the variation of mass weight. It is noted that the mass weight is limited within 20% of the mass weight under the assumption of small lateral vibration level.

## EFFECT OF SENSORS ON BEAM RESPONSE

As discussed previously, the concentrated mass is a passive control element and play an important role on affecting the beam response and the control effectiveness. The selection of error sensors is also a key factor to perform the system control. As shown in Figures 3 and 5, an accelerometer and an ideal distributed acceleration sensors are used as error sensors to accomplish the beam vibration control respectively. In this section, the effect of the location and number of accelerometers will be discussed and compared to that of the ideal distributed acceleration sensor. Figure 6 shows the displacement distribution of the beam attached a concentrated mass 50 grams at x=0.19m subjected to the harmonically excited point force at f=33Hz. Active vibration control is performed while the piezoelectric actuator is located at $x_1$ =0.285m and $x_2$=0.3485m, and several accelerometers are located at 0.1m (S1), 0.255m (S2) and 0.35m (S3) applied individually. The solid line and dashed line indicate the beam response for the disturbance alone and for applying the passive control element respectively. Each accelerometer is used as the error sensor individually to perform the active vibration control. One can observe that the beam displacement is driven to zero at the sensor's location after control for the three cases. It is the cause that the LMS control tends to minimize the cost function which is the least mean square of the acceleration. In addition to the different patterns of residual beam response, the different sensor's location result in different level of reduction of vibration energy. As shown in Table 4, S1 sensor gives 28.92 dB reduction better than S2 (26.40 dB) and S3 (-5.07 dB). The negative number of reduction means a spillover of vibration response which can also be observed in Figure 6. The control voltage to the piezoelectric actuator for the case of S3 is as high as 672 volts which is over the limit of the piezoceramic patch in practice. Therefore, the poor location of sensor not only requires a high control effort but also can not achieve control at all. For the case of the ideal sensor, the reduction of vibration energy is 30.33 dB which is the highest reduction among those cases. The residual response is exhibited the second modeshape since the first mode is controlled. In summary, the ideal sensor required a reasonable control effort gives the best vibration control. On the other hand, carefully selected single accelerometer can also achieve the control; however, the control effectiveness is highly dependent on the location of sensors.

Figure 7 shows the results for the different number of sensors. The solid and dashed lines are for the disturbance alone and applying a concentrated mass only, the same as the previous figures. One can observe that only single sensors case (S1) will result in a dip at sensor's location for beam lateral response. For the cases of two sensors (S1+S2) and three sensors (S1+S2+S3) and the ideal sensor (SC), the displacement distribution are more or less overlap each others only a slight difference, and no dips occur at the sensor's location. This phenomenon had been discussed by (Burdisso and Fuller, 1991) and (Fuller and Jones, 1987). From Table 5, the reductions of vibration energy are almost the same. This means that for the first mode excitation more than one accelerometer used as error sensor is a similar approach as the ideal distributed acceleration sensor. Although the response at the sensor's location is not driven to zero (instead the response at the sensors' location are equivalently reduced), the most significantly contributed mode is controlled leading to the remainder of the next significant mode response, i.e., the second mode. From the above discussion, it is noted that the increase of the number of sensors will not always increase the reduction of vibration energy. The control mechanism may also be different due to the different number of sensors.

## EFFECT OF PIEZOELECTRIC ACTUATORS ON BEAM RESPONSE

Figure 8 shows the displacement distribution of the beam subjected to a harmonically excited point force at f=33Hz, i.e., near the first natural mode, for the cases of applying multiple actuators and sensors. The solid and dashed lines again show the raw response of the beam and the beam attached mass respectively. When one actuator and one sensor are applied as the case of P1S1 denoted in Figure 8, the beam response at the sensor's location is driven to zero. For the case of P13S13, i.e., applying P1 and P3 piezoelectric actuators and S1 and S3 accelerometers as shown on the top of Figure 8, the beam response at S1 and S3 location are again minimized to zero due to the LMS control effect. Similar results can be observed for the case of P123S123, i.e., three actuators and three sensors are applied simultaneously. In comparison to the results of Figure 7 in which the number of sensors is greater than the number of actuators, the beam response at the sensors' location can not be minimized to zero. When the number of actuators and sensors are the same as shown in Figure 8, the beam response at the sensors' location can be minimized as much as possible.

Table 6 summarizes the reduction of vibration energy and control voltages to be applied to piezoelectric actuators for multiple actuators and sensors corresponding to the cases in Figure 8. One can see that as the increase of the number of actuators, the reduction of vibration energy increases, i.e., better vibration control can be achieved for applying more piezoelectric actuators. The control voltages to be applied to actuators for the cases of P12S12 and P12S13 is smaller than the case of P13S13; however, the cases of P12S12 and P12S13 give better vibration control than the case of P13S13. The control effort required by the cases having the better reduction of vibration energy is generally smaller than that required by the cases having less reduction of vibration energy. Therefore, when an optimization procedure is applied to determine the optimal location of actuators, the reduction of vibration energy can be a criterion such that the optima will guarantee the least control effort as shown by Wang et al. (1991).

## CONCLUSIONS

This paper analytically studies the feasibility of the hybrid active and passive structural vibration control in conjunction with the use of LMS feedforward control. When piezoelectric actuator is used as the active control force, the concentrated mass attached the beam is used as the passive control element. Two types of discrete and distributed acceleration sensors are considered. The results show that hybrid active and passive control generally perform better vibration control than passive or active control individually. However, the selection of number and location of actuators and sensors are critical to achieve the system control as well as the location and weight of concentrated mass. This paper layouts a primary design concept for intelligent material structure systems application to structural vibration control. A future work on optimally selecting the location and number of passive control element, active control actuators and sensors are substantial.

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Table 1. Physical properties of the steel beam

| | |
|---|---|
| $E_b = 207 \times 10^9 (N/m^2)$ | $\rho_b = 7870 (Kg/m^3)$ |
| $L = 0.38 (m)$ | $t_b = 2 (mm)$ |

Table 2. Natural frequencies of the simply-supported beam

| n-th mode | Natural frequencies(Hz) |
|---|---|
| 1 | 32.21 |
| 2 | 128.84 |
| 3 | 289.89 |
| 4 | 515.36 |
| 5 | 805.25 |
| 6 | 1159.56 |
| 7 | 1578.29 |
| 8 | 2061.44 |
| 9 | 2609.00 |
| 10 | 3220.99 |

Table 3. Physical properties of the G-1195 piezoceramic patch (Piezo System, 1987)

| | |
|---|---|
| $E_a = 6.3 \times 10^{10} (N/m^2)$ | $\rho_a = 7870\ Kg/m^3$ |
| $t_a = 1.905 (mm)$ | $\nu_a = 0.28$ |
| $d_{31} = d_{32} = 166 \times 10^{-12} (\frac{m}{volt})$ | |

Table 4. Summary for different sensor locations, f=33Hz

| case | vibration energy $\Phi_y$ (dB) | reduction of vibration energy (dB) | control voltage (volt) |
|---|---|---|---|
| Disturbance | 132.15 | | |
| Mass | 112.17 | 19.98 | |
| S1 | 103.23 | 28.92 | -27.31 |
| S2 | 105.75 | 26.40 | -47.17 |
| S3 | 137.22 | -5.07 | -672.32 |
| SC | 101.82 | 30.33 | -34.01 |

Table 5. Summary for different sensor numbers, f=33Hz

| case | vibration energy $\Phi_y$ (dB) | reduction of vibration energy (dB) | control voltage (volt) |
|---|---|---|---|
| Disturbance | 132.15 | | |
| Mass | 112.17 | 19.98 | |
| S1 | 103.23 | 28.92 | -27.31 |
| S1+S2 | 101.85 | 30.30 | -34.86 |
| S1+S2+S3 | 101.86 | 30.29 | -34.96 |
| SC | 101.82 | 30.33 | -34.01 |

Table 6. Summary for multiple actuators and sensors, f=33Hz

| case | vibration energy $\Phi_y$ (dB) | reduction of vibration energy (dB) | control voltage (volt) |
|---|---|---|---|
| Disturbance | 132.15 | | |
| Mass | 112.17 | 19.98 | |
| P1S1 | 103.23 | 28.92 | -27.31(P1) |
| P12S12 | 87.19 | 44.96 | -1.42(P1)<br>-16.31(P2) |
| P12S13 | 87.21 | 44.94 | -1.53(P1)<br>-16.24(P2) |
| P13S13 | 99.64 | 32.51 | -11.44(P1)<br>-56.11(P3) |
| P123S123 | 86.87 | 45.26 | -1.62(P1)<br>-16.10(P2)<br>-0.49(P3) |

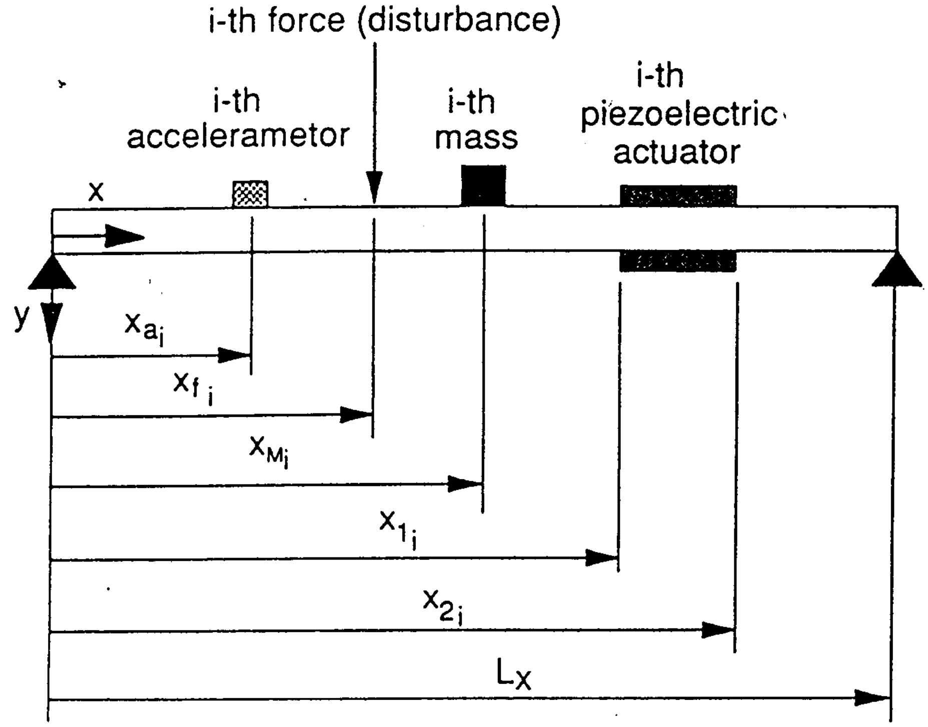


Figure 1. The arrangement and coordinates of simply-supported beam

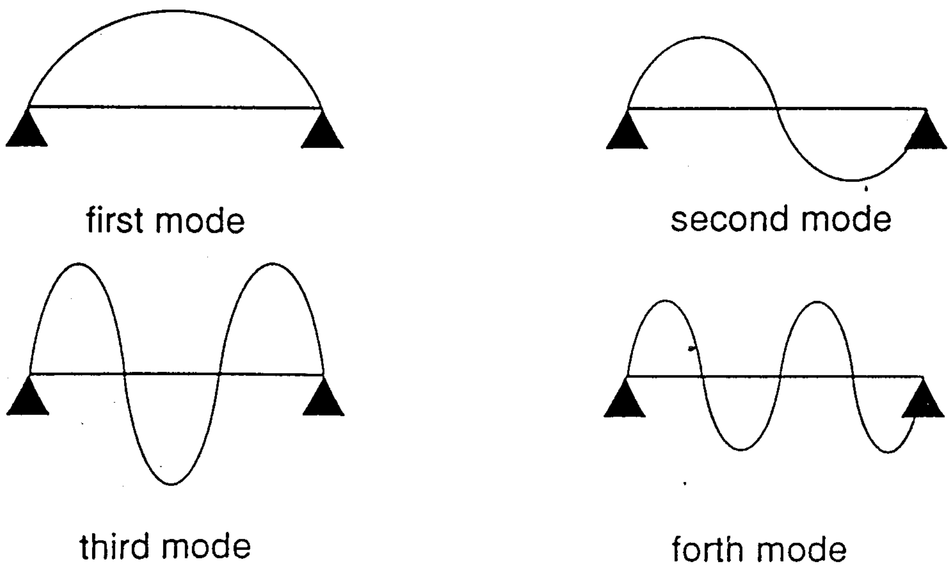


Figure 2. Illustration of modeshapes of the simply-supported beam

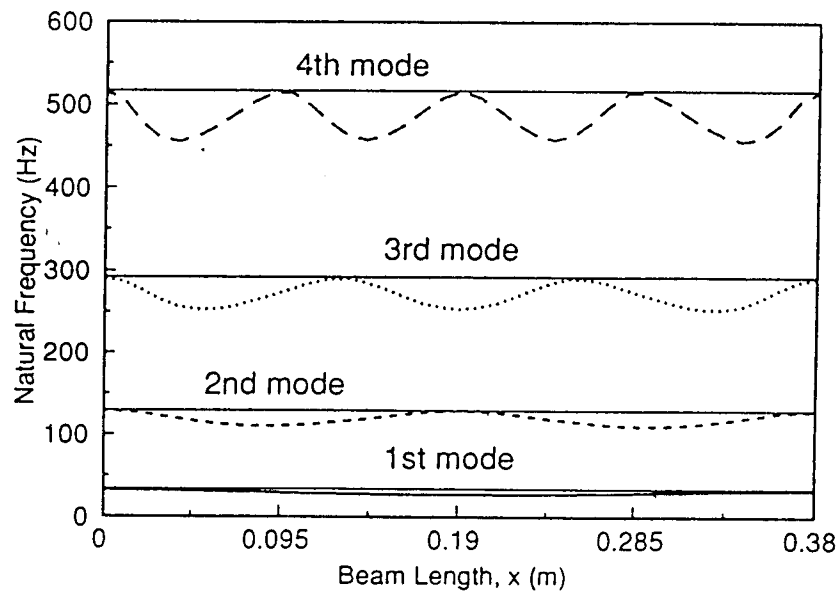


Figure 3. Natural frequencies of the beam with a concentrated mass at various location

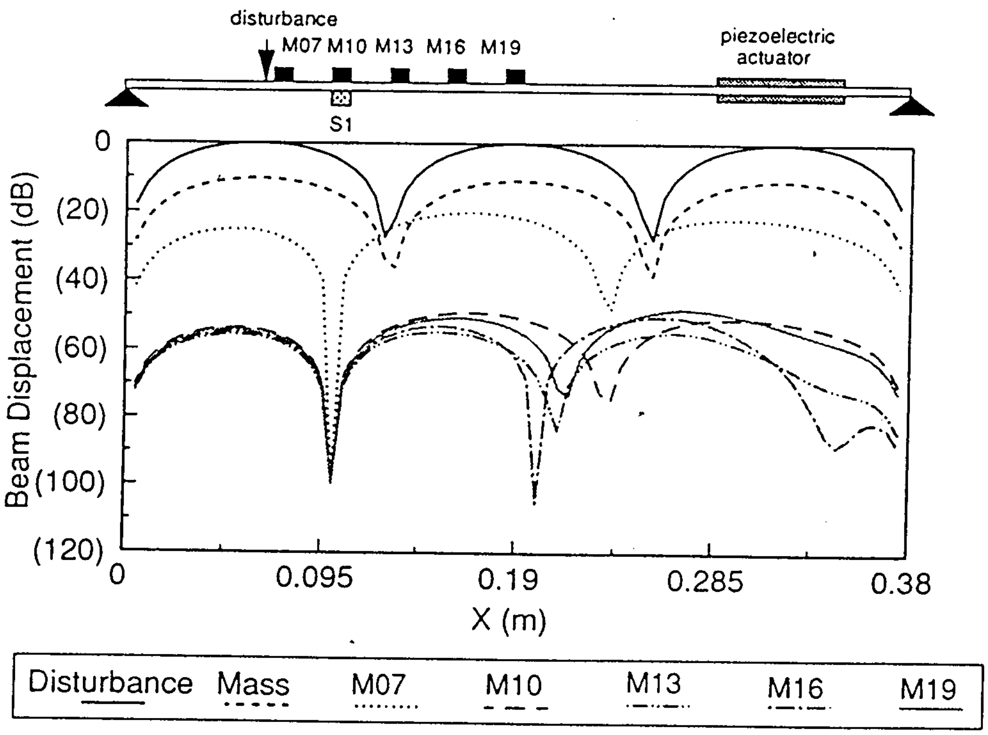


Figure 4. Displacement distribution of the beam with a concentrated mass at different location under active control

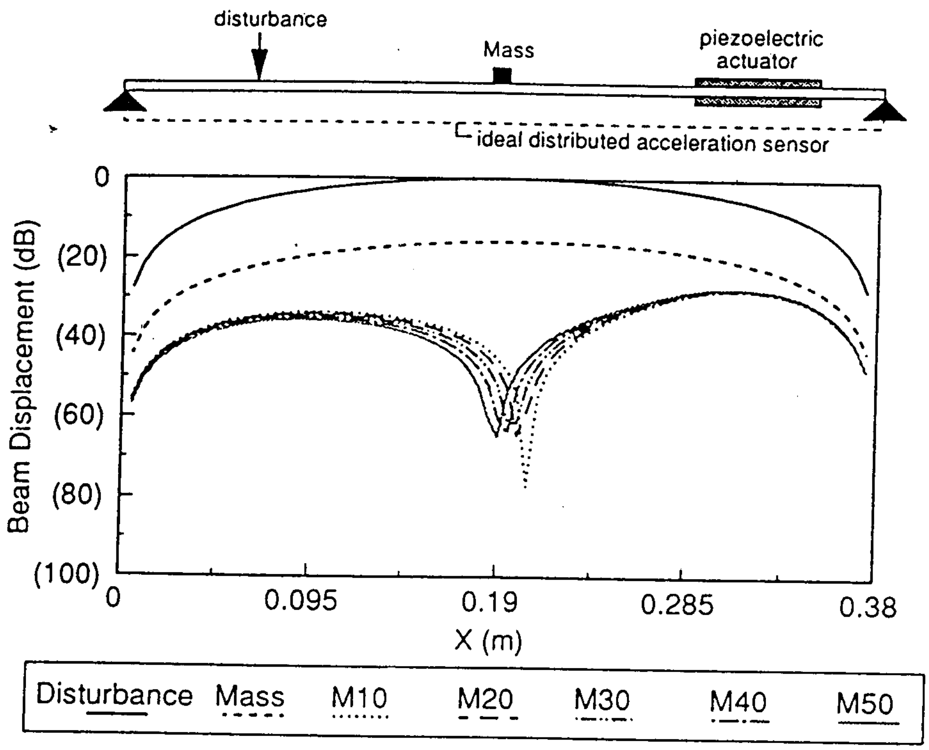


Figure 5. Displacement distribution of the beam with a concentrated mass by varying mass weight under active control

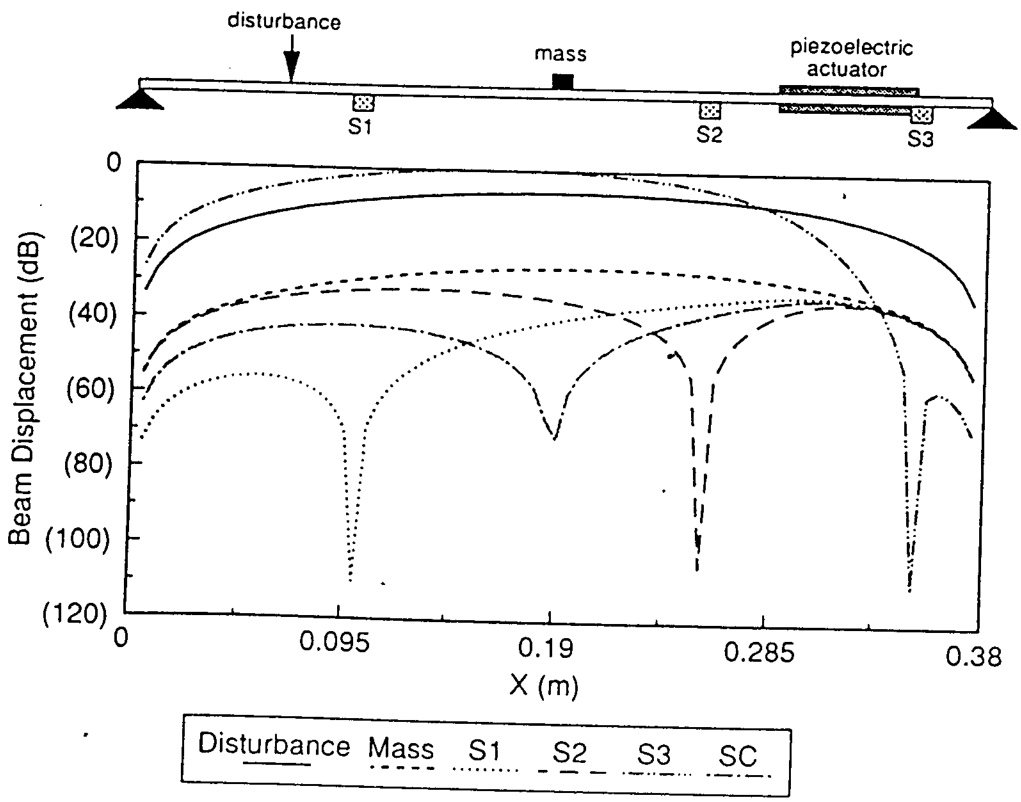


Figure 6. Displacement distribution of the beam for different location of accelerometers, f=33Hz

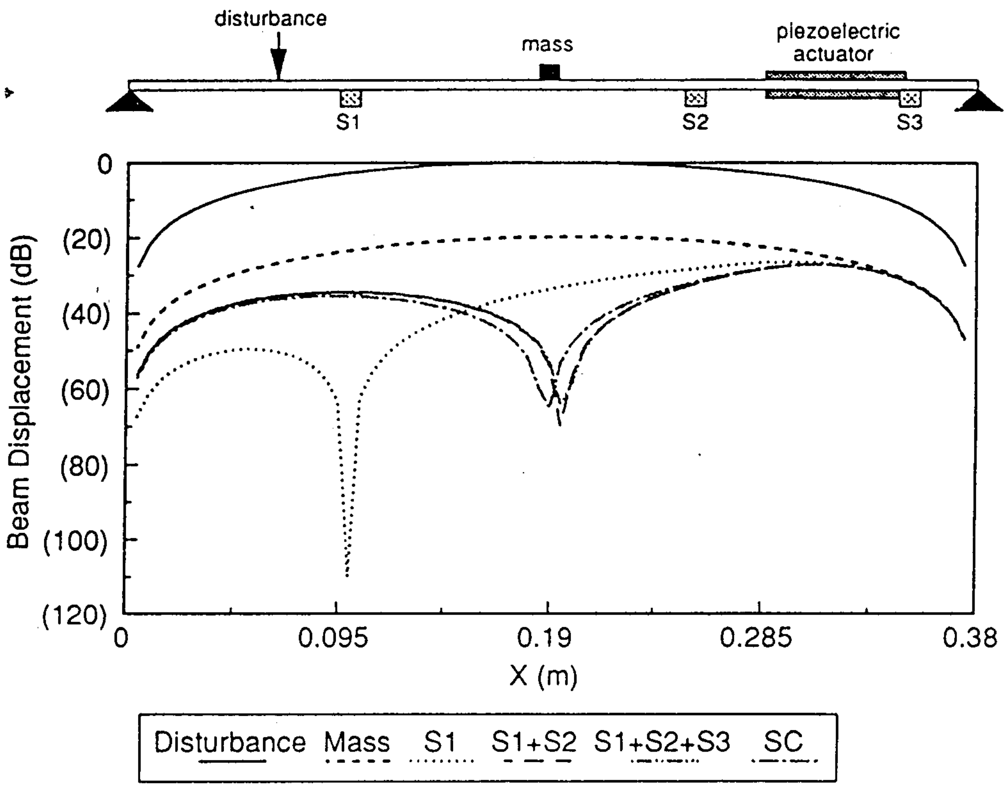


Figure 7. Displacement distribution of the beam for different number of accelerometers, f=33Hz

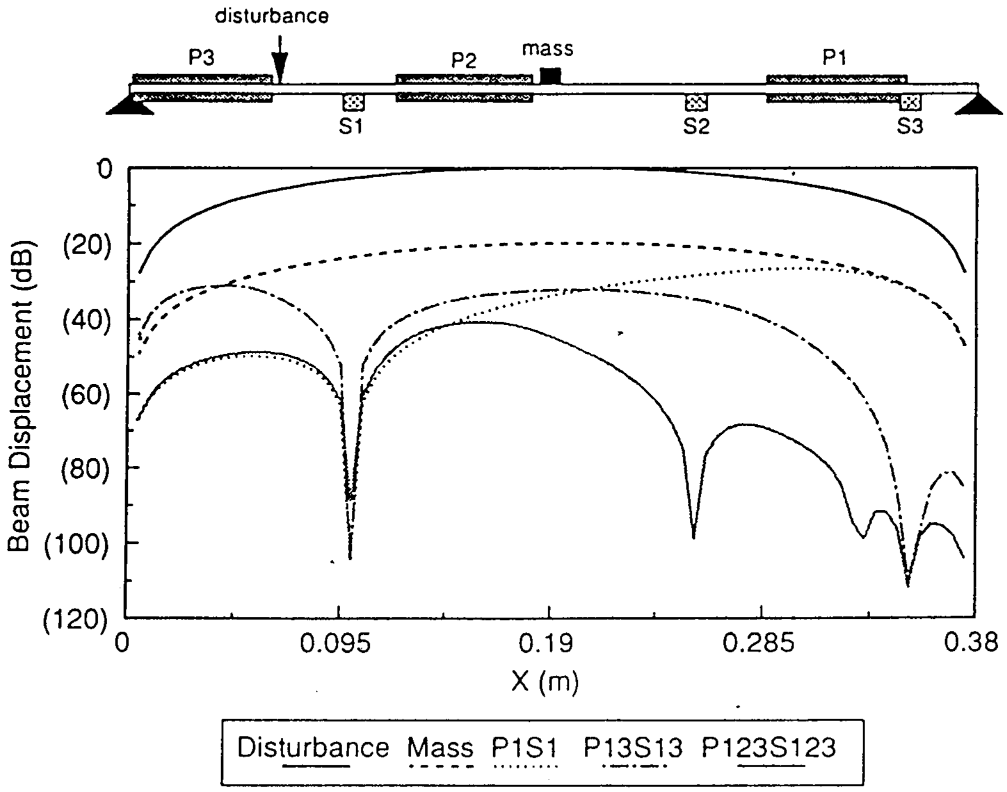


Figure 8. Displacement distribution of the beam for multiple piezoelectric actuators and sensors, f=33Hz

# 混合結構振動控制之可行性分析

王 栢 村[1]

## 摘 要

本研究在探討樑振動之混合主動和被動控制的可行性。考慮之系統乃是一簡支樑受協振點力之干擾激振，利用簡潔的均佈驅動器，如壓電驅動器，為主動控制源，來減小樑的橫向振動，並採用最小平方前饋控制方式，同時，也加上被動控制元件之一的阻塊適當地置於樑上，因而藉著改變樑的物理性質來減小樑之橫向振動。利用線性平方法求小值理論，可計算最佳的控制電壓，以供給壓電驅動器，使能得到最小的成本函數，此成本函數乃是由一串加速度感應器測量而得的加速度最小平方和或為理想之均佈加速度感應器所測得之最小平方和。本研究討論了主動壓電驅動器控制源以及被動控制元件阻塊在樑之振動控制效果，結果顯示樑之振動在混合主動與被動控制是可行的。本研究提供了智慧型材料結構系統的設計原則。

(關鍵詞：樑、振動、阻塊、壓電驅動器、最小平方前饋控制、混合主動和被動控制、智慧型材料結構系統)

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